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MODUS TOLLENS

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There is an established logical procedure according to which one is justified in proceeding as follows. Suppose that three people are discussing Smith; the question is put where he is this evening. Speaker A claims that it is certain that Smith is not drinking; A has no idea, however, as to where Smith is. Speaker B claims that it is likely that Smith is in the Faculty Club. Speaker C then claims that if Smith is in the Faculty Club, it is quite possible that he is drinking. I presume that it will be granted that no conflict is given rise to due to the three claims which have been made.

If speaker C is justified in making the third claim, then according to *modus tollens* he would also be justified in claiming that if there is no possibility of Smith's drinking, then Smith is not in the Club. We recognize in the example an instance of arguing that if one is justified in inferring that if p, q, one is also justified in inferring that if not-q, then not-p.

With respect to the example which I have given one may be moved to object to the application of the procedure. To understand what the objection is let us suppose that one is justified in claiming that it is certain that Smith is not drinking if, and only if, one is also justified in claiming that there is no possibility of Smith's drinking. Let us further suppose that the claim whose justification is in question is in fact made: if there is no possibility of Smith's drinking, then Smith is not in the Club. So if A is justified in claiming that it is certain that Smith is not drinking, one would also be justified in claiming that Smith is not in the Club.

In light of the inference one would wonder whether there is any conflict between B's claim that it is likely that Smith is in the Faculty Club and the last claim that Smith is not in the Club. I think that with respect to ordinary circumstances in which one would make such a claim one would say that there is a conflict. In claiming that he is not in the Club one would not lead one's hearer to think one meant that it is only likely or possible that he is not in the Club. One must rather mean to imply that it is certain that he is not in the Club. In that case, of course, there is a conflict between the two claims. But since there was not a conflict among the three original claims, there must be something wrong either in the interpretation

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which I have given of the final claim or in the application of the procedure of *modus tollens*. Since the interpretation is reasonable, it is reasonable to assume that it is the procedure itself which is not perfectly adaptable to coping with cases in which the expressions "it is certain," "it is likely" and "it is possible" assume positions usually reserved for "it is true". I propose to lay out a procedure of contraposition in order to accommodate such cases as I have now given an example of.[1]

Let us call the expressions "it is certain," "it is likely" and "it is possible" R-expressions ("R" having been drawn from the word "ranking"). Let "*Rc*" stand for "it is certain," "*Rl*" for "it is likely" and "*Rp*" for "it is possible". And let "*Rx*," "*Ry*" or "*Rz*" stand for an R-expression which does not contain a negative particle; I shall use the symbol "*N*" in place of the word "not" and so write, for example, "*RNp*" for "it is not possible".

A reason may be called for why I have associated the three expressions "certain," "likely" and "possible" rather than the three expressions "necessary," "true" and "possible". Suppose we begin to build up a list of expressions by taking "possible" to be given. I think that "true" is neither a competing nor an alternative expression because of the possibility of saying, "It is possible that it is true that he is not in the Club". Iteration of R-expressions, on the other hand, is at least controversial. Some of those who would not reject "possibly true" would reject "possibly likely".[2] The expression "necessary" is sometimes added to the list by definition. But I submit that claiming that it is not possible that he is in the Club carries us into saying that it is certain that he is not in the Club. Similarly, we pass from saying that it is not certain that he is there to saying that it is possible that he is not there. In order to give a table of opposition let "*S*" stand for a sentence which is not introduced with an R-expression. Sentence-negation will be signaled by writing "*SN*". The members of each of the following pairs are meant to be equivalent:

Rc S	*RNp SN*
Rl S	*RNl SN*
Rp S	*RNc SN*

In order to give some rules for coping with *modus tollens* let us suppose in what follows that the speakers make very elementary claims; they are restricted either to claiming, for example, that it is certain that Smith is drinking or to claiming, for example, that if he is in the Club, it is possible that he is not drinking. One is restricted, in short, either to claiming, "*Rx S*" (an option for negation, of course, left open) or to claiming, "If S_1, *Rx* S_2".

In giving the second possibility I left no room for introducing "S_1" with an R-expression. I argue that one is justified in so proceeding because of two premisses. The first is that we are justified in not allowing the iteration of R-expressions; in this paper I shall not argue for the premiss. The second is that I am concerned only with those cases of making claims in which, when it is claimed that if S_1, then *Rx* S_2, it is implied that it is possible that S_1. It accordingly follows that an R-expression may not introduce the if-clause; for otherwise it would be implied that it is possible that Rx that. . . . And that is not permitted.

Let us speak of assigning a rank to a claim. Just as a price is given to an article in order to make certain comparisons and calculations so we say that a rank is assigned to a claim in order to determine the R-expression of claims we argue we can make simply because of other claims already made. If we were to use only my original list of R-expressions, then it would do to make a list of three ranks:

3 points (it is certain)
2 points (it is likely)
1 point (it is possible)

Between any two ranks listed above it is obviously possible to list one new rank; doing such a thing can be carried on as long as one likes, well beyond the possibility of saying whether this is more or less likely than that. Let us suppose that on some list we list n ranks; leaving aside the word "point" we can say that n is the highest rank, that $\frac{1}{2}(n + 1)$ is the middle rank and that 1 is the lowest rank.[3] For assigning ranks the following guideline is given. To a claim of certainty (with or without an if-clause) the highest rank is to be given; to a claim of likelihood the middle rank is to be given, and to a claim of possibility the lowest rank is given. In assigning a rank I shall use the symbols "a," "b" and "c" in place of numerals.

The first rule I shall give let us call the *Teetertotter Rule* (*TR*). Since the first part of it is trivial, I shall set it forth now and simply take it for granted in what follows. Suppose that a claim is made; for purposes of stating the rule we may ignore if-clauses:

(1) *RNx S*

Suppose further that to the claim we assign the rank a. According to the Rule the speaker is in a position to make another claim:

(2) *Ry SN*

To that claim the same rank is assigned. This part of the rule covers the cases which I mentioned in laying out a square of opposition. To state the second part of the Rule let us consider two conflicting claims:

(1) *Rx S*
(2) *RNx S*

Let us suppose that to the first we assign the rank a. To find a we proceed from the top of the given list and count down by v; if a is n, v, of course, is 0. Let us in any case say that a is $n - v$. Let us further assign to (2) the rank b. According to the second part of the Rule b is $1 + v$. It follows that $a + b$ is $n - v + 1 + v$. So $a + b$ is $n + 1$. I shall make some use of this consequence of applying the rule.

The next rule I propose to give is a revised version of *modus ponens* (*MR*). Suppose that the speaker makes a number of claims:

(1) 1. $Rx\ S_1$

. . . . . .

m-1. $Ry\ S_{m-1}$

The speaker makes a further claim, turning the above *S*-sentences into if-clauses:

(2) if $(S_1 \& \ldots \& S_{m-1})$, $Rz\ S_m$

For example, the following two claims may now have been made:

(1) It is possible that Smith is at home.
(2) If he is at home, it is certain that he would be drinking.

To determine whether we are in a position to make a further claim, a claim based simply upon what has already been claimed, we assign ranks to the claims. With respect to the general case above we assign to 1. the rank a, . . ., to m-1. the rank b and to (2) the rank c. We then use the formula "$(a + \ldots + b + c) - (m - 1)n = d$" to determine what the value of d is, given that m is the number of claims and n the number of ranks on the chosen list. Suppose that $1 \leq d \leq n$. In that case there is a further claim to be made to which the rank d is to be assigned:

(3) $Rw\ S_m$, (1), (2) and *modus ponens*

With respect to the example which I began to give above we assign the rank 1 to the first claim and the rank n to the second one. By means of the formula we learn that d is 1; one is therefore entitled, according to the rule, to claim that it is possible that Smith is at home.

As revised the rule is applicable to standard cases of *modus ponens,* provided that the apodosis in if-then-sentences is introduced with an appropriate R-expression.

The third rule I propose to give is that of *contraposition* (*RC*). Suppose that someone claims that if Smith is at home, it is possible that he is drinking. However we are to practice contraposition, it will surely be thought reasonable to insist that our practice not allow us to proceed from a claim of a given rank to a claim of a higher rank. It may further be argued that the ranks of the two claims should be the same. We do conform to the demand of the argument by reasoning as follows. If someone is justified in claiming that if Smith is at home, it is possible that he is drinking, and if he is justified in rejecting the claim that it is possible that Smith is drinking, then he is also justified in rejecting the claim that it is certain that he is at home. I can accordingly state a rule of contraposition as follows: suppose that one is justified in claiming:

(1) if S_1, $Rx\ S_2$.

According to the rule one would be justified in making another claim:

(2) if SN_2, Rx SN_1, (1) and the rule of contraposition

With respect to the example we may argue that since the speaker is justified in claiming that if Smith is at home, it is possible that he is drinking, he would also be justified in claiming that if Smith is not drinking, it is possible that he is not at home.

With three rules stated let us again consider the problem which I mentioned at the beginning of my paper; it arose in considering two claims of the form:

(1) if S_1, Rx S_2
(2) RNx S_2

I was complaining that traditional practice would make it possible to infer that it is not true that S_1. Under an obvious interpretation of the inference, however, its rank would be too high. To correct the mistake I propose to show that what follows is only that it is possibly not true that S_1 or, put differently, that it is possible that SN_1. By contraposition we proceed from (1) to (3):

(3) if SN_2, Rx SN_1, (1) and RC

And by the Teetertotter Rule we proceed from (2) to (4):

(4) Ry SN_2, (2) and TR

Let us assign to the claims (1) and (3) the rank a and to the claims (2) and (4) the rank b. Since (3) and (4) fit a pattern covered by *modus ponens,* we use the formula "$a + b - n = c$" in order to determine the value of c. But we know already from the Teetertotter Rule that $a + b = n + 1$; c is therefore 1, and so we are entitled to conclude:

(5) Rp SN_1

If we are justified in using the rules as I have stated them, then traditional practice allows us to go too far in inferring the falsity of the antecedent; not, of course, that the practice is always misleading. There is a special case for which it works correctly:

(1) if S_1, Rc S_2
(2) Rc SN_2

It may be claimed, for example, that if Smith is at home, he is certainly drinking; it is further claimed to be certain that he is not drinking. In that case one can simply conclude the falsity of his being at home by saying, "Then he must not be at home." But the rank of the conclusion is a function of the ranks of the premisses and not simply of the existence of a conflict between saying that it is certain that he is at home and saying that it is certain that he is not at home.

In order to put the question more seriously whether my formulations of the rules are satisfactory I propose to develop the notion of a conflict somewhat. Let us coin the expression "an amount of conflict" and so make possible inquiring what the amount of conflict is between two claims.[4] In order to provide an answer let us also say that one point is an amount, that two points are an amount and that three points are an amount, given my original list of ranks. In order to say in general what the amount is we may say that 1 point is an amount, . . ., that $\frac{1}{2}(n + 1)$ is an amount, . . . and that n points are an amount. Suppose that two claims are made:

(1) $Rx\ S$
(2) $Ry\ SN$

To the first claim we assign the rank a and to the second one b; to say what the amount of conflict is between them, if any, use the formula "$a + b - n = c$," whereby c is the amount of conflict. It turns out, of course, that we have the greatest amount of conflict when speaker A claims that it is certain that Smith is at home and speaker B claims that it is certain that he is not at home. We have the least amount of conflict when speaker A claims that it is certain that Smith is at home and speaker B claims that it is possible that he is not at home. My earlier complaint against the practice of *modus tollens* can then be made by saying that the procedure is really only suited for cases in which every conflict is maximal.

In order to test the rules as stated I propose to use the following method and then to make a special demand, made in light of the notion of conflict now developed. Suppose that speakers A and B make two claims and that speaker A argues that one is in a position to make a further claim. He goes on to make the third claim, arguing that it follows from the other two. If a fourth claim is made which conflicts with the conclusion, we allow, of course, that it gives rise to a further conflict with the premisses. In light of this allowance I make the demand that there be a way of showing that there is also a conflict between a premiss and either the fourth claim made or a consequence of making it within the context of the various claims made.

It will not do simply to grant that a conflict with the conclusion gives rise to a conflict with the premisses; I submit that we are justified in making a more exacting demand upon the conduct of our arguments, namely that the amount of conflict remains the same.

For example, a case which offers smallest resources for applying *modus ponens* is one in which speaker A claims that it is likely that Smith is at home and speaker B claims that if he is at home, it is likely that he is drinking. In that case we are entitled to infer that it is possible that he is drinking. So suppose that the rule is thus applied:

(1) $Rl\ S_1$
(2) if S_1, $Rl\ S_2$
(3) $Rp\ S_2$, (1), (2) and *MP*

Speaker A contradicts (3) and makes a claim conflicting with (3) in the amount of 1:

(4) $RNp\ S_2$

(5) $Rc\ SN_2$, (4) and TR
(6) if SN_2, $Rl\ SN_1$, (2) and RC
(7) $Rl\ SN_1$, (5), (6) and MP

If speaker A is otherwise willing to apply the rules as I have identified them in (5), (6) and (7) above, he is committed to making claims (1) and (7) between which there is a conflict in the amount of 1. So although he loses the argument, his reasoning meets the demand made for maintenance of the amount of conflict.[5] It can, in fact, be shown in general that by reasoning thus with respect to applying *modus ponens* the amount of conflict remains constant. Suppose that two claims are made:

(1) $Rx\ S_1$
(2) If S_1, $Ry\ S_2$

Let us assign to (1) the rank a and to (2) the rank b. Let us suppose that $a + b - n = c$ and that $1 \leqslant c \leqslant n$. So we assign c to:

(3) $Rz\ S_2$

The objector creates a conflict by making a claim to which we assign d, whereby $c + d - n = e$ and $1 \leqslant e \leqslant n$:

(4) $Rw\ SN_2$
(5) if SN_2, $Ry\ SN_1$, (2) and RC
(6) $Rv\ SN_1$, (4), (5) and MP

We again allow that the rank f is to be assigned to (6) and that $d + b - n = f$ and that $1 \leqslant f \leqslant n$. What is to be shown is that $a + f - n = e$. That can be shown as follows:

$$\begin{array}{r}
a + b - n = c \\
\underline{-(d+b-n) \quad - f} \\
a - d = c - f \\
\underline{+(c+d-n) \quad + e} \\
a - n = e - f \\
a + f - n = e
\end{array}$$

It is similarly possible to show that an argument with respect to applying the rule of contraposition can be given in conformity with the demand to maintain the amount of conflict. Suppose speaker A makes a claim and applies the rule:

(1) if S_1, $Rx\ S_2$
(2) if SN_2, $Rx\ SN_1$, (1) and RC

Speaker B makes two claims in order to object:

(3) $Rc\ SN_2$
(4) $RNx\ SN_1$
(5) $Rx\ SN_1$, (2), (3) and MP

Between (4) and (5) there is a conflict in the minimal amount.

(6) $Ry\ S_1$, (4) and TR
(7) $Rz\ S_2$, (1), (6) and MP

We assign a to (1) and b to (4) and (6); since $a + b - n = c$ and $a + b = n + 1$, c is 1, the rank to be assigned (7); there is therefore the same amount of conflict between (3) and (7) as there is between (4) and (5).

It is not likely, however, that it is possible to meet the demand of maintaining the same amount of conflict when we follow the traditional practice of contraposition. For suppose the following claim is made:

(1) if S_1, $Rp\ S_2$

Ignoring worries about how to construct an if-clause we write:

(2) if $RNp\ S_2$, SN_1

The objector further claims:

(3) $RNp\ S_2$

By *modus ponens* he infers:

(4) SN_1

and contradicts (4) by claiming (5):

(5) S_1

It is not evident how to answer the question by how much (4) and (5) conflict. Since some philosophers have placed truth between necessity and possibility, as it were, it is tempting to say that the missing R-expression is "likely". The conflict between (4) and (5) can then be construed as minimal, but then the amount of conflict in the argument will not be preserved. And if the conflict between (4) and (5) is maximal, the amount of conflict will again not be maintained. To solve such difficulties it is sufficient to use the revised rule of contraposition.

NOTES

1. The motivation for doing so has two sources: the first is accounting for such difficulties as have been discussed; the second is to effect a closer relationship between the concept of possibility in logic and the concept of certainty in epistemology. In order to bring them more closely into alignment we may take as basic the concept of likelihood, presupposing the use of "likely" either in making a comparison or in saying that such and such is very likely. We then define "certain" and "possible" in the following way: certain is what is as likely as possible; possible is what is as slightly likely as possible. Since we distinguish the use of "possible" in "It is possible for us to allow that this is yet more likely" and the use of "possible" in "It is possible that he will come", and since it is in the former sense in which I say, "certain is what is as likely as possible", the use of the word to be explained is not presupposed in giving the explanation.

   One may have reservations about the project of bringing such concepts under the same rules; it may be argued that we have both a modal logic and a calculus of probabilities. They ought not be run together. It may be argued further that traditional formulations of *modus* *ponens* and *modus tollens* are not meant to cover such examples as the one with which I opened my paper.

Regarding the last point it may be observed that Aristotle in fact argues for one case of *modus ponens* which my version of the rule will cover as a special case, namely, if it is possible that p, that if, if p, q must be, then it is possible that q. So it is clear that *modus ponens* was not formulated with a view to excluding the use of such words as "must" and "may". And it may also be replied with respect to the first point that if there are formulations of rules which make the rules equally applicable to cases taken to be disparate, so much the better. It must be conceded that a certain adjustment is necessary in order to effect the unification, and that is that the notion of logical necessity be given up in favor of that of certainty. (That such an adjustment is called for is argued in my book *Aspects of Aristotle's Logic*, Assen, 1974.)

2. Iteration of R-expressions is rejected on general grounds and not with an eye on formulating certain rules. It would take us too far afield to give the grounds in this paper.

3. There are obviously many ways of constructing a suitable scale: we simply require that there be a mid-point (from which we can begin to make comparisons as to how likely something is) and that there be end-points (so that we can place limits now by saying, "It is certain" and now by saying, "It is barely possible").

4. Aristotle makes use of such a notion when he contrasts contraries and contradictories; for he does have occasion to mark a difference between the pair of claims " . . . must be so . . . " and " . . . must not be so . . . " and the pair of claims " . . . must be so . . . " and " . . . may not be so . . . ." In the former case we say that there is a conflict in the greatest amount possible and that in the latter the conflict is of the smallest amount possible. We gain further help with the concept when we consider the two cases of disagreeing: A says that such and such is certainly so and B says that such and such is certainly not so. In this case the conflict is maximal. In the second case B disagrees by arguing that such and such is not certain. The conflict would then be minimal. To whatever claim is made a skeptic would presumably reply by introducing conflict only of the minimal amount. Notice, finally, that to a claim of possibility another claim can be opposed but only in the minimal amount.

5. The procedure for showing whether the same amount of conflict is maintained was initially developed in order to settle a dispute as to whether anything follows from such premisses as the following ones:

(1) $Rp\ S_1$
(2) if S_1, $Rp\ S_2$

Suppose that speakers A and B make the first two claims; A further claims that a third claim follows:

(3) $Rp\ S_2$, (1) and (2)

We now apply the procedure outlined: speaker B contradicts (3) in the amount of 1:

(4) $RNp\ S_2$
(5) $Rc\ SN_2$, (4) and TR
(6) if SN_2, $Rp\ SN_1$, (2) and RC
(7) $Rp\ SN_1$, (5), (6) and MP

But between (1) and (7) there is no conflict; A is therefore wrong in claiming that (3) follows from (1) and (2), for it cannot be shown that in contradicting (3) but affirming the premisses one maintains a conflict with respect to the premisses.

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