Editorial

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Recent Advances in Boundary Element Methods

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Since 2003 together with Martin Schanz (Graz) and Wolfgang L. Wendland (Stuttgart) we have been organizing a series of annular workshops on Fast Boundary Element Methods in Industrial Applications in the Söllerhaus at Hirschegg, Kleinwalsertal, Austria [12]. The topics include, but are not limited to, the formulation, analysis, and implementation of fast boundary element methods such as wavelets, the fast multipole method, panel clustering, Adaptive Cross Approximation, and hierarchical matrices; efficient preconditioned iterative solution methods, parallelization and domain decomposition methods; stability and a priori and a posteriori error analysis of boundary element methods and adaptivity; transient boundary value problems and space-time discretization methods; the coupling of different physical fields and different discretization techniques such as finite and boundary element methods; the application of boundary element methods in shape optimization, optimal control, and inverse problems, and in industrial and engineering applications. The success of this workshop series is mainly due to the interdisciplinary audience from mathematics, engineering, and industry, the inspiring talks by the participants, and the discussions in the special atmosphere at the Söllerhaus. As a success of the Söllerhaus workshops several new cooperations were established, and joint publications were initiated over the years. While in the first years the focus was more on industrial applications, nowadays it is more on the formulation and mathematical analysis of boundary integral equation and boundary element methods as well as related methods, but always with an engineering or industrial application in mind. In addition to regular publications in journals and proceedings some of these results were published in a previous special issue [6] and in a Springer book [7] devoted to the Söllerhaus workshops.

At the occasion of the 20th Söllerhaus workshop in 2022 we decided to edit another special issue on recent advances in boundary element methods. This issue covers nine contributions presenting some state of the art contributions to boundary element methods.

A. Aimi, C. Guardasoni, L. Ortiz–Gracia, and S. Sanfelici use a coupling of a Fourier cosine series with a boundary element method for a fast evaluation of barrier option prices in the Heston model; see [1]. The improvement of the computational efficiency makes this an alternative to more traditional approaches as used by practitioners in finance.

In [2], M. Dambrine, H. Harbrecht, and B. Puig consider Bernoulli's exterior free boundary problem where the interior boundary is random. The analysis is based on a new regularity result on the map from the parametrization of the inner boundary to a parametrization of the outer boundary.

The coupling of a curved virtual element method and a boundary element method for the simulation of wave fields scattered by obstacles in homogeneous infinite media is presented by L. Desiderio, S. Falletta, M. Ferrari, and L. Scuderi in [3]. In particular, a damped wave equation in time-domain with Dirichlet boundary conditions is considered, where a Crank–Nicolson scheme in the interior domain is coupled with an interface boundary integral equation.

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New results on the stability of the non-symmetric coupling of finite and boundary element methods for a free space transmission problem are described by M. Ferrari in [4]. These results rely on an particular linear combination of the boundary integral equation describing the solution of the Laplace equation in the exterior domain, and the domain variational formulation of the diffusion equation in the interior domain.

The symmetric coupling of finite and boundary element methods is analysed by H. Gimperlein and E. P. Stephan in [5] in the case of p-Laplacian type Hencky materials with an unbounded stress-strain relation, and nonlinear transmission conditions to describe contact with friction.

The reconstruction of magnetic fields from measurements is the topic of the work [8] by M. Liebsch, S. Russenschuck, and S. Kurz. Due to their accuracy, boundary element methods are well suited for the computation of higher-order partial derivatives and local expansions of magnetic fields as required in particle tracking.

The numerical computation of electrostatic forces which is based on the use of the Maxwell stress tensor may fail in the case of dielectric materials. P. Panchal, N. Ren, and R. Hiptmair consider in [9] an alternative approach employing the adjoint method from shape optimization to evaluate interface-based force functionals by using boundary integral equations.

In [10], R. Watschinger and G. Of propose and analyze a new time-adaptive fast multipole method for the boundary element solution of the heat equation. The particular focus is on an efficient treatment of tensor product meshes which are adaptive in time. The efficiency of the involved operations and the approximation quality of the related kernel expansions are confirmed by numerical experiments.

The modified Hilbert transformation plays an important role in the discretization of time-dependent partial differential equations in finite time intervals. Since its behavior is similar to that of the hypersingular boundary integral operator of second-order elliptic partial differential equations, similar techniques as known from boundary element methods can be used in the mathematical and numerical analysis, and in the computation and implementation of the involved singular integrals, as considered in the contribution of M. Zank in [11].

The careful reviewing process was only possible with the help of the anonymous referees who did an invaluable work that helped the authors to improve their contributions. Furthermore, we would like to thank the Editorial Board and the Editor-in-Chief, Carsten Carstensen, for the possibility to edit this special issue, and the Executive Secretary, Almas Sherbaf, for the excellent support while preparing this special issue.

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