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CIRCULAR DEMONSTRATION AND VON WRIGHT-GEACH ENTAILMENT

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Students of the literature on entailment [7], [1] are familiar with the von Wright-Geach definition of entailment [6], [2]: p entails q if, and only if, by means of logic,[1] it is possible to come to know the truth of $\ulcorner p \supset q \urcorner$ without coming to know the falsehood of p or the truth of q.

A competitor of the Lewis account of entailment as strict implication, the von Wright-Geach definition has enjoyed little success. Perhaps one reason is that the definition is essentially epistemic, and entailment is not widely thought to be an epistemic notion.[2] The notion of entailment aside, however, the von Wright-Geach notion does intriguingly seem to be applicable to one interesting epistemic concept of obvious logical interest. In this article,* we suggest that the von Wright-Geach *definiens*, in effect, partially defines the concept of a *non-circular demonstration,* and we argue that the von Wright-Geach *definiens* can be extended in a natural way to yield a full definition for non-circular demonstration. Thus we think of the latter notion in a frankly epistemic way. We have argued in [8] and [9] that circularity of argument (*petitio principii*) is best thought of as an epistemic matter, and for those who agree with us on this point it may not seem too surprising that there is a connection between circularity and von Wright-Geach "entailment". For those who disagree with our thesis that *petitio* is essentially epistemic, we hope that establishing the connection in question may serve to diminish the disagreement. In either case, we think that this new application of the von Wright-Geach framework is interesting in its own right.

In [9] we argued that the history of the subject, which has largely followed in the tradition set by Aristotle in various remarks in the *Topics*, *De Sophisticis Elenchis*, and the *Rhetoric*,[3] suggests the wisdom of

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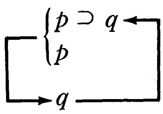
recognizing two broad types of circularity-conditions, neither of which however is as theoretically well-behaved as we might like. According to the *equivalence conception* an argument is circular when the conclusion is equivalent to (or even, in some versions, identical to) some premiss.[4] According to the *dependency conception*, an argument is circular where some premiss depends on the conclusion, i.e., where one cannot know that the premiss is true without knowing that the conclusion is true. Both types of conditions have been stated in both an epistemic and also a purely alethic idiom, but we cite the epistemic variant of the dependency-type condition to fit the framework of von Wright [6]. An epistemic dependency condition (C) in the style of von Wright would read as follows: an argument, 'p therefore q' is non-circular if, and only if, it is possible to come to know the truth of p without coming to know the truth of q. One may know the truth of q, as a matter of fact, but one must have some *other* means of knowing the truth of p—that is, some means independent of q.

How does (C) fit the von Wright-Geach definition of entailment? We can find the answer by recognizing that (C) needs to be modified because there is another kind of circularity that can occur where classical connectives are used.[5] The material conditional $\ulcorner p \supset q \urcorner$ has the property of being true where the consequent, q, is true. But if one were to propose an argument of the form *modus ponens* on the basis that the conditional premiss $\ulcorner p \supset q \urcorner$ obtains because the conclusion q obtains, one would have committed a blatant dependency-*petitio*. We could diagram this state of affairs as follows:



Not only are we obliged to pass from the premisses to the conclusion as intended, but we are likewise obliged to pass from the conclusion to a premiss, thus closing the "circle". Now the von Wright-Geach definition requires that it must be possible to come to know the truth of $\ulcorner p \supset q \urcorner$ without coming to know the truth of q, thus preventing this form of circularity from arising. Thus (C) and the von Wright definition seem to complement each other. Can they be put together to define something like perhaps 'non-circular entailment'?

To accomplish this, two major differences of orientation need to be smoothed out. First, the von Wright-Geach definition, unlike (C), is restricted to what we come to know "by means of logic". Second, (C) is concerned with how we come to know the truth of p (the premisses), whereas the von Wright-Geach definition merely requires the possibility that we come to know the truth of $\ulcorner p \supset q \urcorner$ without coming to know the falsehood of p. Plainly if we are to have a concept of non-circular argument that avoids both kinds of dependency-circularity mentioned above, we will have to take into account the question of whether it is possible to know the truth of p without coming to know the truth of q. So we will smooth out the second difference by broadening the von Wright-Geach definition to take into account the possibility of coming to know the truth of p. On the first difference however, we will narrow the notion of argument to arguments that are concerned with "the means of logic" alone, i.e., we will consider only demonstration (proof), that species of argument where p and $\ulcorner p \supset q \urcorner$ are possible to come to be known true by means of logic,[6] where the premiss-set and conclusion consist exclusively of theorems.[7] Thus we propose a definition of non-circular demonstration. We do not, however, wish to reject the possibility that the sort of definition we offer may be extended to arguments where premisses cannot come to be known to be true by means of logic.

Now it is interesting to observe that if we put (C) and the von Wright-Geach definition together as they stand, an inconsistency is yielded. The conjunctive definition, suitably modified as suggested in the previous paragraph, would read: for theorems ϕ and ψ, $\ulcorner \phi \supset \psi \urcorner$ is a *non-circular demonstration for* ψ if and only if, by means of logic, it is possible to come to know the truth of ϕ and $\ulcorner \phi \supset \psi \urcorner$ without coming to know the falsehood of ϕ or the truth of ψ. But, given what von Wright adopts as axioms (as shown below), if it is possible to know that p is true then p is true. Likewise, if it is possible to know that p is false, then p is false. Thus the phrase "the falsehood of ϕ or" is not only redundant in the definition, but is actually the occasion of its inconsistency. Accordingly, the fully modified definition may be given as follows: ϕ, $\ulcorner \phi \supset \psi \urcorner$ is a *non-circular demonstration for* ψ if, and only if, by means of logic, it is possible to come to know the truth of ϕ and $\ulcorner \phi \supset \psi \urcorner$ without coming to know the truth of ψ. We think this "definition" gives a quite satisfactory account of dependency-circularity in the context of demonstration, and that its closeness to and interdependence with the von Wright-Geach definition of entailment is what gives the latter its plausibility. Yet whether our "definition" really provides a helpful or adequate analysis of circularity of demonstration depends ultimately on what sort of account we can give of the three undefined terms that occur in it: "demonstration", "it is possible to come to know the truth of ϕ", and "without". von Wright has some interesting suggestions to make in this regard, and we will comment on these briefly.

According to von Wright, the following two equivalences are true: 1. p is *demonstrable* (*provable*) if, and only if, it is possible to come to know the truth of p by means of logic, and 2. p is *demonstrated* if, and only if, it is possible to demonstrate p. Thus he thinks it important to distinguish between a proposition's being demonstrable and its being demonstrated. The latter entails the former, but the converse does not obtain.

The term "possible" in the second equivalence is meant by von Wright in the rather unusual way that if it is possible to come to know that p is true, then p *is* true. In a perhaps more usual sense of "possibility", if p is false we would say that it is possible that it might have become known to be true simply because (if it is a contingent proposition) it is possible that it might have been true. But in von Wright's (unusual) sense of "possible", if p is false it is *impossible* that it should become known to be true. This new sense of "possible" is the key to the analysis we propose of a non-circular demonstration. We would conjecture, although we are not yet very confident about it, that "possible" here is not used in the usual sense[8] but in the context of coming to know the truth of a *theorem* (a non-contingent proposition). On our account of the matter, von Wright ([6], p. 186) proposes what amount to the following axioms for $\mathbf{D}p$ (p is demonstrated) and $\mathbf{MD}p$ (p is demonstrable).

(A1) $\mathbf{D}\phi \supset \phi$
(A2) $\mathbf{MD}\phi \supset \phi$
(A3) $\mathbf{D}(\phi \wedge \psi) \equiv (\mathbf{D}\phi \wedge \mathbf{D}\psi)$
(A4) $\mathbf{MD}(\phi \wedge \psi) \equiv (\mathbf{MD}\phi \wedge \mathbf{MD}\psi)$
(A5) $\mathbf{D}\phi \supset \mathbf{MD}\phi$
(A6) $\mathbf{D}\phi \supset \mathbf{DMD}\phi$ (demonstrated $\supset$ demonstratedly demonstrable).
(A7) $\mathbf{D}\phi \supset \mathbf{MDMD}\phi$ (demonstrated $\supset$ demonstrably demonstrable).

Finally, concerning "without", the third undefined term, von Wright proposes the following equivalence: ϕ is demonstrable independently of ψ if, and only if, it is possible that ϕ is demonstrated and ψ is undemonstrated. Thus given $\mathbf{D}$ and $\mathbf{M}$ (for possibility) understood after the fashion of von Wright, we can express our definition of '$\phi, \ulcorner\phi \supset \psi\urcorner$ *is a non-circular demonstration for* ψ' (i.e., $\phi, \phi \supset \psi \ominus\!\!\dashv \psi$) as follows:

$$\phi, \phi \supset \psi \ominus\!\!\dashv \psi =_{df} \mathbf{M}(\mathbf{D}\phi \wedge \mathbf{D}(\phi \supset \psi) \wedge \neg\mathbf{D}\psi).$$

No essentially epistemic concept is purely alethic (i.e., truth-theoretic or proof-theoretic); therefore an epistemic analysis of entailment is misdirected. We think that entailment is, so to speak, a "purely formal" matter. But we do not think that the notion of a non-circular demonstration is a purely formal matter,[9] but rather better viewed epistemically or information-theoretically.

However, we do not wish to become enmeshed in fruitless debate over what is or is not "purely formal". What we hope to have shown is that non-circular demonstration is at least not a purely subjective matter, and is indeed open to analysis.

NOTES

1. In Geach [2], p. 165, the phrase "by means of logic" is changed to "by *a priori* methods".

2. Intuitionists may be the exception. Kripke's semantics for intuitionistic logic [5] may fairly admit of the adjective 'epistemic'. Whether Kripke might identify entailment with the intuitionistic calculus is of course another matter, but the point is that some intuitionists may be ready to make this identification.

3. For more on this, see also Hamblin [3].

4. DeMorgan in his *Formal Logic* defended a variant of this conception. See Douglas Walton, "Mill and DeMorgan on whether the syllogism is a *petitio*" (summary), *Historia Mathematica*, vol. 2 (1975), pp. 336-337.

5. See also the discussion of disjunctive syllogism in Woods and Walton [9].

6. Theorems in some system or other. We concede with von Wright that the phrase "means of logic" is vague, see [6], p. 181. But a vague definition is not a bad one if its vagueness matches the vagueness of its *definiendum*. As von Wright notes, [6], p. 183, the definition allows for a pluralistic interpretation of this phrase as "by *some* means of logic".

7. This way of proceeding fits in very nicely with von Wright's remarks in [6], p. 185: "Truth which has become established 'by means of logic', is, moreover (logically) *necessary* truth."

8. By which we, and we think von Wright, mean that concept (or concepts) of possibility, namely logical possibility, that T, S4, S5, and the other standard systems of modal logic are often said to represent. It has been conjectured by Lemmon (see Hughes and Cresswell [4], p. 80) that S5 is the most adequate system to explicate this sense of possibility.

9. Rather it is a matter of "applied logic", or perhaps "informal logic".

REFERENCES

[1] Bennett, J., "Entailment," *Philosophical Review*, vol. 78 (1969), pp. 197-236.

[2] Geach, P. T., "Entailment," *Proceedings of the Aristotelian Society*, supplementary vol. 32 (1958), pp. 157-172.

[3] Hamblin, C. L., *Fallacies*, Methuen, London, 1970.

[4] Hughes, G. E., and M. J. Cresswell, *An Introduction to Modal Logic*, Methuen, London, 1968.

[5] Kripke, S., "Semantical analysis of intuitionistic logic I," *Formal Systems and Recursive Functions*, ed., J. N. Crossley and M. A. E. Dummett, North-Holland, Amsterdam, 1965, pp. 92-130.

[6] von Wright, G. H., *Logical Studies*, Routledge and Kegan Paul, London, 1957.

[7] Woods, J., "On arguing about entailment," *Dialogue*, vol. III (1965), pp. 405-421.

[8] Woods, J., and D. Walton, "*Petitio* and relevant many-premissed arguments," *Logique et Analyse*, to appear.

[9] Woods, J., and D. Walton, "*Petitio principii*," *Synthese*, vol. 31 (1975), pp. 107-127.

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