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ANT–BASED EXTRACTION OF RULES IN SIMPLE DECISION SYSTEMS OVER ONTOLOGICAL GRAPHS

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In the paper, the problem of extraction of complex decision rules in simple decision systems over ontological graphs is considered. The extracted rules are consistent with the dominance principle similar to that applied in the dominance-based rough set approach (DRSA). In our study, we propose to use a heuristic algorithm, utilizing the ant-based clustering approach, searching the semantic spaces of concepts presented by means of ontological graphs. Concepts included in the semantic spaces are values of attributes describing objects in simple decision systems.

Keywords: ant-based clustering, decision systems, DRSA, ontological graphs, rule extraction.

1. Introduction

Rough set theory delivers useful methods for knowledge discovery and data mining (cf. Pawlak, 1991). Starting points for various data mining algorithms, including rough set ones, are information (decision) tables consisting of vector descriptions of objects. In rough set theory, information (decision) tables are tabular representations of mathematical entities called information (decision) systems. In information (decision) tables, rows represent objects whereas columns correspond to attributes (features) of objects. Entries of the table (intersections of rows and columns) are values of attributes (corresponding to columns) describing objects (corresponding to rows). In classic approaches, values in information (decision) tables can be both symbolic and numeric. To compare objects by means of such attribute values, a lot of measures have been defined. First of all, there are various similarity measures of objects. A generalized definition of approximations of sets, based on similarity, was proposed, for example, by Slowinski and Vanderpooten (1996). Some approaches use also data semantics, e.g., the dominance-based rough set approach (Greco *et al*., 2001).

Intelligent systems play an important role in modern computer science (Tadeusiewicz, 2010b; 2011) and related fields. The recent research in the area of intelligent systems shows that, in many situations, data alone are not sufficient. There is a need to add some expert knowledge about relationships within data expressing the meaning of data. Such knowledge is included in ontologies. In the works of Pancerz (2012b; 2013b), ontologies were incorporated into information (decision) systems, i.e., attribute values were considered in the ontological (semantic) spaces. Similar approaches have been considered in the literature, e.g., DAG-decision systems (Midelfart and Komorowski, 2002), the dominance-based rough set approach (DRSA) (Greco *et al*., 2001), rough ontology (Ishizu *et al*., 2007), etc. In our approach, we replace, in a classic definition

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of information (decision) systems, simple sets of attribute values by ontological graphs which deliver us some new knowledge about meanings of attribute values. For this case, decision rules in decision systems can be seen from different perspectives, for example, taking into consideration synonymy, generality or some more sophisticated properties determining meanings of attribute values.

Formally, the ontology can be represented by means of graph structures. The graph representing the ontology is called the ontological graph. In such a graph, each node represents one concept from the ontology, whereas each edge represents a relation between two concepts. Relations are very important components in ontology modeling as they describe the relationships that can be established between concepts. There have been proposed two ways for creating information (decision) systems over ontological graphs. In the first approach, attribute values of a given information (decision) system are concepts from ontologies assigned to attributes. Such a system is said to be a simple information (decision) system over ontological graphs. In the second approach, attribute values of a given information (decision) system are local ontological subgraphs of ontological graphs assigned to attributes. Such a system is said to be a complex information (decision) system over ontological graphs. It means that the term "simple" should be understood as the word describing the property of the decision system, not the size of it. In simple decision systems over ontological graphs, attribute values are single concepts. The opposed system is "complex". In complex systems, attribute values are graph structures.

An important problem concerning decision systems is extracting the knowledge hidden in such systems. This knowledge can be expressed in the form of decision rules. The topic of rule definition and extraction in various decision systems has been widely considered in the literature. Therefore, Pancerz (2013a) defined decision rules and related notions in simple decision systems over ontological graphs analogously to those defined for classic decision systems in rough set theory.

Another look, this time based on the dominance-based rough set approach (Greco *et al*., 2001), at decision rules for simple decision systems over ontological graphs was presented by Pancerz (2012a), too. In that paper, elementary decision rules, similar to that defined in the DRSA, were considered. An elementary rule is said to be a rule with one condition descriptor. Moreover, the exhaustive algorithm for mining the most general elementary rules, in a given simple decision system over ontological graphs, with respect to their condition parts for fixed decision parts, was proposed. That algorithm was based on the depth-first search technique with pre-pruning.

The idea of incorporating the DRSA into decision systems over ontological graphs, proposed by Pancerz (2012a), is recalled in Section 3. In this paper, we continue the discussion on decision rules consistent with the dominance-based rough set approach. Our investigations are extended to the decision rules with complex condition parts, i.e., the rules having multi-descriptor left-hand sides. However, only a special case is taken into consideration, when descriptors appearing on the left-hand sides of rules are linked by the *and* logical connectives. The complex decision rules in simple decision systems over ontological graphs are considered in Section 4. In the case of complex rules, the rule extraction becomes a more complicated process than in classic rough set theory. The space of possible condition descriptors appearing in the rules is significantly greater. Descriptors can include not only attribute values (concepts) present in decision systems, but also values which are more general concepts than the ones mentioned, according to defined ontological graphs. Therefore, to solve the problem considered, we propose, in Section 5, to use some heuristic algorithm utilizing the ant-based clustering approach.

2. Simple decision systems over ontological graphs

Information (decision) systems were proposed by Pawlak (1991) as knowledge representation systems. An information (decision) system represents a set of objects described by attribute value vectors. Pancerz (2012b) proposed to consider attribute values describing objects in the ontological spaces, where ontologies are constructed on the basis of controlled vocabularies and the relationships of the concepts in the controlled vocabularies (cf. definitions given by Neches *et al.* (1991) and Köhler *et al*. (2006)). In that approach, we use formal representations of ontologies by means of graph structures. Such structures are called ontological graphs. For a given ontology $\mathcal{O}$, an ontological graph includes nodes representing concepts from $\mathcal{O}$ and edges representing relations between concepts from $\mathcal{O}$.

Definition 1. Let $\mathcal{O}$ be a given ontology. An *ontological graph* is a quadruple $OG = (\mathcal{C}, E, \mathcal{R}, \rho)$, where $\mathcal{C}$ is a nonempty, finite set of nodes representing concepts in the ontology $\mathcal{O}$, $E \subseteq \mathcal{C} \times \mathcal{C}$ is a finite set of edges representing relations between concepts from $\mathcal{C}$, $\mathcal{R}$ is a family of semantic descriptions (in natural language) of types of relations (represented by edges) between concepts, and $\rho : E \to \mathcal{R}$ is a function assigning a semantic description of the relation to each edge.

Semantic relations describe the relationships that can be established between concepts. In the literature, a variety of taxonomies of different types of semantic relations has been proposed (Brachman, 1983; Chaffin and Herrmann, 1988; Milstead, 2001; Storey, 1993; Winston *et al.*, 1987). In our approach, we will use the taxonomy of types of semantic relations modeled on the project called Wikisaurus (Wikisaurus, 2013) aiming at creating a thesaurus of semantically related terms.

There are four main types of semantic relations distinguished in the project: synonymy, antonymy, hyponymy/hyperonymy, meronymy/holonymy. Synonymy concerns concepts with a meaning that is the same as, or very similar to, other concepts. Antonymy concerns concepts which have the opposite meaning to others. Both of these relations are nonhierarchical. Hyponymy/hyperonymy determines narrower/broader meaning. Hyponymy concerns more specific concepts than others. Hyperonymy concerns more general concepts than others. Meronymy and holonymy define part/whole relations. Meronymy concerns concepts that denote parts of the wholes that are denoted by other concepts. Holonymy concerns concepts that denote wholes whose parts are denoted by other concepts.

Further, we will be interested only in the hyperonymy. This relation will be marked with $R_{\triangleright}$ and $(v, v') \in R_{\triangleright}$ is read "v is a hyperonym of v'". This label will be used, for simplicity, instead of a semantic description (in natural language) of hyperonymy assigned to edges in ontological graphs. In drawing ontological graphs, for readability, we will omit reflexivity of hyperonymy. However, a given concept is a hyperonym of itself.

In this paper, we will use the definition of a simple decision system over ontological graphs given by Pancerz (2012b).

Definition 2. A *simple decision system* SDS^{OG} over ontological graphs is the septuple

$$SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d),$$

such that

- U is a nonempty, finite set of objects,
- C is a nonempty, finite set of condition attributes,
- D is a nonempty, finite set of decision attributes,
- $\{OG_a\}_{a \in C}$ is a family of ontological graphs associated with condition attributes from C,
- $V_d = \bigcup_{a \in D} V_a$, where V_a is a set of values of the decision attribute $a \in D$,
- $f_c : C \times U \to \mathcal{C}$, where $\mathcal{C} = \bigcup_{a \in C} \mathcal{C}_a$, is an information function such that $f_c(a, u) \in \mathcal{C}_a$ for each $a \in C$ and $u \in U$, where $\mathcal{C}_a$ is a set of concepts from the graph OG_a,
- $f_d : D \times U \to V_d$ is a decision function such that $f_d(a, u) \in V_d$ for each $a \in D$ and $u \in U$.

Remark 1. It is not necessary for an information function to be a total function, i.e., $f_c : C \times U \to \mathcal{C}^* \subseteq \mathcal{C}$.

The character of sets of attribute values differentiates simple information (decision) systems over ontological graphs from information (decision) systems proposed by Pawlak (1991). Now, attribute values are not singular (individual) values, but they are placed in the graph structures expressing relationships between these values.

Pancerz (2014) developed complex information systems over ontological graphs in which attribute values are local ontological graphs of ontologies assigned to attributes.

Example 1. (*Decision system*) Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d)$ be a simple decision system, represented by a decision table (see Table 1), over ontological graphs shown in Figs. 1 and 2. In this system,

$$U = \{u_1, u_2, \ldots, u_{15}\}$$

is a set of fifteen objects described with respect to the development level. $C = \{Sector, Region\}$ is a set of condition attributes describing an economy sector and a continental region. $D = \{Level\}$ is a set of decision attributes, D consists of one attribute evaluating the development level. $OG_{Sector} = (\mathcal{C}_S, E_S, \mathcal{R}, \rho_S)$ is an ontological graph associated with the attribute $Sector$ (see Fig. 1). $OG_{Region} = (\mathcal{C}_R, E_R, \mathcal{R}, \rho_R)$ is an ontological graph associated with the attribute $Region$ (see Fig. 2). As mentioned earlier, only hyperonymy is taken into consideration, i.e., $\mathcal{R} = \{R_{\triangleright}\}$. $V_d = \{Low, Medium, High\}$ is a set of decision values. f_c is an information function and f_d is a decision function, both defined in the tabular form in Table 1.

Obviously, ontological graphs used in this example are simplified in comparison to ontological graphs expressing real-world relations between concepts.

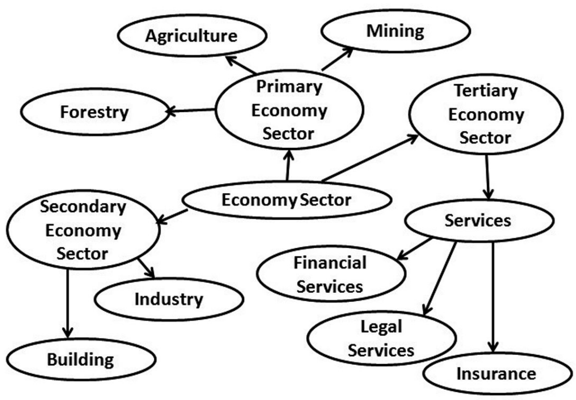

Fig. 1. Ontological graph OG_{Sector} associated with the attribute $Sector$.

♦

Table 1. Simple decision system over ontological graphs.

$U/C \cup D$	*Sector*	*Region*	*Level*
u_1	*Forestry*	*Northern America*	*High*
u_2	*Forestry*	*Caribbean*	*Medium*
u_3	*Forestry*	*Latin America*	*Medium*
u_4	*Forestry*	*Middle East*	*Low*
u_5	*Mining*	*Middle East*	*High*
u_6	*Financial Services*	*Northern America*	*High*
u_7	*Legal Services*	*Northern America*	*High*
u_8	*Insurance*	*Northern America*	*High*
u_9	*Financial Services*	*Latin America*	*Medium*
u_{10}	*Legal Services*	*Latin America*	*Medium*
u_{11}	*Insurance*	*Latin America*	*Medium*
u_{12}	*Industry*	*Far East*	*High*
u_{13}	*Industry*	*Asia Pacific*	*Medium*
u_{14}	*Industry*	*Middle East*	*Medium*
u_{15}	*Mining*	*Far East*	*Low*

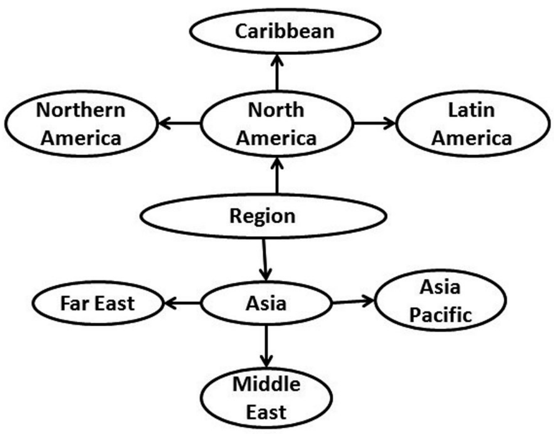

Fig. 2. Ontological graph OG_{Region} associated with the attribute $Region$.

Let $OG = (\mathcal{C}, E, \mathcal{R}, \rho)$ be an ontological graph. In further definitions, we will use the following notation: $[c_i, c_j]$ is a simple path in OG between $c_i, c_j \in \mathcal{C}$, $\mathcal{E}([c_i, c_j])$ is a set of all edges from E belonging to the simple path $[c_i, c_j]$, and $\mathcal{P}(OG)$ is a set of all simple paths in OG. In the literature, there are different definitions for a simple path in the graph. In this paper, we follow the definition in which a path is simple if no node or edge is repeated, with the possible exception that the first node is the same as the last. Therefore, the path $[c_i, c_j]$, where $c_i, c_j \in \mathcal{C}$ and $c_i = c_j$ can also be a simple path in OG.

Definition 3. Given an ontological graph $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$ associated with the attribute a in a simple decision system, where $\mathcal{R} = \{R_{\triangleright}\}$ and $v \in \mathcal{C}_a$ a *hyperonymous meaning relation* $HprMR_a^v$ is a set of all pairs $(c_1, c_2) \in \mathcal{C}_a \times \mathcal{C}_a$ satisfying the following conditions:

$$\underset{[v,c_1]\in\mathcal{P}(OG_a)}{\exists}\left(\underset{e\in\mathcal{E}([v,c_1])}{\forall}\rho(e) \in \{R_{\triangleright}\}\right)$$

and

$$\underset{[v,c_2]\in\mathcal{P}(OG_a)}{\exists}\left(\underset{e\in\mathcal{E}([v,c_2])}{\forall}\rho(e) \in \{R_{\triangleright}\}\right).$$

If two concepts are in the hyperonymous meaning relation, then in a more general meaning, they can be treated as the same concept, for example, car and bus are, in a more general meaning, $vehicle$.

3. Dominance-based rough set approach in simple decision systems over ontological graphs

Pancerz (2012a) proposed to consider simple decision systems over ontological graphs in terms of the dominance-based rough set approach (Greco *et al.*, 2001). It seems to be natural because hyperonymy can be considered in terms of dominance relations.

Let an ontological graph $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$ be associated with the attribute a in a simple decision system and $c_1, c_2 \in \mathcal{C}_a$. It is said that c_1 dominates c_2, written as $D^{\geq}(c_1, c_2)$, if c_2 is a hyperonym of c_1 (or, in other words, c_1 is a hyponym of c_2).

Let an ontological graph $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$ be associated with the attribute a in a simple decision system and $c_1, c_2 \in \mathcal{C}_a$. It is said that c_1 is dominated by c_2, written as $D^{\leq}(c_1, c_2)$, if c_1 is a hyperonym of c_2 (or, in other words, c_2 is a hyponym of c_1).

Remark 2. According to the definitions given earlier, we will further denote the hyponymy relation by $D^{\geq}(c_1, c_2)$ and the hyperonymy relation by $D^{\leq}(c_1, c_2)$.

Example 2. (*Domination*) Consider the ontological graph OG_{Region} given in Example 1. For instance, $Middle\ East$ dominates $Asia$, i.e., $D^{\geq}(MiddleEast, Asia)$, whereas $Asia$ is dominated by $Middle\ East$, i.e., $D^{\leq}(Asia, Middle\ East)$. ♦

In the dominance-based rough set approach (Greco *et al.*, 2001), an outranking relation S_q (Roy, 1985) corresponding to a criterion q is used. In this case, $S_q(x, y)$ means that "x is at least as good as y with respect to the criterion q". In our approach, covering hyperonymy, the meaning will be quite similar, i.e., "x is at least y with respect to a given ontological graph OG".

Definition 4. Consider a simple decision system

$$SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$$

over ontological graphs, $a \in C$ and $u \in U$. An a-*dominating set* with respect to u is the set

$$D^{+}_{f_c(a,u)} = \{u' \in U : D^{\geq}(f_c(a, u), f_c(a, u'))\}.$$

An a-*dominated set* with respect to u is a set

$$D^{-}_{f_c(a,u)} = \{u' \in U : D^{\leq}(f_c(a, u), f_c(a, u'))\}.$$

Remark 3. Consider a simple decision system

$$SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$$

over ontological graphs be given, $a \in C$ and $v \in \mathcal{C}_a$ of OG_a. We use the following notation:

- by D_a^{+v} we denote the set

$$\{u \in U : D^{\geq}(f_c(a, u), v)\},$$

  i.e., the set of all objects u in U for which $f_c(a, u)$ dominates v,

- by D_a^{-v} we denote the set

$$\{u \in U : D^{\leq}(f_c(a, u), v)\},$$

  i.e., the set of all objects u in U for which $f_c(a, u)$ is dominated by v.

The a-dominating set with respect to u is a set of all objects from U having concepts assigned to them by the attribute a whose hyperonym is the concept assigned to u by the attribute a according to an ontological graph OG_a. Analogously, the a-dominated set with respect to u is a set of all objects from U having concepts assigned to them by the attribute a which are hyperonyms of the concept assigned to u by the attribute a according to an ontological graph OG_a.

4. Decision rules consistent with the dominance principle

In this section, we use definitions related to the dominance-based rough set approach, given, among other things, by Greco *et al.* (2001), to provide notions for decision rules in a simple decision system over ontological graphs.

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs, $a_d \in D$ and $\mathbf{Cl}_{a_d} = \{Cl_t : t \in T\}$, where $T = \{1, \dots, n\}$, be a set of classes of U determined by a_d, such that $u \in U$ belongs to one and only one class $Cl_t \in \mathbf{Cl}_{a_d}$. Moreover, suppose that we can define a complete preorder, i.e., a strongly complete and transitive binary relation S_{a_d}, for a decision attribute $a_d \in D$ in SDS^{OG}. For $r, s \in T$, $r > s$ means that each element of Cl_r is preferred (strictly or weakly) to each element of Cl_s. For $u, v \in U$, $(u, v) \in S_{a_d}$ means "u is at least as good as v".

For a family $\mathbf{Cl}_{a_d}$ of classes, we define an upward union of classes:

$$Cl_t^{\geq} = \bigcup_{s\geq t} Cl_s,$$

where $Cl_t, Cl_s \in \mathbf{Cl}_{a_d}$.

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs, $a \in C$ and $Cl_t^{\geq}$ be an upward union of classes determined by a given decision attribute from D. We define

- the a-lower approximation of $Cl_t^{\geq}$:

$$\underline{a}(Cl_t^{\geq}) = \{u \in U : D_a^{+}(u) \subseteq Cl_t^{\geq}\},$$

- the a-upper approximation of $Cl_t^{\geq}$:

$$\overline{a}(Cl_t^{\geq}) = \bigcup_{u\in Cl_t^{\geq}} D_a^{+}(u),$$

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs. Let $\mathcal{C} = \bigcup_{a\in C} \mathcal{C}_a$, where $\mathcal{C}_a$ is a set of concepts from the graph OG_a associated with a given $a \in C$. For the decision system SDS^{OG}, we define

- condition descriptors, which are expressions $(a, v)^{\geq}$ over C and $\mathcal{C}$, where $a \in C$ and $v \in \mathcal{C}$, read as "a is at least v" according to OG_a,

- decision descriptors, which are expressions $(a, v)^{\geq}$ over D and V_d, where $a \in D$ and $v \in V_d$, read as "a is at least v" according to a complete preorder defined for a.

In a given simple decision system

$$SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, f_c, f_d)$$

over ontological graphs, we will consider a $D_{\geq}$-decision rule in the form

$$(a_{c_1}, v_{c_1})^{\geq} \wedge (a_{c_2}, v_{c_2})^{\geq} \wedge \cdots \wedge (a_{c_k}, v_{c_k})^{\geq} \Rightarrow (a_d, v_d)^{\geq},$$

where $a_{c_1}, a_{c_2}, \ldots, a_{c_k} \in C$, $v_{c_1} \in \mathcal{C}_{a_{c_1}}$ of $OG_{a_{c_1}}$, $v_{c_2} \in \mathcal{C}_{a_{c_2}}$ of $OG_{a_{c_2}}$, ..., $v_{c_k} \in \mathcal{C}_{a_{c_k}}$ of $OG_{a_{c_k}}$, $a_d \in D$, $v_d \in V_d$ and $\phi_1 \wedge \phi_2 \wedge \cdots \wedge \phi_k \Rightarrow \psi$ is read: "if ϕ_1 and ϕ_2 and ... and ϕ_k, then ψ".

The above decision rule can be read in the following way: "if a_{c_1} is at least v_{c_1} and a_{c_2} is at least v_{c_2} and ... and a_{c_k} is at least v_{c_k}, then a_d is at least v_d". This rule is true (valid, certain) in SDS^{OG} if and only if

$$D^{+v_{c_1}}_{a_{c_1}} \cap D^{+v_{c_2}}_{a_{c_2}} \cap \cdots \cap D^{+v_{c_k}}_{a_{c_k}} \subseteq Cl^{\geq}_{v_d}$$

and

$$D^{+v_{c_1}}_{a_{c_1}} \cap D^{+v_{c_2}}_{a_{c_2}} \cap \cdots \cap D^{+v_{c_k}}_{a_{c_k}} \neq \emptyset,$$

where Cl_{v_d} denotes the class of objects $u \in U$ such that $f_d(a_d, u) = v_d$.

Remark 4. If, in the above rule, $k = 1$, then the rule is called a $D_{\geq}$-elementary decision rule, i.e., it has the form

$$(a_c, v_c)^{\geq} \Rightarrow (a_d, v_d)^{\geq},$$

where $a_c \in C$, $v_c \in \mathcal{C}_{a_c}$ of OG_{a_c}, $a_d \in D$, $v_d \in V_d$.

Investigations into $D_{\geq}$-elementary decision rules in simple decision systems over ontological graphs were carried out by Pancerz (2012a).

From rough set theory, we know that lower approximations generate rules true in decision systems. In our case, each nonempty B-lower approximation of $Cl^{\geq}_t$ generates $D_{\geq}$-elementary decision rules true in a given decision system, where $B = \{a_{c_1}, a_{c_2}, \ldots, a_{c_k}\}$.

5. Generation of decision rules consistent with the dominance principle

Pancerz (2013a) extended notions related to decision rules in classic decision systems, given by Pawlak (1991), to analogous notions for simple decision systems over ontological graphs. Another look, this time based on the dominance-based rough set approach (DRSA) (Greco *et al.*, 2001), at decision rules for simple decision systems over ontological graphs was presented also by Pancerz (2012a). However, in the second case, we considered only the most general elementary rules true in a given simple decision system over ontological graphs.

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs. An elementary rule $(a_c, v_c)^{\geq} \Rightarrow (a_d, v_d)^{\geq}$, where $a_c \in C$, $v_c \in \mathcal{C}_{a_c}$ of OG_{a_c}, $a_d \in D$, $v_d \in V_d$, is said to be the most general rule with respect to its condition part for a fixed decision part $(a_d, v_d)^{\geq}$ if and only if the rule $(a_c, v_c)^{\geq} \Rightarrow (a_d, v_d)^{\geq}$ is true in SDS^{OG}, but the rule $(a_c, v'_c)^{\geq} \Rightarrow (a_d, v_d)^{\geq}$, where v'_c is a hyperonym of v_c according to OG_{a_c}, is not true in SDS^{OG}.

In this paper, we consider a more general case, i.e., decision rules have the form mentioned in Section 4. Such a situation complicates the problem of extraction of rules, consistent with the dominance principle, from simple decision systems over ontological graphs. In the remaining part of this section, we propose some heuristic algorithm (see Algorithm 1) using the ant-based clustering process for extraction of decision rules true in a given simple decision system over ontological graphs. At the beginning, we start with some auxiliary notions used in the ant-based algorithm.

Let

$$r : (a_{c_1}, v_{c_1})^{\geq} \wedge (a_{c_2}, v_{c_2})^{\geq} \wedge \cdots \wedge (a_{c_k}, v_{c_k})^{\geq} \Rightarrow (a_d, v_d)^{\geq}$$

be a $D_{\geq}$-decision rule and

$$U^{\not\geq}_r = (D^{+v_{c_1}}_{a_{c_1}} \cap D^{+v_{c_2}}_{a_{c_2}} \cap \cdots \cap D^{+v_{c_k}}_{a_{c_k}}) - Cl^{\geq}_{v_d},$$

$$U^c_r = D^{+v_{c_1}}_{a_{c_1}} \cap D^{+v_{c_2}}_{a_{c_2}} \cap \cdots \cap D^{+v_{c_k}}_{a_{c_k}}.$$

For the rule r, we define its accuracy $acc(r)$ as

$$acc(r) = \frac{card(U^c_r) - card(U^{\not\geq}_r)}{card(U^c_r)},$$

where $card(X)$ denotes the cardinality of X.

It is easy to see that if r is true in a given simple decision system over ontological graphs, then $U^{\not\geq}_r = \emptyset$ and $acc(r) = 1$.

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs, where $D = \{a_d\}$. For each ontological graph $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$, where $a \in C$, for each concept (attribute value) $v \in \mathcal{C}_a$, we define the goodness δ_v of v for a fixed $v_d \in V_d$ as

$$\delta_v = \frac{card(D^{+v}_a) - card(D^{+v}_a - Cl^{\geq}_{v_d})}{card(D^{+v}_a)}. \tag{1}$$

It is easy to see that the goodness of a given concept v is equal to 1 with respect to a fixed $v_d \in V_d$ if and only if a $D_{\geq}$-elementary decision rule

$$(a, v)^{\geq} \Rightarrow (a_d, v_d)^{\geq}$$

is true in SDS^{OG}.

Example 3. (*Elementary rules*) Let us consider the simple decision system over ontological graphs SDS^{OG} given in Example 1. For a fixed decision part in the form $Cl^{\geq}_{Medium}$, the concepts from ontological graphs OG_{Sector} and OG_{Region} with the goodness equal to 1 are marked in gray in Figs. 3 and 4, respectively.

According to Figs. 3 and 4, we obtain the following $D_{\geq}$-elementary decision rules true in SDS^{OG}:

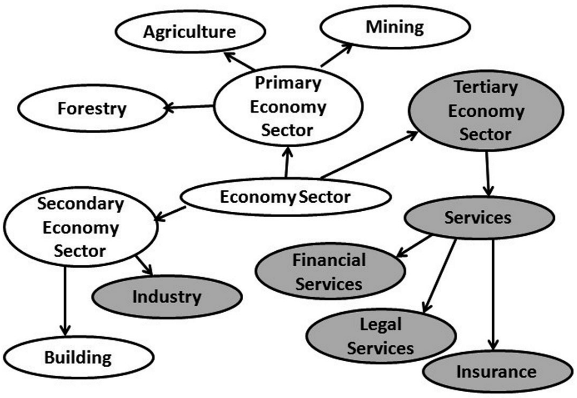

Fig. 3. Concepts with the goodness equal to 1 (marked in gray) in ontological graph OG_{Sector}.

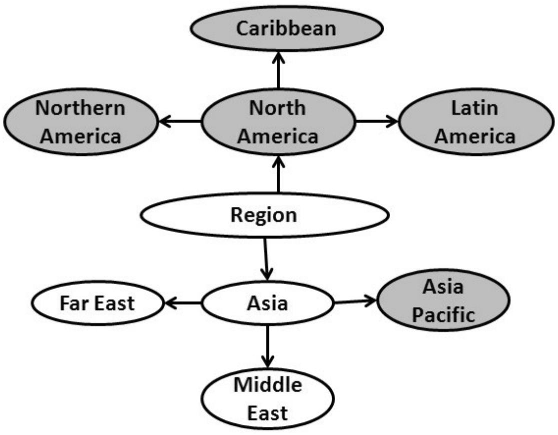

Fig. 4. Concepts with the goodness equal to 1 (marked in gray) in ontological graph OG_{Region}.

- $(Sector, Tertiary\ Economy\ Sector)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Services)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Financial\ Services)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Legal\ Services)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Insurance)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Industry)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, North\ America)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, Northern\ America)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, Caribbean)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, Latin\ America)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, Asia\ Pacific)^{\geq} \Rightarrow (Level, Medium)^{\geq}$.

It is worth noting that we can select, from the above $D_{\geq}$-elementary decision rules, the most general ones:

- $(Sector, Tertiary\ Economy\ Sector)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Sector, Industry)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, North\ America)^{\geq} \Rightarrow (Level, Medium)^{\geq}$,
- $(Region, Asia\ Pacific)^{\geq} \Rightarrow (Level, Medium)^{\geq}$.

♦

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d)$ be a simple decision system over ontological graphs. Analogously to Pancerz (2012a), we are interested in mining decision rules, in a given simple decision system over ontological graphs, with respect to their condition parts for fixed decision parts $(a_d, v_d)^{\geq}$. In the formal description of Algorithm 1, we can distinguish two main parts. In the first one, goodness for all concepts occurring in ontological graphs associated with condition attributes in SDS^{OG} is calculated. Next, all concepts with the goodness equal to 1 are extracted from the graph because they generate elementary decision rules true in SDS^{OG}. These elementary rules are recorded. The remaining concepts participate in creating multi-condition descriptor decision rules in the second part of the algorithm. This part has a heuristic character utilizing the ant-based clustering process. The algorithm is mainly based on those proposed earlier by Deneubourg *et al.* (1991), Lumer and Faieta (1994) as well as Handl *et al.* (2006). Formulas for picking and dropping decisions are proposed to adjust the process to our specific problem. The aim of the ant-based clustering process is to join together descriptors including concepts from ontological graphs to build decision rules with complex condition parts, i.e., consisting of descriptors linked by the *and* logical connectives, but possibly with the smallest numbers of descriptors. Such rules are called minimal. The problem of extraction of minimal rules has been earlier considered in the literature (e.g., Fernández *et al.*, 2001; Skowron and Rauszer, 1992).

The algorithm has the polynomial complexity. Let $Descr(p)$ denote the set of descriptors occupying a given place p because, in our approach, a given place p can be occupied by more than one object, i.e., some heaps can be created. Picking and dropping decisions for

Algorithm 1. Mining decision rules in SDS^{OG}.

Require: $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, f_c, f_d)$: a simple decision system over ontological graphs, $v_d \in V_d$ – a fixed decision value of the attribute $a_d \in D$

$Rul(SDS^{OG}) \longleftarrow \emptyset$;
$Descr \longleftarrow \emptyset$;
for each attribute $a \in C$ **do**
    for each concept $v \in \mathcal{C}_a$, where $\mathcal{C}_a$ is a set of concepts from OG_a **do**
        Calculate the goodness δ_v of v according to Eqn. (1);
        if $\delta_v = 1$ **then**
            Create a rule $r : (a, v)^{\geq} \Rightarrow (a_d, v_d)^{\geq}$;
            $Rul(SDS^{OG}) \longleftarrow Rul(SDS^{OG}) \cup \{r\}$;
        else
            Place (a, v) randomly on a grid G;
            $Descr \longleftarrow Descr \cup \{(a, v)\}$;
        end if
    end for
end for
for each $ant_j \in Ants$ **do**
    Place ant_j randomly on a grid place occupied by one of descriptors from $Descr$;
    Set ant_j as not loaded;
end for
for $k \in \{1, \ldots, N\}$ **do**
    for each $ant_j \in Ants$ **do**
        if ant_j is not loaded **then**
            Select randomly one (a, v) dropped in a place indicated by ant_j;
            Draw a random real number $r \in [0, 1]$;
            if $r \leq p_{pick}(a, v)$ **then**
                set (a, v) as picked;
                set ant_j as carrying the descriptor (a, v);
                move ant_j randomly to another place occupied by one of descriptors from $Descr$;
            else
                move ant_j randomly to another place occupied by one of descriptors from $Descr$;
            end if
        else
            Draw a random real number $r \in [0, 1]$;
            if $r \leq p_{drop}(a, v)$, where (a, v) is carried by ant_j, and there is no descriptor (a', v') in a place indicated by ant_j such that $a = a'$ **then**
                move (a, v) to a place indicated by ant_j;
                set (a, v) as dropped;
                set ant_j as not loaded;
            end if
        end if
    end for
end for
for each non-empty place p of a grid G **do**
    if $acc(r_{Descr(p)}) = 1$ **then**
        $Rul(SDS^{OG}) \longleftarrow Rul(SDS^{OG}) \cup \{r_{Descr(p)}\}$;
    end if
end for
return $Rul(SDS^{OG})$

the descriptor (a, v) can be formally expressed by the following formulas:

$$p_{pick}(a, v) = \begin{cases} 0 & \text{if } acc_r = 1, \\ 1 - acc_{r'} & \text{otherwise,} \end{cases} \quad (2)$$

and

$$p_{drop}(a, v) = \begin{cases} 0 & \text{if } acc_r = 1, \\ acc_{r'} & \text{otherwise,} \end{cases} \quad (3)$$

where

- r is a rule built on descriptors from $Descr(p)$, i.e.,

$$r : (a_{c_1}, v_{c_1})^{\geq} \wedge (a_{c_2}, v_{c_2})^{\geq} \wedge \cdots \wedge (a_{c_k}, v_{c_k})^{\geq} \Rightarrow (a_d, v_d)^{\geq},$$

- r' is a rule built on descriptors from $Descr(p)$ and (a, v), i.e.,

$$r' : (a_{c_1}, v_{c_1})^{\geq} \wedge (a_{c_2}, v_{c_2})^{\geq} \wedge \cdots \wedge (a_{c_k}, v_{c_k})^{\geq} \wedge (a, v)^{\geq} \Rightarrow (a_d, v_d)^{\geq},$$

- $Descr(p) = \{(a_{c_1}, v_{c_1}), (a_{c_2}, v_{c_2}), \ldots, (a_{c_k}, v_{c_k})\}$.

It is easy to see that, if a rule built on descriptors occupying a given place p has the accuracy equal to 1, then another descriptor is neither picked up nor dropped at this place. This enables us to obtain true decision rules with a minimal number of descriptors on the left-hand sides of the rules.

Remark 5. In Algorithm 1, $r_{Descr(p)}$ denotes a rule built on descriptors from $Descr(p)$, i.e., if $Descr(p) = \{(a_{c_1}, v_{c_1}), (a_{c_2}, v_{c_2}), \ldots, (a_{c_k}, v_{c_k})\}$, then

$$r_{Descr(p)} : (a_{c_1}, v_{c_1})^{\geq} \wedge (a_{c_2}, v_{c_2})^{\geq} \wedge \ldots \wedge (a_{c_k}, v_{c_k})^{\geq} \Rightarrow (a_d, v_d)^{\geq}.$$

Example 4. (*Rules*) Let us consider the simple decision system over ontological graphs SDS^{OG} given in Example 1 and concepts with the goodness equal to 1 determined in Example 3. For mining multi-descriptor decision rules true in SDS^{OG}, we use the ant clustering process with the initial set of descriptors (the remaining descriptors, after removing those with the goodness equal to 1, from ontological graphs OG_{Sector} and OG_{Region}, i.e., those marked in gray in Figs. 3 and 4, respectively). After execution of the second part of Algorithm 1, we obtain, for example, the following $D_{\geq}$-decision rule true in SDS^{OG}, for fixed $(Level, High)^{\geq}$:

$$(Sector, Mining)^{\geq} \wedge (Region, Middle\ East) \Rightarrow (Level, High)^{\geq}.$$

♦

6. Conclusions and further work

In the paper, we have proposed a heuristic algorithm based on the ant clustering for mining decision rules consistent with the dominance-based rough set approach in simple decision systems over ontological graphs. The algorithm consists of two stages. At the first stage, the algorithm is, in fact, deterministic and it extracts all $D_{\geq}$-elementary decision rules. At the second stage, the algorithm is properly heuristic and it extracts the $D_{\geq}$-decision rules, this time, with multiple descriptors on the left-hand sides of the rules. One can see in the paper that the presented approach refers to computations proposed years ago by Zadeh (1996) and called "computing with words". In further work, we will examine more complicated picking and dropping decision formulas enabling us to obtain decision rules, as general as possible, and we will consider another heuristics (e.g., genetic algorithms) for mining decision rules in simple decision systems over ontological graphs, and compare the quality of decision rules on real-life data.

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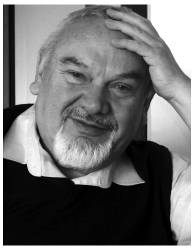

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