

Numerical and practical experiments for maximally stiff structure based on the topology optimization theory and the FEM

Kenta Yoshihara¹, Takahiko Kurahashi¹ and Masanari Kobayashi²

¹ Deparment of Mechanical Engineering, Graduate School of Nagaoka Uiversity of Technology, 1603-1 Kamitomioka-machi, Nagaoka-shi, Niigata 940-2188, Japan

 2 Oiles Corporation, 8 Kirihara-cho, Fujisawa-shi, Kanagawa 252-0811, Japan

E-mail s163100@stn.nagaokaut.ac.jp

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Abstract

In this study, numerical studies for a maximally stiff structure based on the topology optimization theory and the FEM are carried out, and the result of several practical tensile tests for the optimized structure was shown. The specimens for tensile testing were made using 3D printer. The numerical studies included filtering radius based examination. The thickness of the optimized model was also investigated, assuming the displacement of the optimized model to be the same as that of the initial model.

Keywords topology optimization, finite element method, adjoint variable method, filtering radius, 3d printer

Research Activity Group Mathematical Design

1. Introduction

The topology optimization was structural optimization theory established in the late 1980s. This method, using a characteristic function expressed as "0" and "1" was applied to represent the shape of a structure [1]. In the 1990s, topology optimization using the homogenization method [2, 3] was proposed and investigated. However, once composite materials were included in topology optimization, the computational algorithm became complicated. To solve this problem, a density method was developed, in which Young's modulus is represented as a function of density [4]. Numerous researcher have investigated topology optimization using this density method.

More recently, a growing body of research related to computational mechanics has focused on topology optimization; and numerous meetings, such as the "World Congress of Structural and Multidisciplinary Optimization 2017 (WCSMO 2017)", have been held. There are also several paper related to topology optimization using the density method [5–7]. Taylor et al. performed the topology optimization of an assembly structure taking additive manufacturing into account, and Hoffarth et al. and Ven et al. focused on the overhang angle related to support structures. Hoffarth et al. investigated topology optimization that included constraints on the overhang angle. Ven et al. also propose a form of topology optimization that takes into account manufacturing limitations using an unstructured finite element mesh. It can be confirmed that technical paper related "additive manufacture" and "topology optimization" gradually increase as shown in Fig. 1. Although papers related to research on the topology optimization and experiments can be seen [8, 9], it is difficult to find research papers



Fig. 1. Relationship between number of papers and published year on "additive manufacturing" and "topology optimization" by paper search service Scopus.

that have considered the weight reduction based on the topology optimization and experiments. This prompted us, in this study, to manufacture the shape obtained by topology optimization using the density method in twodimensional model. Tensile load testing was carried out on the initial and optimized models as investigated in three-dimensional model. We also investigated the relationship between the performance function at the final iteration and the filtering radius as part of the filtering technique [10, 11].

2. Numerical simulation for derivation of maximally stiff structure

2.1 Derivation of Lagrange function gradient with respect to non-dimensional density

For deformation problems of the elastic body, a discretized equation can be derived based on the finite eleA I*

$$[K]\{u\} = \{f\},\tag{1}$$

where [K], $\{u\}$ and $\{f\}$ indicate the stiffness matrix, the displacement vector and the force vector, respectively. Here, the performance function is defined under consideration of minimization of work by external force, and is shown in (2).

$$J = \frac{1}{2} \{f\}^T \{u\} = \frac{1}{2} \{u\}^T [K] \{u\}.$$
 (2)

Due to minimizing (2) under the constraint condition of (1), the Lagrange function shown in (3) is introduced by using the adjoint variable vector $\{\lambda\}$.

$$J^* = \frac{1}{2} \left[\{f\}^T \{u\} + \{\lambda\}^T \left([K] \{u\} - \{f\} \right) \right].$$
(3)

The gradient of the Lagrange function with respect to the non-dimensional density is also obtained as shown in (4). Here, index "(e)" denotes the element number.

$$\frac{\partial \delta}{\partial \rho_{(e)}} = \frac{1}{2} \left[\{f\}^T \frac{\partial \{u\}}{\partial \rho_{(e)}} + \{\lambda\}^T \left(\frac{\partial [K]}{\partial \rho_{(e)}} \{u\} + [K] \frac{\partial \{u\}}{\partial \rho_{(e)}} \right) \right] \\
= \frac{1}{2} \left[\{\lambda\}^T \frac{\partial [K]}{\partial \rho_{(e)}} \{u\} + \left(\{f\}^T + \{\lambda\}^T [K]\right) \frac{\partial \{u\}}{\partial \rho_{(e)}} \right].$$
(4)

Considering the stationary condition in (4), the relationship equation, $\{-\lambda\}^T[K] = \{f\}^T$, is obtained, and the equation is represented as $[K]^T\{-\lambda\} = \{f\}$. In addition, taking into account the symmetry of the stiffness matrix [K], the relation equation, i.e., $\{-\lambda\} = \{u\}$, is satisfied. Therefore, (4) is represented as (5).

$$\frac{\partial J^*}{\partial \rho_{(e)}} = \frac{1}{2} \{\lambda\}^T \frac{\partial [K]}{\partial \rho_{(e)}} \{u\} = \frac{1}{2} \{\lambda\}^T [B]^T \frac{\partial [D]}{\partial \rho_{(e)}} [B] \{u\}$$
$$= -\frac{1}{2} \{u\}^T [B]^T \frac{\partial [D]}{\partial \rho_{(e)}} [B] \{u\}.$$
(5)

Here, [B] and [D] indicate the relationship matrix between strain and displacement and the elastic matrix [12]. Also, the Young's modulus of each element is defined by (6). In this equation, it is assumed that the Young's modulus is given by a non-dimensional density function.

$$E_{(e)} = (E_0 - E_{\min})\rho_{(e)}^p + E_{\min}.$$
 (6)

The parameters E_{\min} , E_0 and p respectively indicate the numerical stability parameter, the Young's modulus and the penalty parameter. A plane stress state is assumed in this analysis, and the gradient of the Lagrange function with respect to the non-dimensional density for an element is represented as (7).

$$\frac{\partial J^*}{\partial \rho_{(e)}} = -\frac{1}{2} p(E_0 - E_{\min}) \rho_{(e)}^{p-1} \{u\}^T [B]^T \frac{1}{1 - \nu_{(e)}^2} \\ \times \begin{bmatrix} 1 & \nu_{(e)} & 0 \\ \nu_{(e)} & 1 & 0 \\ 0 & 0 & \frac{1 - \nu_{(e)}}{2} \end{bmatrix} [B] \{u\}.$$
(7)

2.2 Filtering for the Lagrange function gradient with respect to non-dimensional density

It is necessary to apply the filtering technique for the Lagrange function gradient with respect to nondimensional density. The filtering equation shown in (8) is employed in this study.

$$\frac{\partial \bar{J}^*}{\partial \rho_{(i)}} = \frac{\sum_{j \in M_i} w(x_{(j)}, y_{(j)}) \rho_{(j)} \frac{\partial J^*}{\partial \rho_{(i)}}}{\rho_{(i)} \sum_{j \in M_i} w(x_{(j)}, y_{(j)})}.$$
(8)

Here, $w(x_{(j)}, y_{(j)})$ indicates the weighting coefficient, and is represented as shown in (9). In this equation, Rindicates the filtering radius.

$$w(x_{(j)}, y_{(j)}) = R - \sqrt{(x_{(j)} - x_{(i)})^2 + (y_{(j)} - y_{(i)})^2}.$$
 (9)

2.3 Computational flow of topology optimization

The computational flow of our topology optimization is shown as follows.

- (1) Input of computational conditions.
- (2) Finite element analysis for the material deformation problem.
- (3) Computation of performance function.
- (4) Check for convergence: if $|J^{(k+1)} J^{(k)}| < \epsilon$ then stop, else go to next step.
- (5) Computation of Lagrange function gradient with respect to non-dimensional density.
- (6) Computation of filtering for Lagrange function gradient with respect to non-dimensional density.
- (7) Update of non-dimensional density by optimality criteria method (See (10)) while the volume constraint (See (11)) and variation limit of the density (See (12)) are satisfied, and return to step 2.

$$\rho_{(e)}^{(k+1)} = \rho_{(e)}^{(k)} \frac{\left(\frac{\partial \bar{J}^*}{\partial \rho_{(e)}}\right)^{(k)}}{-\Lambda^{(k)} \left(\frac{\partial V}{\partial \rho_{(e)}}\right)^{(k)}},\tag{10}$$

$$\frac{1}{nx}\sum_{e=1}^{mx}\rho_{(e)} - V_f = 0,$$
(11)

$$\rho_{(e)}^{(k)} - \rho_{limit} \le \rho_{(e)}^{(k+1)} \le \rho_{(e)}^{(k)} + \rho_{limit}.$$
 (12)

Here, mx is number of finite elements, V_f is normalized volume, Λ is Lagrange multiplier coefficient and index "(k)" is number of iterations.

3. Investigation of maximally stiff structure based on practical experiments

The tensile test model and the target domain in the topology optimization, i.e., a 1/4 model, are shown in Fig. 2. Comparisons of numerical and experimental results are shown in this section.

3.1 Numerical experiments

The computational conditions are shown in Table 1.

Numerical experiments are carried out by changing the filtering radius R. The filtering radius R is set at 0.75 mm (Case 1), 1.50 mm (Case 2), 2.50 mm (Case 3).



Fig. 2. Tensile test model.

Table 1. Computational conditions.

Number of elements	3000
$(x \text{ direction } \times y \text{ direction})$	(30×100)
Element shape	quadrangle
Mesh size, mm	0.5
Initial non-dimensional density ρ	0.50
Volume constraint $V, \%$	50
Penalty parameter p	3
Variation limit of the density ρ_{limit}	0.15
Convergence criterion ϵ	10^{-5}
Numerical stability parameter E_{\min} , MPa	10^{-3}
Young's modulus E_0 , MPa	$1800 \ (=1.8 \text{GPa})$
Poisson's ratio ν	0.37

The variations of normalized performance function and volume constraint are shown in Fig. 3. In all cases, the value of performance function minimize under the volume constraint condition. In addition, it is found that the value of performance function increases due to the constraint condition of density update at initial iterations. Fig. 4 shows the non-dimensional density distribution in the initial model and the optimized model in each case, respectively. It is apparent that an indistinct and shapeless form is obtained by increasing the value of filtering radius R.

3.2 Examination by means of practical experiments

The result of the practical tensile test for the initial model and the optimized model in Case1 is shown below. The test piece made using the 3D printer is shown in Fig. 5 (a). The total mass of the initial model (3.1)g) is the same as that of the optimized model. The relationship between load and displacement for the initial and optimized model is shown in Fig. 5 (b). It is found that the displacement of the optimized model is smaller than that of the initial model under the same load value.

Next, we found the appropriate thickness at which the displacement of the initial model is the same as the displacement of the optimized model. The experimental conditions are shown in Table 2. The thickness of the specimen for the optimized model was reduced as shown in Table 3. In Tables 2 and 3, case A is same as the optimized model in case 1. Consequently, it is found that the total mass for the optimized model case D can be reduced by approximately 10% in comparison with that of the initial model.



(a) Variation of normalized performance function.



(b) Variation of volume constraint.

Fig. 3. Variations in normalized performance function and volume constraint.





(b) Case1 (R=0.75 mm): (a) Non-dimensional density distribution in initial model.



distribution in optimized

model.

Non-dimensional density distribution in optimized model.



Non-dimensional density distribution in optimized model.

Fig. 4. Results of topology optimization

Conclusions **4**.

In this study, a topology optimization analysis of the tensile test model performed, followed by a practical tensile test. In the analysis of the topology optimization, the finite element method was applied to carry out the deformation analysis, and numerical experiments were carried out with respect to the filtering radius R were carried



(a) Test pieces. (Left : initial model, Right : optimized model)



(b) Relationship between load and displacement for initial and optimized models. Fig. 5. Test piece and experimental result

Table 2. Experimental conditions.

Specimen	Thickness, mm	Mass, g
Initial model	1.06	3.1
Optimized model Case A	2.12	3.1
Optimized model Case B	2.03	3.0
Optimized model Case C	1.95	2.9
Optimized model Case D	1.90	2.8

Table 3. Comparison of experimental results under a load of 200 N.

Specimen	Thickness, mm	Displacement, mm
Initial model	1.06	0.68
Optimized model Case A	2.12	0.61
Optimized model Case B	2.03	0.64
Optimized model Case C	1.95	0.65
Optimized model Case D	1.90	0.68

out. In addition, a practical tensile test was also carried out on the optimized model to investigate whether the performance function can minimized. It was found that the displacement of the optimized model was smaller than that of the initial model under the same loading conditions. Next, we investigated the thickness of the optimized model such that the displacement of the optimized model was identical to that of the initial model. The result showed, that a 10% material reduction could be applied to the optimized model.

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