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Linear Network Coding and Parallel Transmission Increase Fault Tolerance and Optical Reach

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Abstract—As optical networks evolve towards more dynamicity and an ever more efficient and elastic spectrum utilization, a more integrated, fault tolerant and system efficient design is becoming critical. To increase efficiency of spectral resource in bit rate per Hz (bit/s/Hz), high-level modulation formats are used, challenged by the accompanying optical impairments and the resulting limitation of optical reach. Previous work has addressed the issue of optical reach and transmission fault tolerance in the physical layer by deploying various FEC schemes and by a careful design of optical transceivers and links. This paper uses a different approach, applicable to link and networking layers. We propose a novel theoretical framework, whereby a randomized linear network coding (LNC) is applied to the main optical path, and in parallel, an auxiliary optical path is used at much lower transmission speeds, i.e., in addition to the main path. With the reception of the auxiliary path, as we analytically show, the system is highly tolerant to bit errors and packet loss caused by optical impairments in the main path, whereby alleviating the constraints on optical transmission quality and indirectly achieving better optical reach and spectral efficiency. The results are shown for a case study of high-speed Ethernet end-system transmitted over optical OFDM networks, which due to the inherent system-level parallelism in both networks, present one of the most interesting candidate technologies for the proposed method to yield best performance.

I. INTRODUCTION

Today, the coarse granularity of DWDM channels, typically in “fixed grid” spectral distances of 50GHz or 100GHz, is showing its fundamental capacity limits. To accommodate a massive Internet traffic growth at an annual rate of more than 30%, new research and development have started towards highly flexible and scalable network technologies beyond DWDM, most notably in the area of *elastic optical networks*. Elastic optical networks have emerged in two distinct flavors: *flexi-grid* single carrier networks with flexible optical spectrum allocation, and multi-carrier *optical Orthogonal Frequency Division Multiplexing (OFDM) networks*. In an optical OFDM network, for instance, each sub-carrier can be modulated with a specific modulation format depending on multiple factors, such as the desired spectral efficiency, the channel condition (physical impairments) and the designed transmission distance (optical reach). To accommodate high-speed Ethernet systems at 100/400Gb/s [1] [2], today’s systems use high-level modulation formats, such as 64-QAM, to increase bit rate per Hz (bit/s/Hz, also referred to as spectrum efficiency).

However, high-level modulation formats are known to be less tolerant to optical impairments, such as chromatic dispersion and polarization mode dispersion, thus requiring higher signal-to-noise ratio (SNR) [3]. In the presence of transmission impairments that cause bit errors, signals may not be correctly detected at the receiver. This eventually leads to packet errors in the data link and the network layers, thus degrading the

overall system performance. To address this issue, current approaches focus on the development of novel transceivers based on coherent detection techniques. In addition, Forward Error Correction (FEC) codes have been commonly applied in physical layer to ensure the BER performance end-to-end. As standard FEC codes continue to evolve, there is comparably less insight into the effects of transmission bit errors on the packet loss in the higher layers, especially in dynamically routed networks, where the proposed physical layer schemes under varying networking conditions cannot always guarantee the correct reception of packets. With the emergence of high-speed Ethernet systems at 100/400Gb/s [1], [2], the design of end-systems in terms of robustness and tolerance to transmission impairments are becoming critical.

In this paper, we propose a novel approach to achieve a fault-tolerant data transmission in optical networks, applicable to link and networking layers. In our framework, randomized linear network coding (LNC) is applied to the main optical channel in the client network layers, and in parallel, an auxiliary optical channel is setup at much lower transmission speeds, i.e., in addition to the main channel. The main optical path, as we analytically show, can be more fault tolerant to physical impairments, and thus use high-level modulation formats with linear network coding, and that independently of, or rather complementary to, the physical layer mechanisms to mitigate bit errors. In our proposed system, we define fault tolerance as the capability of the network system losing packets (for any reasons) and still correctly receive the original information. We propose a case study of today’s high-speed Ethernet systems over optical OFDM networks, a combination which due to the inherent system-level parallelism in both networks, present one of the most interesting candidate technologies for the proposed method to yield best performance. The results show that for a data block as large as the maximum Ethernet frame size (1500 bytes), the 100GE transmission can tolerate up to 10^{-3} bit error rate with 64-QAM.

The rest of the paper is organized as follows. Section II provides a literature review and emphasizes our contributions. Section III presents a reference system architecture. Section IV presents the linear network coding model and describes the design factors. Section V presents the analytical results and Section VI concludes the paper.

II. RELATED WORK

In optical OFDM networks, the high-level modulation formats can be used to improve spectral efficiency. To this end, past work focused on the development of novel transceivers based on coherent detection techniques. [4] proposed to use asynchronous detection strategy and asynchronous filter adjustment algorithms, commonly seen in wireless communica-

tions, here applied to optical OFDM systems to increase the tolerance against the bit errors caused by noise and other fiber effects. [5] analytically evaluated BER performance of OFDM system that are intensity modulated in presence of baseband distortion at the transmitter and noise at the receiver. It showed that maintaining the system performance at a specific BER level is a tradeoff between the transmitting power and high bit resolution at the receiver.

The emerging high-speed Ethernet standards, 100 Gbps [1], and more recently, 400 Gbps [2], are fueling the already rapid development of elastic optical networks. Specifically, the new high-speed Ethernet standards, such as IEEE 802.3ba [1], defining Multiple Lane Distribution (MLD) systems with parallel interfaces, is fundamentally suitable to interface with optical OFDM networks and systems, which by the virtual of parallel multiple carriers are also parallel in nature. To this end, Routing and Spectrum Assignment (RSA) algorithms are required to find and set up a connection for the Ethernet packet flow. Whereas single path RSA can utilize numerous Routing and Wavelength Assignment (RWA) algorithms from the vast body of already existing literature, less attention has been paid to the RSA algorithms in optical parallel transmission systems suitable for the Ethernet standards ([6], [7], [8]).

Forward Error Correction (FEC) codes are commonly applied in physical layer to ensure the BER performance. In [9], an FEC codes based on block turbo code was proposed for long-haul transmission in a DWDM network. Recently, LDPC codes have been proposed which have been shown to outperform the turbo product codes in terms on improving BER performance [10]. In our approach, we neither use nor modify any FEC codes applied in the physical layer, and assume that the best possible data rate and optical reach has already been achieved in the physical layer, which we further attempt to improve by the parallel transmission of an auxiliary channel and network coding. We consider linear network coding, which means that the codes we use are *composable without the need for decoding* and can operate at *arbitrary code rates*. Concerning the latter aspect, linear network codes do not have the restrictions, common to structured codes, be they algebraic codes such as Reed-Solomon codes, or modern codes such as Turbo codes or low-density parity-check codes (LDPCs) of being limited to certain given coding rates, thus significantly restricting the design parameters. RLNCs are maximum-distance separable (MDS), as Reed-Solomon codes, and, while our analysis in this paper considers a block-style operation, they may be used in a rateless fashion.

Previous work on linear network coding focused in general on improving network throughput and reliability [11]. In optical networks, linear network coding was mainly applied to multicast connections. Manley et al [12] provided a comprehensive study on all-optical network coding. Work by Kamal et al [13] comprehensively surveyed network coding for protection in the optical layer.

The novel contribution of this paper is in a new application scenario of linear network coding where a parallel transmission scheme is designed to improve effectively the fault tolerance of optical networks against bit errors caused by physical impairments. The case study is unique and considers high-speed Ethernet over optical OFDM networks. For fairness, OFDM networks with 12.5 GHz spacing can be considered rather similar to wavelength spacing in WDM networks in our theoretical framework. However, WDM networks generally are

designed based on the worst case scenarios, i.e., they do not adapt modulation format according to the path properties, and usually deploy uniform transceivers with equidistant carriers. Here, we show that the true potential of the proposed method is in its capability to transmit two channels in parallel, one with high bit rate and the other much lower bit rate. Such concept is not possible in an operational WDM network and is best suitable in optical OFDM network.

The major difference between this work and related work is that it explicitly considers the use of RLNC over OFDM networks for the purpose of resilience to packet losses, and analyses the buffer size issues as a consequence. While our analysis is, to obtain a first assessment of the feasibility of this technology, in the setting of coding only at the ingress of the network, the approach we consider is, unlike that of traditional block or rateless codes that are, per their structure, only end-to-end, not limited to being implemented thus, but lends itself to a distributed implementation.

III. SYSTEM ARCHITECTURE

Fig. 1 illustrates the basic idea on an example of Ethernet data blocks network coded and transmitted over optical OFDM networks. According to IEEE 802.3ba, the serial stream of Ethernet frames is encoded into 66b blocks, which we also refer to as *data blocks*, or simply *packets*. Each data block is marked with an order identifier and is distributed to parallel Ethernet lanes in a round robin fashion. The resulting incoming K packets or data blocks shown here (e.g., from Ethernet virtual lanes) are encoded into N packets and transmitted over the selected optical paths. The basic idea here is to transmit the encoded packets from the source over two parallel optical paths simultaneously, hence the term *parallel transmission*. As an example, one can assume that the optical paths are based on OFDM optical transmission or a DWDM transmission with mixed line rates and all-optical switching in the network. Two spectrum paths are used, i.e., a main path (p_{main}) at the required and high transmission speed, and an auxiliary path (p_{aux}) at a comparably much lower speed. At the receiver, any K out of N packets are sufficient for correct decoding. The main optical path, once network coded, is now able to use a high-level modulation format over longer optical reach, since there is an auxiliary path available to guarantee the tolerance to bit errors caused by optical impairments due to the increased optical reach. We note that the fault tolerance provided in this fashion is independent, and comes in addition to any BER mitigation mechanisms from the physical layer, such as FEC. It is a mechanism applicable in the higher layers, to be used when lower layer mechanisms either fail or yield insufficient performance. At the same time, it is not a mechanism to be used in case of fiber cut: it merely protects about any type of packet loss due to signal quality degradation.

In this system architecture, the goal is to increase the spectral efficiency of the main path by applying high-level modulation formats to the path denoted as p_{main} . At the same time, we assume that p_{aux} is at a much lower transmission speed than the main path, i.e., p_{main} , thus can use a low-speed and consequently a low-cost transceiver. In general, our goal is to show that the total spectrum consumption of both p_{main} and p_{aux} is not larger than if a high-level modulation format were not used. In the results section, we show that despite allocating an additional path, the overall spectrum consumption is lower than if lower level modulation formats

packets to the next link based on either predefined next hop information shared by the routing protocol, or with one simple operation, that is a vector multiplication coding process at an intermediate node v at time $t + 1$, i.e.,:

$$\forall e_j : \text{tail}(e_j) = v : Y^{t+1}(e_j) = \sum_{e_i: \text{head}(e_i)=v} f_{e_i, e_j} \cdot Y^t(e_i) \quad (2)$$

Destination node: At the receiver, decoding is performed by Gaussian elimination on the received packets [16]. Any K out of N packets received from p_{main} and p_{aux} , are sufficient for decoding. The system can loose up to $N - K$ packets per generation (i.e., the redundancy of the system) and still be able to decode correctly. In other words, the system can tolerate $N - K$ erroneous packets per generation without any impact on the correct reception of the packets at the receiver. Decoded information at time t on the output lane i is modeled as follows:

$$Z^t(d, i) = \sum_{e_j: \text{head}(e_j)=d} \sum_{u=0}^{\delta_{t+1}} b_{i, e_j} \cdot Y^{t-u}(e_j) \quad (3)$$

, where a_{i, e_j} , f_{e_i, e_j} and b_{i, e_j} are randomly chosen from the finite field F_{2^q} and collected in the matrices \mathcal{A} , \mathcal{F} and \mathcal{B} , respectively. A triple $(\mathcal{A}, \mathcal{F}, \mathcal{B})$, referred to as a *linear network code* [17], specifies the parallel transmission process between a pair of Ethernet switches over an optical core network.

A. Buffer Requirement

The delay differences between the chosen main spectrum path p_{main} and the auxiliary path p_{aux} makes the issue of buffering quite challenging in high speed systems. If we define the path delay and capacity of the main and auxiliary paths as pd_{main} , c_{main} and pd_{aux} , c_{aux} , respectively, the difference between the path delay as $\delta_t = |pd_{main} - pd_{aux}|$. In this case, the capacity of the buffer required at the receiver for compensation of the differential delay (Bu) is:

$$Bu = \begin{cases} c_{main} \cdot \delta_t & \text{if } pd_{main} \leq pd_{aux} \\ c_{aux} \cdot \delta_t & \text{if } pd_{main} \geq pd_{aux} \end{cases} \quad (4)$$

Again, the receiver requires at least one generation to start decoding. The decoding interval δ_d thus contributes to the total size of required buffer [15]. Two cases are of interest.

1) When the decoding interval is not shorter than the differential delay, i.e., $\delta_d \geq \delta_t$, the total buffer size (B) is:

$$B = \begin{cases} c_{main} \cdot \delta_d & \text{if } pd_{main} \leq pd_{aux} \\ c_{aux} \cdot \delta_d & \text{if } pd_{main} \geq pd_{aux} \end{cases} \quad (5)$$

2) When the decoding interval is shorter than the differential delay, i.e., $\delta_d < \delta_t$, the total buffer size (B) is:

$$B = \begin{cases} c_{main} \cdot \delta_t + (N - K) \cdot Mq & \text{if } pd_{main} \leq pd_{aux} \\ c_{aux} \cdot \delta_t + K \cdot Mq & \text{if } pd_{main} \geq pd_{aux} \end{cases} \quad (6)$$

Buffering is the price to pay at the receiver in the proposed system. Let us take an example of $c_{aux} = 10\text{Gb/s}$ in case of $pd_{main} \geq pd_{aux}$ and assume decoding interval is shorter than differential delay ($\delta_d < \delta_t$). Per Eq.(6), the increase of δ_t leads to a larger difference between the arrival time of packets from the same generation on parallel links. More packets need to be stored at the receiver before the decoding can start. Impact of buffer requirements can be minimized by system design. In the next section, we show that a trade-off exist between the best performance and the resulting buffer size.

B. System Design

We define that if any bit in a packet can not be correctly detected, the packet is considered as an error packet and is dropped; in the network layer, this is usually referred to as *packet loss*. Hence, the BER can be directly translated into packet error rate (PER) and indirectly interpreted as a measure of packet loss. The relation between PER and BER is defined as follows:

$$PER = 1 - (1 - BER)^{Mq} \quad (7)$$

where Mq is the packet size.

Proposition 1. *For the effectiveness of the proposed system, the redundant packets from a generation transmitted on p_{aux} should not arrive after the packets from the same generation on p_{main} to avoid a prohibitively large buffer at the receiver.*

Proof: As previously mentioned, linear network coding requires a complete generation for a successful decoding. Therefore, packets from the same generation need to be buffered at the receiver until one complete generation is successfully decoded and released. When parallel paths are used, the differences in the transmission delay between these two paths makes the issue of queue size and buffering rather challenging. All packets from one generation need to be buffered until the entire generation is received for a correct decoding. However, buffering packets at a very high-speed line rate, for instance, 100 Gb/s Ethernet traffic or higher, would not be practical, especially considering the time required for decoding. Hence, it is more effective to buffer packets from the path with lower capacity, which is the reason why lower speed channel yields best performance. As shown in Eq. (5) and Eq. 6, smaller size queue is required when $pd_{main} \geq pd_{aux}$, given the condition that $c_{main} \geq c_{aux}$, as per our proposed scheme. ■

To measure the system performance in terms of fault tolerance, we define a parameter *Affordable Packet Loss (APL)*. APL is defined as the number of packets that can be lost, and/or contain bit errors during the transmission, without affecting the system performance in terms of correct packet reception. When packet loss (error) is smaller than APL, the system can correctly receive all packets and the transmission is fault tolerant. The value of APL is calculated as the function of BER, see Eq. 7. For instance, if the maximum acceptable BER of the system is 10^{-6} and packet size $Mq = 1500$ byte (i.e., the maximum size of an Ethernet frame), the maximum acceptable $PER = 0.0119$. At that PER value the system is free of any errors in reception, and be guaranteed for a 100% correct reception. In other words, if a receiver can tolerate some faulty packets, the affordable PER can be even higher.

If we denote the difference between the propagation delay of two spectrum paths as δt , $\delta t = |pd_{main} - pd_{aux}|$, the following theorem applies.

Theorem 1. *Parallel transmission system with linear network coding and a parallel auxiliary path can tolerate packet loss ratio upper bounded by $APL \leq \frac{c_{aux}}{c_{main}} - \frac{c_{aux}\delta t}{KMq} \leq 1$ if $pd_{main} \leq pd_{aux}$; otherwise, $APL \leq \frac{c_{aux}}{c_{main}} + \frac{c_{aux}\delta t}{KMq} \leq 1$, if $pd_{main} \geq pd_{aux}$.*

Proof: Assume at time t_0 , N_{main} ($N_{main} \geq K$) and N_{aux} encoded packets from one generation are sent to p_{main} and p_{aux} , respectively, where $N = N_{main} + N_{aux}$. N_{main} packets on p_{main} would arrive at the receiver at $t_{main} =$

$\frac{N_{main}Mq}{c_{main}} + pd_{main}$, while N_{aux} packets on p_{aux} would arrive at the receiver at $t_{aux} = \frac{N_{aux}Mq}{c_{aux}} + pd_{aux}$, where Mq is the packet size. As discussed earlier, it is required that $t_{aux} \leq t_{main}$, i.e., the redundant packets from a generation transmitted on p_{aux} should not arrive after the packets from the same generation on p_{main} . Since the minimum value of N_{main} is K , we can hereby derive the upper bound of $APL = \frac{N_{aux}}{N_{main}} \leq \frac{c_{aux}}{c_{main}} - \frac{c_{aux}\delta_t}{KMq}$. The latter value becomes $\frac{c_{aux}}{c_{main}} + \frac{c_{aux}\delta_t}{KMq}$ when $pd_{main} \geq pd_{aux}$. In both scenarios, maximum affordable packet loss on p_{main} is 100%, i.e., APL is upper bounded by 1. ■

V. NUMERICAL RESULTS

In this section, we analyze the performance of the proposed system. We assume that $c_{main} = 100$ Gb/s and packet size is set to $Mq = 12,000$ bits, which corresponds to the maximal Ethernet frame size of 1,500 bytes. In all results, we assume that the decoding interval is smaller than the differential delay, i.e., $\delta_d < \delta_t$. As shown in Fig. 2, when $pd_{main} \geq pd_{aux}$, an auxiliary path with higher capacity, i.e., higher transmission rate, makes it possible to include more redundant packets per generation, ultimately leading to a higher APL. When two spectrum paths are characterized by the same end-to-end delay, i.e., $\delta_t = 0$, an auxiliary path selected for a transmission rate of 10 Gb/s can guarantee that all packets are correctly received when packet loss ratio is not larger than 10%. Per Eq. (7), the system can now tolerate bit error rate up to 8.78×10^{-6} . Further increase of the capacity of auxiliary path can increase the fault tolerance of the system.

The same trend can be observed in case of $pd_{main} \leq pd_{aux}$, as shown in Fig. 3. When the auxiliary path has half capacity of the main path, i.e., $c_{aux} = \frac{1}{2} \cdot c_{main} = 50$ Gb/s, the system can tolerate 50% packet loss/error on the main path per generation. However, this ratio does not need to be maintained in practice. According to Eq. (7), with $APL = 50\%$, the system can tolerate BER up to 5.776×10^{-5} , for the case of packet size of 1500 bytes. In an optical transmission system, this is a relatively high BER value, i.e., not very common. If we assume more common values of BER as it appears in optical networks, the auxiliary path can run at a much lower transmission rate, thus consuming less spectral resource overall. In both cases of $pd_{main} \geq pd_{aux}$ as well as $pd_{main} \leq pd_{aux}$, it is obvious that an auxiliary path with larger capacity can guarantee a higher level of fault tolerance.

In parallel transmission systems, the differential delay is one of the key factors, which has an impact on the system performance. As shown in Fig. 2, a larger δ_t leads to a larger value of APL, regardless of c_{aux} , when $pd_{main} \geq pd_{aux}$. It is especially phenomenal when the number of redundant packets per generation is high, i.e., C_{aux} is large. For instance, with $K = 100$, and $c_{aux} = 10$ Gb/s, the system can tolerate 10 % packet loss when $\delta_t = 0$. In the same setting, the system can tolerate 19.17% packet loss on main path with $\delta_t = 11\mu s$. Per Theorem 1, APL is constrained by 1, indicating that the performance reaches the optimum at a certain point. An example can be seen in the analysis shown in Fig. 2, where the affordable packet loss can not be improved anymore when $\delta_t = 12\mu s$ with $c_{aux} = 50$ Gb/s and generation size of 100.

When $pd_{main} \leq pd_{aux}$, the value of APL decreases with the increased value of δ_t . This is determined by design, as per requirement defined in Proposition I. To ensure that the redundant packets routed on the auxiliary path arrive before

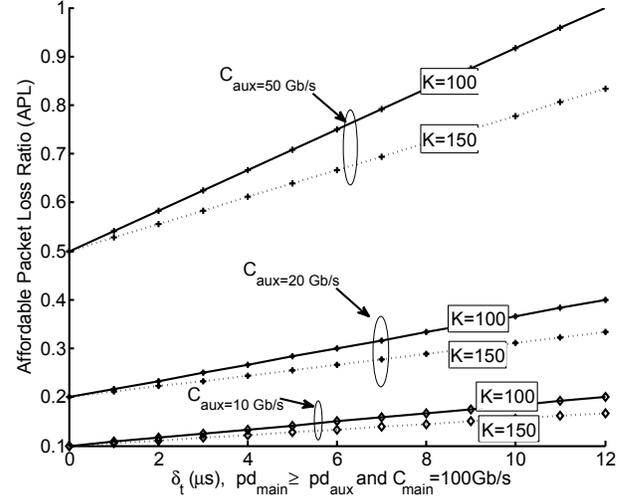


Fig. 2: Affordable packet loss vs. generation size (K) and C_{aux} , packetsize = 1500 bytes, $pd_{main} \geq pd_{aux}$

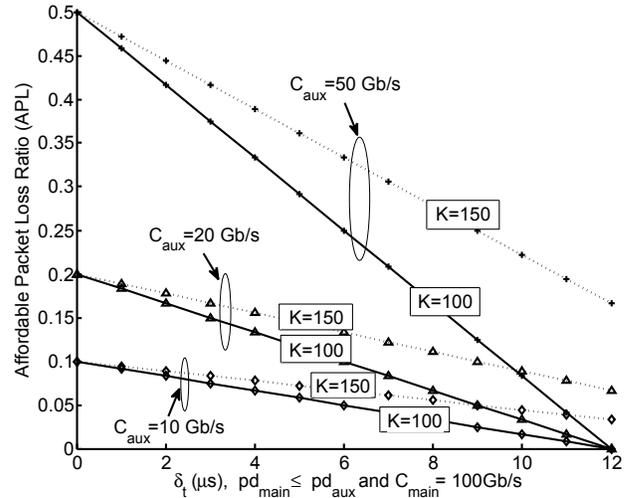


Fig. 3: Affordable packet loss vs. generation size (K) and C_{aux} , packetsize = 1500 bytes, $pd_{main} \leq pd_{aux}$

the main path's packets of the same generation, the number of redundant packets has to be very small, especially when c_{aux} is small. As shown in Fig. 3, with $K = 100$ and $c_{aux} = 10$ Gb/s, the system can tolerate 10 % packet loss when $\delta_t = 0$. In the same setting, the system can only tolerate 2.5% with $\delta_t = 9\mu s$. Finally, the system cannot tolerate any packet loss despite of parallel transmission when $\delta_t = 12\mu s$.

It should be noted here that this example shows that attention needs to be paid to two important parameters, namely c_{aux} or δ_t , when an optical OFDM transmission system is designed with $pd_{main} \geq pd_{aux}$. This is because the improvement of system performance in terms of APL directly depends on the methods to increase either of the parameter of c_{aux} or δ_t . An increase of c_{aux} indicates that attention needs to be paid to the overall spectral resource consumption, as an increased c_{aux} typically consumes more spectral resource. However, as we will show later, the total consumption of spectral resource is still less compared to the case where the system does not utilize high-level modulation formats, which would be due to the constraint of maximum affordable BER. Second, an increase of δ_t can be achieved by either finding a pd_{main} with

| Acceptable BER | Required APL | p_{main} with OSNR=25db | p_{aux} with OSNR=15db |
|----------------|--------------|---------------------------|--------------------------|
| 10^{-5} | 0.113 | 32-QAM, 2 sub-carriers | BPSK, 1 sub-carrier |
| 10^{-4} | 0.6988 | 64-QAM, 2 sub-carriers | QPSK, 1 sub-carrier |
| 10^{-3} | 0.999993 | 64-QAM, 2 sub-carriers | 16-QAM, 1 sub-carrier |

TABLE I: Modulation formats vs. BER; $c_{main} = 100$ Gb/s, $Mq = 1500$ bytes; 12.5 GHz/ sub-carrier; $pd_{main} \geq pd_{aux}$

larger delay, or a pd_{aux} with smaller delay. Put simply, when the shortest path is used as the auxiliary path, a path longer than the shortest path has to be set up as the main path, thus increasing the total consumption of network resource. Hence, a careful consideration is required to find a trade-off between fault tolerance and network resource consumption when setting up the proposed fault tolerant transmission system in practice.

Generation size has impact on system performance in terms of fault tolerance, and it also has direct impact on the buffer size required for decoding. As shown in Fig. 2, the value of APL decreases when generation size is larger when $pd_{main} \geq pd_{aux}$. For instance, with $c_{aux} = 20$ Gb/s, APL is 40% when $\delta_t = 12\mu s$, which is 6.67% higher comparing with $K = 150$ in the same setting. When $pd_{main} \leq pd_{aux}$, larger generation size can lead to a better performance in terms of APL. This is due to the fact it requires more time to send packets to the main path with a larger K , which essentially reduces the difference between arrival time of packets from two paths at the receiver. Per Theorem 1, a larger K is preferred when $pd_{main} \leq pd_{aux}$.

Finally, we analyze the spectral efficiency of the proposed transmission scheme. Table 1 shows the relation between the acceptable BER and the corresponding highest modulation format can be used on p_{main} with given OSNR budget according to the data measured in [5]. Here, we assume that size of a sub-carrier is 12.5GHz. The spectral efficiency of the p_{main} can be improved by using the high-level modulation format, while the redundancy carried on the auxiliary path can guarantee the correct reception at the end. For instance, when the acceptable BER= 10^{-5} in the network, the affordable packet loss ratio is 11.3% per Eq. (7). Hence, p_{main} can use 32-QAM modulation format by allocating 25 GHz (2 sub-carriers). In networks without network coding and parallel transmission, where QPSK is commonly used, it requires 4 sub-carriers instead of 2, which would be spectrally much less efficient. In our case, p_{aux} is modulated with BPSK using 1 sub-carrier. The total used spectral resource is 3 sub-carriers, which is still comparably less spectral resource used overall.

VI. CONCLUSION

This paper proposed a novel optical transmission scheme to guarantee correct reception of data while alleviating the requirements on transmission quality, especially in the context of optical transmission with high-level modulation formats. On a case study of high-speed Ethernet over optical OFDM networks, we showed that optical OFDM systems can be spectrum efficient with high-level modulation formats, while being fault-tolerant with linear network coding and over the main and auxiliary paths. We theoretically derived the upper bound of affordable packet loss for a variety of scenarios. The results showed that a 100GE Ethernet transmission system based on optical OFDM can be fault tolerant using high-level modulation formats, while remaining spectrally efficient with a comparably small system overhead induced by a low-speed auxiliary path and network coding. We showed that the

combined consumption of spectrum resource by both main and auxiliary path is comparably less than without using high-level modulation formats. To the best of our knowledge, this is the first study that shows that a fault tolerance transmission in optical network system can be achieved by designing a novel transmission scheme with an auxiliary path. Our future work will include consideration of physical layer parameters as measured in the practical systems, as well as consideration of routing and spectrum assignment algorithms suitable to support the system parameters necessary to maintain fault tolerance for a variety of modulation and transmission formats.

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