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Credit scoring method using estimated forward financial statements based on purchase order information

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Abstract

In this article, we propose a credit scoring method using purchase order information on target borrower firms. First, we introduce a time-series model which captures purchase order volume transitions of a target firm. Then, we execute credit scoring based on estimated financial statements reflecting expected values of future purchase orders obtained from the purchase order models. We demonstrate the applicability of our method to practical credit risk monitoring with a case study. One of the advantages of our method is its abilities to capture changes of credit risk timely reflecting the firms' business conditions.

Keywords credit risk, credit scoring, purchase order information

Research Activity Group Mathematical Finance

1. Introduction

Financial institutions assess and monitor their borrowers' credit risk, which is the risk associated with financial losses caused by defaults, for their lending businesses. For traditional lending application screening, borrower firms are mainly evaluated using disclosed financial statements which are made regularly, e.g. annually. However, there are several problems with this traditional method of lending application judgment. That is, financial institutions cannot recognize possible changes in a firm's condition timely, when using only the financial statements obtained annually. On the other hand, purchase order (PO) information, which includes date of PO receipts and PO volumes, reflects precise business conditions of firms on a real-time basis. In this paper, we propose a credit scoring method based on monthly estimated forward-looking financial statements using PO information. Our method enables us to monitor the business conditions more frequently than traditional credit risk assessment based on only annually or quarterly disclosed financial statements.

In order to realize frequent credit monitoring, a structural type of credit risk model using PO information is proposed by [1]. The model suggested in [1] is constructed under the concept of the structural framework of credit risk model (refer to [2].) In the model, the value of the firm's asset, which is the sole determinant of default probability, is obtained using PO.

On the other hand, in financial practice, statistical framework, so called credit scoring method, is widely employed since Altman's Z-score model [3] appeared. In the credit scoring method, linking between financial statements and future default occurrence is specified by some statistical model and the possibility of default is quantified as credit score. Therefore, we propose a PO based credit risk model which has affinity for common credit scoring model.

Table 1. Data format of the sample PO information.

Buyer	Date of PO receipts	Product number	Unit price /thousand yen	Quantity
Firm 1	Aug. 2014	X-031	1682.4	24
Firm 2	Aug. 2014	X-043	21023.1	3
Firm 3	Sep. 2014	X-032	4823.5	13
Firm 1	Sep. 2014	X-034	2418.9	210
Firm 2	Sep. 2014	X-033	10523.1	15

We demonstrate a case study that shows the effect of our method with real PO samples. First, we estimate forward financial statements using a PO volume time-series model. There, we calibrate model parameter to implied growth rate by customer's stock prices observed in stock market. Then, we obtain credit score by using estimated forward financial statements as an input of a credit scoring model. Financial institutions can monitor the actual business conditions of borrower firms by using purchase order information. Since calculated credit score is derived from estimated forward financial statement, our method enables us forward looking credit risk assessment.

This paper is organized as follows. Section 2 illustrates PO sample data provided by a sample firm. Section 3 introduces a PO based credit scoring model and illustrates some empirical results. Section 4 concludes.

2. PO data

The sample data we use for our modelling is a PO information which includes attributions of buyers, date of purchase order receipts, product attributions, product prices and PO amounts. Table 1 shows the format of the sample PO information. Our sample data is the historical PO records of Kojima Industries Co. Kojima Industries Co. is an unlisted firm, which manufactures interior and exterior automobile components. The main customers (buyers) of the firm are leading auto manufacturers such as Toyota Motor Company, Toyota Auto Body Company, Toyota Motor East Japan Incorporated, Hino Motors, and Daihatsu Motor Company. The sample data are the monthly PO records from June 2011 to December 2014. We recognize seasonality in PO volumes, for example, there is a relative decrease in PO volume every August and December. In order to handle the seasonality of PO volumes, we employ an autoregressive-type model of the PO volume ratio, which is calculated as the ratio of the PO volume to the PO volume of the same month in the previous year. We model the POs of each customers respectively. Hereafter we number each customer in order of the ranking of PO volumes. The customers for Kojima Industry Co. are ranked in the top nine PO volumes ($i = 1, 2, 3, \dots, 9$). The sum of the top nine PO volumes is accounting for approximately 96% of all PO volumes. In addition, we model the aggregated remainder ($i = 10$).

3. Model and empirical results

This section provides our credit scoring method based on PO information and some empirical results. First, we construct a time-series model of PO volume transition for each buyer. Next, we obtain expected future PO volume for the next one year from calculation date with the model. Then, expected values of proceeds of sales are calculated by the sum of the expected PO volumes with a time lag between receiving POs and collecting proceeds of sales. Next, we calculate the expected cost of producing and supplying products. Then, we obtain expected profits or losses from the difference between the sales and the costs. Obtained profits and losses are added to asset values. In this way, we obtain estimated forward-looking profit and loss statement (P/L) and balance sheet (B/S). Finally, we calculate credit scores putting estimated financial statements into a credit scoring model. Hence calculated credit score is derived from estimated future P/L and B/S, our method realizes, so to say, forward-looking credit risk assessment.

We model uncertainty in the economy on a filtered complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in\mathcal{T}}, \mathbb{P})$ where$\{\mathcal{F}_t\}_{t\in\mathcal{T}}$ is a complete filtration with discrete time space $\mathcal{T} = \{0, 1, 2, \dots, \infty\}$, which associated with our monthly PO sample data and the time unit implies one month. The target firm of credit risk assessment is the seller side of POs. We denote by $\mathcal{I} = \{1, 2, 3 \dots, I\}$ the set of the corresponding buyer.

Our model is simpler than the actual practical processes of generating firm assets from a PO, because we intend to clarify our concept of deriving financial statements from PO information. The steps of obtaining our credit scoring model are the following:

Modeling steps

(1) Introduce a stochastic process for PO volumes

(2) Estimate P/L

(3) Estimate B/S

(4) Construct credit scoring model

3.1 Introduce a stochastic process for PO volumes

PO volumes ordered by buyer $i \in \mathcal{I}$ are denoted by $\{O_t^i\}_{t\in\mathcal{T}}$, which is an $\{\mathcal{F}_t\}$-adopted stochastic process. As we saw in section 2, data of monthly PO volume often have seasonal effects. To handle seasonal effects, we model the one-year difference of log-PO volumes $R_t^i = \log(O_t^i) - \log(O_{t-12}^i)$. We employ AR (Auto-Regressive) model of R_t^i described as follows:

$$R_t^i = \alpha_i + \beta_i R_{t-1}^i + \epsilon_{i,t}. \tag{1}$$

Here, $\epsilon_{i,t} \sim N(0, \sigma_i^2)$and they are independent of time, where $N(m, v)$ is normal distribution with mean m and variance v. PO volumes $\{O_t^i\}_{t\in\mathcal{T}}$ are obtained from $\{R_t^i\}$ as

$$O_t^i = \left(O_{t-12}^i \times \exp(R_t^i)\right) 1_{\{t\le T_i\}}.$$

Here, T_i is the time to break off business connections with buyer i. We assume the break-off of business connections occurs only when the buyer defaults. Then, T_i is the same as the default time of buyer i.

We are able to calibrate the PO model to the stock prices observed in stock market, which reflects the expectation of future earnings of customers. Here, customers which are unlisted firms are out of scope. Our calibration procedure is as follows. First, we derive the growth rate of dividend from observed stock prices. In particular, we estimate growth rate of dividend k from market stock prices by employing Dividend Discount Model (DDM):

$$P_t^{stock} = \sum_{n=1}^{\infty} \frac{D_{t+1}(1+k)^{n-1}}{(1+r)^n} = \frac{D_{t+1}}{r-k}.$$

Here, D_t denotes dividend on time t, constant k denotes the growth rate of dividend, constant r denotes the cost of equity capital. Dividends D_t is calculated by $D_t = EF_t \times DPR$, using earnings forecast EF_t and dividend payout ratio DPR. Then, assuming that the growth rate of dividend and the growth rate of earnings are the same, in addition, assuming the growth rate of earnings and growth rate of PO volumes are the same, we obtain implied growth rate of PO volume. With our PO volume model, growth rate of PO volumes is $\log(O_t^i) - \log(O_{t-12}^i) = R_t^i$. Then, expcted growth rate of PO volumes $\mathbb{E}[R_t^i]$ is obtained recursively calculating by

$$\mathbb{E}[R_n^i] = \alpha_i + \beta_i \mathbb{E}[R_{n-1}^i].$$

Thus we obtain

$$\mathbb{E}[R_t^i] = \alpha_i + \alpha_i\beta_i^1 + \alpha_i\beta_i^2 + \cdots + \alpha_i\beta_i^{10} + \alpha_i\beta_i^{11}\mathbb{E}[R_{t-12}^i].$$

Finally, we calibrate the parameter of PO model to implied growth rate of PO volume by minimizing absolute difference $|\mathbb{E}[R_t^i] - k|$. Here, we have fixed parameter β_i as $\hat{\beta}_i$, and obtain calibrated parameter α_i as

$$\hat{\alpha}_i = \frac{k}{1 + \hat{\beta}_i^1 + \hat{\beta}_i^2 + \cdots + \hat{\beta}_i^{10} + \hat{\beta}_i^{11}\mathbb{E}[R_{t-12}^i]}.$$

We estimate the PO model of the buyers for every months of Jan. 2014 to Dec. 2014. The average estimated parameters of the PO model are described in Table 2. Because most of the estimated values of auto-regressive

Table 2. Averages of estimated parameter values of the model (1) of one-year difference of log-PO.

i	1	2	3	4	5
β	0.294	0.730	0.645	0.295	0.756
α	-0.019	0.001	0.068	0.036	0.032
$\hat{\alpha}$	0.055	–	–	0.059	0.024
σ^2	0.011	0.024	0.019	0.036	0.024

i	6	7	8	9	10
β	0.548	0.117	0.638	0.507	0.766
α	0.060	-0.052	0.044	-0.043	-0.030
$\hat{\alpha}$	–	0.057	0.030	0.052	–
σ^2	0.019	0.071	0.027	0.027	0.089

Note: Firm2, 3, 6 are unlisted firms.

coefficients $\{\beta_i\}$ are positive, we recognize the existence of PO trends in our sample data. In addition, we execute Ljung-Box test and the results are no significant auto-correlation in the residuals, and the model is not rejected with under 5%-significant level (all p-values are over 0.05).

With the estimated PO model, we calculate expected PO on $t \in \{s, s+1, \dots, s+11\}$. Considering the distribution of $\log(O_t^i)$ as normal distribution, expected PO is calculated by

$$\hat{O}_t^i = \mathbb{E}[O_t^i|\mathcal{F}_s] = \exp\left(m_{s,t}^i + \frac{\nu_{s,t}^i}{2}\right) PS^i(s,t)$$

where $m_{s,t}^i$ and $\nu_{s,t}^i$ denote expectation and variance of $\log(O_t^i)$ under information $\mathcal{F}_s$ respectively. $PS^i(s,t)$ denotes survival probability from s to t and hereafter we assume $PS^i(s,t)$ is constant. With our PO model, expectation $m_{s,t}^i := \mathbb{E}[\log(O_t^i)|\mathcal{F}_s]$ is obtained recursively by

$$m_{s,t}^i = \alpha_i + m_{s,t-12}^i + \beta_i m_{s,t-1}^i - \beta_i m_{s,t-13}^i.$$

Also, variance $\nu_{s,t}^i := \mathrm{Var}[\log(O_t^i)|\mathcal{F}_s]$ is obtained recursively by

$$\nu_{s,t}^i = \nu_{s,t-12}^i + \beta_i^2 \nu_{s,t-1}^i + \beta_i^2 \nu_{s,t-13}^i + \sigma_i^2.$$

We calculate survival probability $PS^i(s,t)$ by $PS^i(s,t) = (1 - PD^i(0,60))^{(t-s)/60}$, where $PD^i(0,60)$ is historical 5-year default rate, associated with the credit ratings of customer i. The historical 5-year default rates are provided by R&I . For unrated customers, we assume a credit rating of BBB.

3.2 Estimate P/L

Next, we calculate proceeds of sales obtained from associated PO volumes. We assume that the firm receives proceeds of sales amounting to PO volumes after some time lags if there are no problems with product delivery and sales collection. If there are canceled POs or buyer defaults before sales collections, cash below the PO amount is collected by cancellation charges and recovery given default. Thus, estimated proceeds of sales at time t are given by

$$S_t^i = \hat{O}_{t-h}^i \times PS^i(t-h,\ t) + \hat{O}_{t-h}^i \times PD^i(t-h,t) \times (1 - LGD).$$

Constant LGD denotes loss rate given default and we set $LGD = 0.7$. The time lag between the order and collection of sales is set to two months ($h = 2$). $PD^i(s,t)$ denotes default probability of customer i from time s to time t. We calculate survival probability $PD^i(s,t)$ by $PD^i(s,t) = PD^i(0,60))^{(t-s)/60}$.

Then, estimated total sales at time t is $S_t = \sum_{i=1}^I S_t^i$ and estimated yearly total sales is $TS_t = \sum_{u=0}^{11} S_{t+u}$.

Next, we estimate ordinary profits and losses. For the purpose, we consider ordinary costs and assume that ordinary costs TC_t is given by the linear function of PO:

$$TC_t := a \sum_{u=0}^{11} \left(\sum_{i=1}^{I} \hat{O}_{t-g+u}^i \right) + b.$$

Here, we tentatively set the time lag between receiving the POs and the corresponding cost defrayments to one month ($g = 1$).

To estimate the parameters of the above function $a,\ b$, we use the historical annual PO volumes and related items from the disclosed P/L in 2008 to 2013. We execute linear regression of the realized operational costs by employing annual sales as explanatory variables. The coefficients obtained by linear regression are annually based values, and we transform them into the monthly based values. The obtained coefficients of operational costs are $a = 0.905$ and $b = 7.83 \times 10^8$ (the unit of the estimated parameter value is Yen.)

Ordinary profits and losses at time t are obtained by the difference in proceeds of sales and operational costs. Earnings before tax (EBT) are obtained by adding ordinary profits and losses, non-operating profit and losses EP_t, and extraordinary profit and losses SP_t:

$$EBT_t := (TS_t - TC_t) + EP_t + SP_t.$$

We set non-operating profit and losses EP_t and extraordinary profit and losses SP_t by historical average of non-operating profit and losses and extraordinary profit and losses obtained from disclosed P/L.

Then, the net earnings are obtained by adjusting tax payments to EBT:

$$P_t := (1 - G) EBT_t 1_{\{EBT_t \geq 0\}} + EBT_t 1_{\{EBT_t < 0\}}.$$

Here, the constant G denotes the corporate tax rate and we set $G = 0.4$.

3.3 Estimate B/S

We reflect estimated yearly net earnings P_s to B/S. Supposing there are no dividends to shareholder, retained profits increases by yearly net earnings. In addition, fixed asset and float asset increases. In particular, we obtain additional fixed asset by

$$\Delta FS_s = \min\left(P_s, \tilde{a} P_s + \tilde{b}\right)$$

and remains $P_s - \Delta FS_s$ are added to cash. To estimate the parameters of the fixed asset function, we use the firm P/Ls in 2008 to 2013. We execute linear regression and obtained coefficients $\tilde{a} = 1.843,\ \tilde{b} = -3.18 \times 10^9$.

For simplicity, we set debt amount as constant.

3.4 Construct credit scoring model

We calculate financial indices listed below from estimated forward financial statements obtained previously. The list of financial indices are followings:

$$z_1 = \frac{\text{Total current assets} - \text{Total current liabilities}}{\text{Total assets}},$$

$$z_2 = \frac{\text{Net income}}{\text{Total assets}}, \quad z_3 = \frac{\text{EBT}}{\text{Total assets}},$$

$$z_4 = \frac{\text{Equity capital}}{\text{Total liabilities}}, \quad z_5 = \frac{\text{Sales}}{\text{Total assets}},$$

$$z_6 = \text{Logarithmic sales}, \quad z_7 = \frac{\text{Operating income}}{\text{Total liabilities}}.$$

These financial indices are inputs for credit scoring model.

We employ a multinomial logit model for credit scoring, which are estimated using credit class samples and financial statements samples of transportation equipment companies listed with first section of the Tokyo Stock Exchange. That is, we obtain probabilities of the credit class of firm i by calculating

$$p_{i,\tilde{c}} = \frac{\exp\left(\beta_{\tilde{c},0} + \sum_{j=1^m} \beta_{\tilde{c},j} z_{i,j}\right)}{\sum_{c=1}^{4} \exp\left(\beta_{c,0} + \sum_{j=1^m} \beta_{c,j} z_{i,j}\right)}$$

where $c \in \{1, 2, 3, 4\}$ is credit class. Then, estimated credit score is given by the expectation of credit classes: $CS_i = \sum_{c=1}^{4} c \times p_{i,c}$. For estimating parameters of the logit model, we use the financial statements during 2000 to 2013 and one-year forward credit classes which are mapped from credit ratings announced by R&I and JCR. Here, class 1 indicates AAA∼AA−, class 2 indicates A+∼A−, class 3 indicates BBB+∼BB−, class 4 indicates BB+∼CCC−. In-sample performance of estimated credit scoring model is 70.1% on classification accuracy.

Fig. 1 shows the obtained credit scores under the realized PO scenario. The level of estimated credit scores are in the range between 2 and 3 due to the high credit quality (low default risk) of the buyers and the stability of the PO volume. In addition, we recognize in Fig. 1 that the increase of the estimated credit scores incurred by the decrease in the PO volumes. Fig. 2 shows the obtained credit scores under the stressed scenarios in which the growth rate of PO volume $\mathbb{E}[R_t^i]$ decreases by 10% and 5%, increases by 5% in July 2014. The results in Fig. 2 indicate that decreases (increases) in PO growth rate immediately cause an increase (decrease) of estimated PDs. These results imply that credit risk modeling based on PO information enables financial institutions to monitor the credit risk affected by a change in business conditions of borrower firms on a real-time basis.

4. Concluding remarks

This paper proposed a credit scoring method based on forward financial statements estimated using PO information. We first introduced the time-series model for PO transitions and process of calculating financial indices from PO amounts. Then, we demonstrate applicability of the model to credit risk monitoring with PO samples of actual firm. It is a future work to examine adequacy of the model with big data of POs obtained from a number of actual firms.

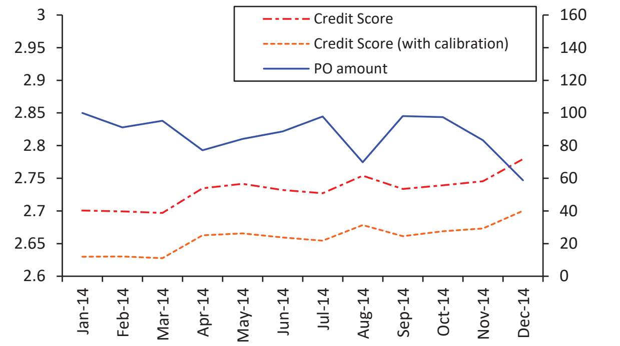

Fig. 1. Estimated credit score without calibration(dash-dot line) and credit score with calibration (dashed line) of Kojima Industries Co. and cumulative one-year PO amounts observed for each months (solid line).

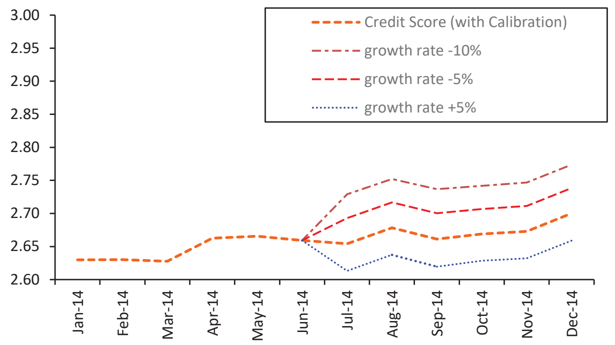

Fig. 2. Estimated credit scores under stress scenarios. The dashed orange line shows the credit scores with calibration in Fig. 1. The dashed red line shows the credit scores under the stress scenario of a 5% decrease of the PO growth rate in July 2014. The dash-dot purple line shows the credit scores under the stress scenario of a 10% decrease of the PO growth rate in July 2014. The dotted blue line shows the credit scores under the stress scenario of a 5% increase of the PO growth rate in July 2014.

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