Notre Dame Journal of Formal Logic
Volume 21, Number 4, October 1980

First-Degree Entailments and Information

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Entailment is sometimes informally explained by saying that A entails B just in case B is contained in A. Pressed to explain this notion of containment, it seems plausible to begin by saying that B is contained in A just in case the information conveyed by B is included in that conveyed by A. This paper presents two interpretations of the first-degree (FD) entailments of propositional logic that are based directly on the notion of inclusion of information.* It is proved in Section 2 that one of these interpretations exactly characterizes the tautological entailments of [2], while the other exactly characterizes the valid arguments of classical truth-functional logic. In Section 3, following a line of reasoning suggested in part by consideration of these interpretations, it is argued that the claim that relevance logic better captures our intuitions about entailment than classical logic is false. Section 1 presents natural-deduction formulations of both classical and relevant FD entailments that are used in subsequent proofs.

1 Two systems of natural deduction rules I take $-$, $\vee$, and & as primitive connectives and assume that sentential letters are specified. Wffs are as usual. I let $A, B, \ldots, F$ (with or without numerical subscripts) range over wffs and let M and N range over finite nonempty sets of wffs.

Given any finite nonempty set of wffs M, an infinite set of wffs X_M is defined recursively as follows:

1. If $A \in M$, then $A \in X_M$.
2. $A \,\&\, B \in X_M$ iff $A \in X_M$ and $B \in X_M$.
3. If $A \in X_M$ or $B \in X_M$, then $A \vee B \in X_M$.

*I wish to thank Carl Brandt, Norman Dahl, Alice de la Cova, LaVerne Shelton, Terrance McConnell, Michael Root, F. Christopher Swoyer, and John Wallace for helpful suggestions and criticisms.

Received July 16, 1979

4. If $A \in X_M$ and if B results from A by replacing a well-formed part of A of the form given on the left (right) of one of the lines a-j below with the wff given on the right (left) of the same line, then $B \in X_M$.

a.	C	$--C$
b.	$C \& D$	$D \& C$
c.	$C \vee D$	$D \vee C$
d.	$(C \& D) \& E$	$C \& (D \& E)$
e.	$(C \vee D) \vee E$	$C \vee (D \vee E)$
f.	$-(C \vee D)$	$-C \& -D$
g.	$-(C \& D)$	$-C \vee -D$
h.	$C \& (D \vee E)$	$(C \& D) \vee (C \& E)$
i.	$C \vee (D \& E)$	$(C \vee D) \& (C \vee E)$
j.	C	$C \vee C.$

Since each of the rules in the above definition is truth preserving under the usual interpretation of the connectives, we can think of them as natural deduction rules. Thus given a wff F and a set of wffs M I shall say that F is *E-derivable* from M iff $F \in X_M$. (The significance of 'E' in this definition will become clear presently.) In view of rule 2 and the fact that M is always a finite nonempty set, it will be sufficient to restrict our attention to cases in which M has just one member. And in these cases we can dispense with the notion of M as a set and speak merely of one sentence being E-derivable from another. In this way E-derivability can be construed as a relation that holds between the consequent and antecedent of some FD entailments. I next specify a normal form for FD entailments and prove a theorem which leads to a decision procedure for E-derivability. A second theorem establishes the equivalence of E-derivability and tautological entailment in the sense of [2].

A FD entailment is in *normal form* iff both its antecedent and its consequent are in conjunctive normal form (CNF). Clearly any wff can be put into CNF by means of replacements based on 4a-i. (These are just the equivalences used in [2] to obtain normal forms.) If we specify that only replacements based on 4a-i may be used to obtain normal forms of a FD entailment, it follows that the consequent of a FD entailment is E-derivable from its antecedent iff the same is true of its normal forms.

By a *primitive disjunction* I mean a disjunction whose disjuncts are all *atoms* (i.e., sentential letters or the denials of sentential letters). A CNF is just a conjunction of primitive disjunctions. For two primitive disjunctions A and B, I say that *B is contained in A* iff every atom of A is an atom of B.

Theorem 1 *Consider a FD entailment in normal form, $A_1 \& \ldots \& A_n \rightarrow B_1 \& \ldots \& B_m$. $B_1 \& \ldots \& B_m$ is E-derivable from $A_1 \& \ldots \& A_n$ iff each B_i is contained in some A_j.*

Proof: The 'if' part is trivial. (But notice the importance of rule 4j here.) The 'only if' part is somewhat more complicated but can be handled by two-valued matrices. The matrices for & and $\vee$ are the usual ones. There is no general matrix for $-$, but $--A$ is treated in the usual way, and $-(A \& B)$ and $-(A \vee B)$ are treated like $-A \vee -B$ and $-A \& -B$, respectively.[1] If there is some B_i that is not contained in any A_j, there will be an assignment of values to atoms that gives A_1 & . . . & A_n the designated value and B_1 & . . . & B_m the undesignated value. But since rules 1-4(a-j) can be shown to preserve the designated value, it follows that B_1 & . . . & B_m is not E-derivable from A_1 & . . . & A_n.

Since containment of one primitive disjunction in another is an effective notion, Theorem 1 provides a decision procedure for E-derivability.

Theorem 2 *The consequent of a FD entailment is E-derivable from its antecedent iff the entailment is tautological in the sense of [2].*

Proof: Consider any FD entailment in normal form A_1 & . . . & $A_n \rightarrow B_1$ & . . . & B_m. By the dual of the lemma of [2] (p. 21), it follows that this entailment is tautological iff for each B_i there is some A_j such that $A_j \rightarrow B_i$ is tautological. But since both A_j and B_i are primitive disjunctions this holds iff B_i is contained in A_j. The desired result now follows by Theorem 1.

The motivation for the term 'E-derivability' is now clear. Hereafter I shall say that a FD entailment is an *E-entailment* iff it is tautological in the sense of [2].

Suppose that rule 4 is now expanded by adding two new pairs of interreplaceable sentence forms.[2] They are:

k.	C	$C \vee ((D \,\&\, -D) \,\&\, E)$, where E may be null
l.	C	$C \,\&\, ((D \vee -D) \vee E)$, where E may be null.

Given a wff F and a set of wffs M, I say that F is *C-derivable* from M iff $F \in X_M$, where X_M is generated by the expanded set of rules. C-derivability can also be construed as a relation holding between consequent and antecedent of some FD entailments. And I say that a FD entailment is a *C-entailment* (i.e., it is classically tautological) iff there is no assignment of truth-values to its sentential letters that makes its antecedent true and its consequent false, given the usual truth-tables for &, $\vee$, and $-$.

Theorem 3 *The consequent of a FD entailment is C-derivable from its antecedent iff the entailment is a C-entailment.*

Proof: The 'only if' part is obvious in view of the truth-preserving nature of the expanded set of rules. The 'if' part follows easily from the relation between the decision procedures for E- and C-entailment given in [2], the relation between E- and C-derivability, and Theorem 2.

We now have two closely related systems of natural deduction rules which capture, respectively, the E- and the C-entailments. In the next section, I present intensional interpretations of E- and C-entailment which reflect, in a certain way, these natural deduction systems.

2 ***The semantics of nonampliativity*** The notion of a valid argument is often informally characterized in two distinct ways. One way is to say that an argument is valid just in case it is impossible for all its premises to be true and its conclusion false. The basic idea here is that the valid arguments are just those in which true premises can never lead us to a false conclusion. I shall call this way of characterizing validity the *truth-preservation account.* The other characterization of validity has to do with the information contained in or conveyed by the sentences that make up the argument. I shall use the term 'content of a sentence' as a convenient abbreviation of the loose notion of the information that a sentence contains or conveys. With this notion of content in mind, it is often said that an argument is valid just in case the content of the conclusion does not exceed the combined contents of the premises. I shall call this way of characterizing validity the *content-nonexpansion account*.

I think most logicians believe that these two accounts, although different, really pick out the same class of arguments. They believe, that is, that any plausible formalization based on either account will sanction exactly the same arguments. Evidence that this view is widely held among logicians can be found in the books they write. For while logic texts often begin by giving both informal accounts of validity, they usually develop their formal semantics solely along the lines suggested by the truth-preservation account. In so doing they implicitly suggest that any adequate formalization of validity construed as content-nonexpansion would lead to the same class of valid arguments. Indeed the case can be made more strongly. Bar-Hillel and Carnap [6] have shown how to define the content of a sentence in such a way that it becomes demonstrable (for a suitably restricted language) that the content-nonexpanding arguments are just the truth-preserving arguments. And most logicians, I believe, would say that this result vindicates their intuitive feeling that truth-preservation and content-nonexpansion ultimately come down to the same thing.

The proponents of relevance logics,[3] however, are not most logicians. They hold that truth-preservation is a necessary but not a sufficient condition for validity, and they cite the so-called paradoxes of strict implication as examples of arguments that are necessarily truth preserving but not valid. They say that in order for an argument to be valid it must preserve truth *and* there must be an appropriate relevance between its premises and its conclusion. So relevance logicians reject the truth-preservation account as by itself insufficient for validity. But do they accept the content-nonexpansion account? In this section I shall present an intensional interpretation of E-entailment that can plausibly be taken to formalize the intuitive notion of content-nonexpansion. So there is a sense in which relevance logicians can be thought of as accepting the content-nonexpansion account of validity. But I shall also present a similar interpretation of C-entailment.[4] Thus it will turn out that whether or not relevance logicians really are to be thought of as accepting the content-nonexpansion account of validity depends on how the intuitive notion of content is explicated. Indeed given these two interpretations it becomes possible to view the dispute[5] between classical and relevance logicians as a dispute over the preferred explication of the notion of the content of a sentence.

Before presenting either of these interpretations I want to indicate how the basic idea behind the notion of the content of a sentence on which both interpretations are based differs from the usual account of content. On the usual account (e.g., [6]) the content of a sentence A is identified with the states of affairs ruled out by A or with the set of sentences of a certain kind (e.g., state-descriptions) incompatible with A. This might be called the negative approach to content since it identifies the content of A with what A denies. The approach given here is positive since it bases the content of A on what A

asserts. The idea is to identify the content of A (or, as I prefer to say, the information conveyed by A) with the set of propositions expressed by all the sentences entailed by A. Given such a definition it is of course easy to prove that A entails B only if the information conveyed by B is included in that conveyed by A. But more interesting than this is the fact that by using the natural deduction rules of the previous section it is possible to prove that the notion of content so defined satisfies some very simple and natural conditions (see Theorem 6 below).

Since the interpretations I wish to develop take propositions as basic and undefined, I need some postulates about propositions and the ways in which they are related to wffs. The postulates are:

A. There is an infinite set of propositions, $\mathcal{P}$, and there are functions P from the set of wffs W into $\mathcal{P}$.

B. For any wffs A, B, C, D, and any function P from W into $\mathcal{P}$, $P \in H_e(P \in H_c)$ iff each of the following conditions holds:

1. If A and B E-entail (C-entail) each other, then $P(A) = P(B)$
2. If $P(A) = P(C)$ and $P(B) = P(D)$, then $P(A \,\&\, B) = P(C \,\&\, D)$ and $P(A \vee B) = P(C \vee D)$
3. If $P(C) = P(A \,\&\, B)$, then there are wffs E and F such that $P(E) = P(A)$, $P(F) = P(B)$, and C E-entails (C-entails) $E \,\&\, F$.

Some comments on B are in order. The members of H_e and H_c are to be thought of as those assignments of propositions to wffs that are acceptable from the points of view of relevance and classical logic, respectively. Thus B1 says that under any assignment acceptable from one of those points of view, the wffs that entail each other according to that point of view must express the same proposition. Turning to B2, it may be thought that its consequent should also say that $P(-A) = P(-C)$. This is indeed plausible but is not included because it is not needed for anything proved in this paper. The idea behind B3 is that if a wff expresses a conjunctive proposition then it must either be a conjunction itself or at least entail some wff that is a conjunction. The power for expressing conjunctions inherent in the language ought not to be thrown away, according to B3. Thus, for example, B3 rules out functions that assign to some sentential letter the same proposition that is assigned to the conjunction of two other sentential letters. Again it may be thought that if B3 is plausible, then similar principles should be adopted for disjunction and negation. No such principles are needed, however, for any of the proofs given below.[6]

By a *message* I mean any finite nonempty set of wffs. Given a message M and a function $P \in H_e$, $I_e^P(M)$ (read 'the information E-conveyed by M under P') is defined as the set of propositions satisfying the following conditions:

1. Only propositions associated with some wff by P are members of $I_e^P(M)$
2. For any wff A, $P(A) \in I_e^P(M)$ (read 'the proposition expressed by A under P is part of the information E-conveyed by M under P') iff there is a wff B such that $P(B) = P(A)$ and M E-entails B.[7]

Given a message M and a function $P \in H_c$, $I_c^P(M)$ (read 'the information C-conveyed by M under P') is defined analogously using C-entailment in place of

E-entailment.[7] The functions I_e^p and I_c^p are the central notions of the interpretations being presented here. Some theorems concerning them will now be stated and proved.

Theorem 4 *For every wff A and message M:* (a) *M E-entails A iff, for every* $P \in H_e$, $P(A) \in I_e^p(M)$; (b) *M C-entails A iff, for every* $P \in H_c$, $P(A) \in I_c^p(M)$.

Proof: I consider only (a), since the proof of (b) is strictly analogous. The 'only if' part is trivial in view of the definition of I_e^p. For the 'if' part consider a function P' from W into $\mathcal{P}$ such that for any wffs B and C, $P'(B) = P'(C)$ iff B and C E-entail each other. It is easy to show that any such $P' \in H_e$. Hence if $P(A) \in I_e^p(M)$, for every $P \in H_e$, it follows that $P'(A) \in I_e^{p'}(M)$. But from this and the transitivity of E-entailment if follows that M E-entails A.

Before presenting the next theorem, two new terms are defined. Given a message M and a wff A, the argument from M to A (i.e., the argument whose premises are the members of M and whose conclusion is A) is said to be: (a) *E-nonampliative* iff, for every $P \in H_e$, $I_e^p(\{A\}) \subseteq I_e^p(M)$; (b) *C-nonampliative* iff, for every $P \in H_c$, $I_c^p(\{A\}) \subseteq I_c^p(M)$. These two definitions are presented as competing formal explications of the intuitive notion of content-nonexpansion discussed earlier.

Theorem 5 *For every message M and wff A, the argument from M to A is:* (a) *E-nonampliative iff M E-entails A;* (b) *C-nonampliative iff M C-entails A.*

Proof: Again the proofs of (a) and (b) are strictly analogous. The 'only if' parts are straightforward given the definitions of E- and C-nonampliativity and Theorem 4. The 'if' parts are trivial in view of the transitivity of E- and C-entailment.

Theorem 5 shows that I_e^p satisfies, from the point of view of relevance logic, an obvious necessary condition of adequacy of any proposed explication of the information conveyed by a message. It also shows that I_c^p satisfies a similar condition from the point of view of classical logic. Of course neither of these results is surprising—indeed they are both quite trivial—in view of the definitions of I_e^p and I_c^p. What is interesting (and perhaps somewhat surprising) is that I_e^p and I_c^p also satisfy the simple and natural conditions embodied in Theorem 6. These conditions seem so natural that I suggest that provability of a strict analogue of Theorem 6 is a necessary condition of adequacy of any account of information which: (1) construes the information conveyed by a message as a set of propositions; and (2) applies to a language containing only conjunction, disjunction, and negation as primitive connectives.[8] Further discussion of Theorem 6 follows its statement and proof.

Theorem 6 *For every wff A, message M, and function P from W into* $\mathcal{P}$:

(a) *if* $P \in H_e$, *then* $P(A) \in I_e^p(M)$ *iff either:*

1. *There is a wff B such that* $P(A) = P(B)$ *and* $B \in M$; *or*
2. *There is a wff B such that* $P(A \,\&\, B) \in I_e^p(M)$ *or* $P(B \,\&\, A) \in I_e^p(M)$; *or*
3. *There are wffs C and D such that* $P(A) = P(C \,\&\, D)$ *and* $P(C) \in I_e^p(M)$ *and* $P(D) \in I_e^p(M)$; *or*

4. *There are wffs C and D such that* $P(A) = P(C \vee D)$ *and* $P(C) \in I_e^p(M)$ *or* $P(D) \in I_e^p(M)$;

(b) *if* $P \in H_c$, *then a strictly analogous result holds with* I_c^p *replacing* I_e^p.

Proof: Again I consider only (a), since the proof of (b) is strictly analogous. The 'if' part is straightforward using postulates B2 and B3. The 'only if' part is facilitated by the following

Lemma *For every wff A, message M, and function* $P \in H_e$, *if M E-entails A, then either 1 or 2 or 3 or 4 as in Theorem 6.*

To prove the lemma assume that M E-entails A. By Theorem 2 it follows[9] that A is E-derivable from M. It can now be proved straightforwardly by induction on the length of the E-derivation of A from M that, for any $P \in H_e$, one of the four clauses of Theorem 6 must hold. This proof makes use of postulates B1 and B2 in those cases involving the replacement rules 4a-j. Given the lemma, the 'only if' part of Theorem 6 follows easily, using postulate B2.

As a statement of necessary and sufficient conditions for membership in $I_e^p(M)$ and $I_c^p(M)$, Theorem 6 constitutes an adequacy result for these notions as much because of what it doesn't say as because of what it does say. Specifically, it contains no clause to the effect that if a message conveys the proposition expressed by a disjunction it must convey the proposition expressed by one or the other of the disjuncts. And surely the absence of such a clause is something we want in this case.[10] Similarly, there is no clause at all for negation. This too is desirable, since we want to be able to say that a message may convey both members or neither member of a pair of contradictory propositions.

Because of Theorems 5 and 6 we are now able, if we wish, to view both E- and C-entailment as at least minimally adequate, although conflicting, formalizations of the intuitive notion of content-nonexpansion. In the next section I shall consider what light is shed on the dispute over E- and C-entailment by viewing them in this way.

3 ***The role of logical intuition*** Relevance logicians have often suggested and sometimes claimed explicitly that relevance logic better captures our intuitions about entailment (i.e., about the validity of arguments) than classical logic.[11] In the present section I argue that this view is false. The reasons I shall advance in favor of this conclusion are based solely on consideration of FD entailments (i.e., in effect, truth-functional arguments). Thus it may be objected that my conclusion is suspect because I have not taken account of our intuitions about higher-degree entailments. I have three reasons for not doing so. First, I believe that our intuitions about higher degree entailments are much less well-developed than those about FD entailments. So there is less to take into consideration. Second, it seems to me that the basic issues are all present at the level of FD entailments and that one's views about entailment in general (including higher-degree entailments) will be largely determined by his views about FD entailment.[12] Finally, the controversy over whether 'entails' is better treated as a connective or a relation complicates and casts doubt upon higher degree examples.[13]

I shall begin by using the interpretations of the previous section to try to help decide whether E- or C-entailment better captures our intuitions about validity when the latter is construed as content-nonexpansion. Interpreting E-entailment as E-nonampliativity, the relevance logician can be expected to argue somewhat as follows. "It is clear from Theorem 4(a) that if neither $P(A)$ nor $P(-A)$ is conveyed by a message, then $P(A \vee -A)$ need not be conveyed, and so there may be a tautological proposition that the message does not convey. On the other hand if both $P(A)$ and $P(-A)$ are conveyed by a message so is $P(A \ \& \ -A)$, but again Theorem 4(a) shows that it does not follow from this that the message conveys all propositions expressible under P. Put more simply these results are that if one construes the information conveyed by a message in terms of I_e^P, then a message may fail to convey tautological information, and it may convey contradictory information without thereby conveying all information. These results are intuitively plausible. And since they are directly reflected in the fact that neither all arguments with tautological conclusions nor all with contradictory premises are E-nonampliative, we have strong evidence that E-nonampliativity accurately reflects our intuitions about information and content-nonexpansion."

The classical logician, interpreting C-entailment as C-nonampliativity, will respond by first making a concession and then offering a counterargument. His response will be something like the following. "It is true that C-entailment does not yield the allegedly intuitive results mentioned by the relevance logician. For by Theorem 4(b) it follows that if the information conveyed by a message is construed in terms of I_c^P, then, for any $P \in H_c$, every message conveys all the tautological information expressible under P, and any message that conveys contradictory information conveys all the information expressible under P. But the price relevance logic pays for its avoidance of these so-called paradoxes is the rejection of disjunctive syllogism, which is intuitively content-nonexpanding. Surely it is bizarre to hold that a message may include A and $-A \vee B$ but fail to convey the information that B. Imagine someone who, under oath, admits to having received a message, but denies that it conveyed the information that B. If it were later revealed that the message in question was $\{A, -A \vee B\}$, perjury charges would be in order. The disjunctive syllogism is C-nonampliative, and this is strong evidence that C-nonampliativity accurately reflects our intuitions about information and content-nonexpansion. Rejecting disjunctive syllogism does greater violence to these intuitions than accepting the so-called paradoxes."

The relevance logician will of course reply that accepting the paradoxes does greater violence to our intuitions about content-nonexpansion than rejecting disjunctive syllogism. He may also argue that the plausibility of disjunctive syllogism is only apparent due to a confusion between truth-functional and intensional 'or'.[14]

The preceding paragraphs contain familiar arguments wearing some new clothes provided by the nonampliativity interpretations. Thus arrayed each becomes more compelling and the problem of choosing between them correspondingly more difficult. In fact when they are presented in this way I think it is impossible to choose between them on any rational basis. For I find the relevance logician's argument about the two senses of 'or' unconvincing,[15] and so the dispute reduces to whether the presence of the paradoxes or the absence of disjunctive syllogism does greater violence to our intuitions about content-nonexpansion.[16] My intuitions are simply incapable of deciding this issue, and I am convinced by extensive (although unscientific) sampling among colleagues and students that this is also the case with the intuitions of most logicians and philosophers, as well as those of "naive, untutored folk"[17]. But if our intuitions about content-nonexpansion fail to decide between the paradoxes and disjunctive syllogism, then they fail to decide between E- and C-entailment. For what other examples or arguments could be used to show that these intuitions significantly favor one system over the other? I know of none, and I can only leave it as a challenge to those who think there are some to produce them.[18]

My conclusion so far is that if logical validity is construed as content-nonexpansion, and if the goal of formal logic is to capture our intuitions about validity, then relevance and classical logic are equally far from the goal. Of course we might be able to show that one of these approaches is closer to the goal by considering some other way of construing logical validity. But what other intuitive notion of validity is there? The only one I know of is truth-preservation, and it is clear that the truth-preserving arguments are just the classically valid arguments. So it begins to look as though the relevance logician is unable to support his position by any appeals to our intuitions about validity. For appeals to content-nonexpansion are inconclusive, and appeals to truth-preservation lead to classical logic.

At this point the relevance logician can be expected to object that our intuitive notion of validity involves relevance, and that my discussion has been defective in not paying sufficient attention to it. I have not done so because, as far as I can see, the way in which relevance figures in our intuitive notion of validity is extremely unclear. Truth-preservation and content-nonexpansion are the only reasonably well-specifiable intuitive notions of validity that I know of. To say merely that our intuitive notion of validity involves relevance, or perhaps that it involves relevance along with truth-preservation or content-nonexpansion is simply too vague to allow anyone to make a fair judgement about whether classical or relevance logic better captures this notion. And so far as I know, no relevance logician has been able to give a pre-formal specification of the intuitive notion that he claims to capture with his formalism that is any clearer than this.

I do not want to be misunderstood at this point. I am not denying that relevance logicians have produced formal systems containing devices which purport to capture an intuitive sense of relevance. The subscripting device of the natural deduction systems of [1], [3], and [5] is an example. But it is not at all clear that these devices really capture our intuitions about relevance rather than merely functioning to assure that the resulting stock of theorems conforms to a prior formulation of relevance logic. For example, the subscripting restrictions on the conjunction introduction rule of [1] and [5] allow the inference of A & B from A and B only if A and B both depend on exactly the same premises. But it is arguable that a more intuitively plausible rule would allow this inference regardless of the premises on which A and B depend, with the stipulation that A & B would then depend on the union of the premises of A and B. Such a rule leads back in the direction of classical logic, but that is just the point. Our intuitions about relevance, such as they are, do not seem to point unequivocally in the direction of either relevance or classical logic.

The main conclusion I wish to draw from the foregoing arguments is that relevance logic does not better capture our intuitions about validity than classical logic. And it is tempting to draw the additional conclusion that classical logic is superior to relevance logic since it clearly captures one of our intuitive notions of validity (i.e., truth-preservation), while relevance logic does not clearly capture any such notion. There is, however, a different additional conclusion that I am more inclined to draw. It is that the central concept involved here–namely that of "our intuitions about validity"–is so vague and its empirical basis so unclear that it is a mistake to think of the primary goal of logic as the production of formal systems that "capture our intuitions about validity".[19] Of course the logician must take his first few steps guided by his intuitions. But intuition quickly deserts him as the rigor of a formal approach forces him to decide questions that "naive, untutored folk" never even ask. Adopting this view defuses somewhat the dispute of the present section and makes it possible to use less competitive standards in evaluating systems of formal logic. Thus we can judge a logical system not just on how it reflects our intuitions, but at least equally importantly on how it measures up to certain theoretical and practical standards. In the former category we might consider such things as whether or not proofs of consistency, an interesting kind of soundness and completeness, and a deduction theorem are available. In the latter category we could ask questions about how and where the system might fruitfully be applied.[20] When matters are viewed in this way it may turn out that relevance logic has its place and its uses. But this place and these uses will have been established by specific results and arguments, not by sweeping claims about our logical intuitions.

NOTES

1. These matrices are essentially what one obtains from the semantics of Dunn [8] by limiting the universe of discourse to a single topic.
2. Only one of these pairs is needed. The effect of each is obtainable from the other in the presence of 4*a*, *f*, and *g*. This was pointed out to me by James Hawthorne.
3. By 'relevance logic' I mean any system whose propositional FD fragment coincides with the E-entailments of the previous section. Thus some systems (e.g., that of W. T. Parry) that might otherwise be called relevance logics will not be considered here.
4. That these interpretations are plausible formalizations of the intuitive notion of content-nonexpansion is due ultimately to that fact that E- and C-entailment can be construed, respectively, as E- and C-derivability. See the use of Theorems 2 and 3 in the proof of Theorem 6 below.

5. I realize that there are at least two disputes between relevance and classical logicians: one over the validity of arguments (or the entailment relation among sentences), and another over the proper analysis of conditionals. In this paper I restrict my attention to the former dispute as it applies to arguments involving only truth-functional sentences.

6. Richard Routley has suggested that if a possible-worlds semantics is adopted and propositions are defined within it as sets of possible worlds, then these postulates may all be dispensible. Indeed B1-2 seem to be satisfied by such a semantics, and A is clearly satisfied if the set of possible worlds is infinite. But B3 is not satisfied by all models in a possible-worlds semantics, and B3 is required for the proof of Theorem 6 below.

7. The terms 'E-entailment' and 'C-entailment' introduced in the previous section are here being extended in an obvious way to apply to truth-functional arguments with several premises.

8. Hence I consider both Theorems 5 and 6 as adequacy results for I_e^p and I_c^p. The reader should compare this situation with [6], p. 149, where Bar-Hillel and Carnap mention only a single adequacy result (an analogue of Theorem 5) for their *Cont*(i). Of course *Cont*(i) is not defined as a set of propositions, but it is curious that Bar-Hillel and Carnap say so little about its adequacy as an explication of "the information conveyed by the statement i".

9. The use of Theorem 2 here (and of Theorem 3 at the corresponding place in the proof of (b)) is crucial. Notice that clauses 1-4 of Theorem 6 reflect rules 1-3 of E- and C-derivability.

10. It is the presence of just such a rule (R3) that prevents the propositional fragment of the sequent calculus LEQ_1 of [4] from being interpreted straightforwardly in terms of the information conveyed by messages. In the absence of R3 each sequent could be thought of as partially specifying the information conveyed by a message, the wffs on the left indicating propositions that are included in the message and those on the right indicating propositions that are excluded. Reading the remaining rules upward then provides a plausible analysis of the message specified by a sequent.

11. See, for example, [3], especially pp. 19-21, 28-30, 33-34, and 50-51, and [9], especially pp. 808-812. Similar passages occur in [5].

12. This point, and to a lesser extent the first, are supported in [12], pp. 335-336:

> Most of the traditional and current disputes come up and can be settled at the first degree level, for example, disputes about the paradoxes and their effects, and as to the adequacy of principles such as Disjunctive Syllogism. Thus choice of a first degree system already fixes in large measure one's position on entailment. Furthermore, for a number of important notions the higher degree structure is either peripheral, as with inclusion of logical content, or, as with causal sufficiency, not well-defined.

13. See [11], pp. 166-168.

14. As in [2], pp. 19-22.

15. It is successfully criticized, I believe, in [7], p. 206.

16. An alternative way of viewing the dispute derives from the following considerations: (a) the definitions of E- and C-nonampliativity differ only in the terms 'E-entails' and 'C-entails'; (b) by Theorems 2 and 3 E- and C-entailment differ only on the question of whether wffs that differ from each other at most in ways allowed by rules 4k-l entail each other; (c) by postulate B1 and Theorem 4, two wffs A and B E-entail (C-entail)

each other iff for all $P \in H_e$ (for all $P \in H_c$), $P(A) = P(B)$. Hence we can view the dispute as reducing to the question of whether wffs that differ from each other at most in ways allowed by rules 4k-l always express the same proposition. Someone who believes, as Quine does, that the main difficulty with propositions is that conditions for their identity can't be specified will find it quite appropriate that the dispute over E- and C-nonampliativity becomes refractory precisely at the point where questions of propositional identity arise.

17. This phrase is borrowed from [3], pp. 20-21.

18. Notice that although much of the dispute presented above was couched in terms of E- and C-nonampliativity, this is not essential to the conclusion being drawn here. For clearly the same points could be made given any plausible way of construing both E- and C-entailment as formalizations of the intuitive notion of content-nonexpansion. Indeed they can be made even in the absence of such formalizations: simply try to decide whether the E-entailments or the C-entailments more nearly coincide with the arguments we intuitively judge to be content-nonexpanding. Thus the arguments of this section, although related to and inspired by the semantics of nonampliativity, are independent of it.

19. Although I think this is a mistake that relevance logicians have often made in the past, there are a few signs things may be changing. For example in [10], Meyer suggests that our intuitions about a key notion (what he calls 'fusion') will develop only as technical work in relevance logic proceeds. This is sensible and a far cry from some of the strong claims that have been made on behalf of relevance logic (cf. the references cited in Note 11).

20. For example, Meyer suggests in [9], pp. 814-815, that relevance logic can be used to resolve difficulties in epistemic and deontic logic. This suggestion deserves serious consideration.

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