Beyzanur Cayir Ervural*, Bilal Ervural* and Özgür Kabak*

Evaluation of Flexible Manufacturing Systems Using a Hesitant Group Decision Making Approach

DOI 10.1515/jisys-2017-0065
Received February 27, 2017; previously published online July 14, 2017.

Abstract: Flexible manufacturing systems (FMS) are capable of processing various parts, styles, and quantities of production in manufacturing systems. It is a quite complex process for companies to decide the appropriate FMS design as it involves multiple and conflicting criteria and multiple decision makers under various uncertainties. The fuzzy set theory offers an efficient tool to cope with vagueness and to define performance measurement of FMS in a multi-attribute group decision making (MAGDM) framework. In this study, we present a MAGDM approach based on hesitant fuzzy sets to evaluate FMS in a fuzzy environment. A practical example is provided to demonstrate the proposed methodology. In addition, the performance of the method is assessed by a comparative study and sensitivity analysis. The results of the analysis show that the MAGDM approach is a useful tool for experts in terms of evaluation of FMS.

Keywords: Fuzzy set theory, decision making, hesitant fuzzy sets, flexible manufacturing systems.

1 Introduction

In today's global market, there is a great competition between companies for producing the highest quality products with the lowest cost and having the best customer satisfaction. In this highly competitive environment, the survival of a company depends on its ability to tackle variations and adapt to changing conditions. Hence, developing new strategies in order to provide high quality customer service is significant to survive in the global market. In this respect, flexible manufacturing systems (FMS) are capable of processing various parts, styles, and quantities of production in a manufacturing system [23]. FMS is easily adapted to changing market conditions and gives some advantages in terms of efficient and quick response to customer needs. FMS is a manufacturing technology; most companies follow this strategy to be flexible in their operations, and then they can satisfy various market segments effectively. The components of the FMS are workstations, material handling and storage systems, computer control system, and system control operators. Flexibility in manufacturing means the ability of producinghigh quality products with low cost considering restricted delivery times. Browne et al. [3]'s study is one of the earliest works that define flexibility in FMS with eight subclasses such as machine flexibility, product flexibility, process flexibility, production flexibility, volume flexibility etc., to show an overall system flexibility.

How to determine the optimal FMS among a number of feasible alternatives is a critical issue. Selecting a suitable FMS is important for manufacturing companies when making capital investment decisions to improve their manufacturing performance [5]. To increase the manufacturing flexibility, manufacturing companies are looking at FMS as a practicable alternative to enhance their competitive superiority [8].

***Corresponding authors: Beyzanur Cayir Ervural, Bilal Ervural, and Özgür Kabak,** Istanbul Technical University, Industrial Engineering Department, Macka, Istanbul, Turkey, Phone: +90 212 2931300/2039, e-mail: cayirb@itu.edu.tr. https://orcid.org/0000-0002-0861-052X (B. C. Ervural); bervural@itu.edu.tr. https://orcid.org/0000-0002-5206-7632 (B. Ervural); kabak@itu.edu.tr, https://orcid.org/0000-0002-5542-309X (Ö. Kabak)

In the literature, manufacturing technology attributes are divided into two subclasses: objective and subjective [15, 21, 27]. Objective attributes are the quantitative effects of manufacturing technology and are determined using some numerical scales. On the other hand, subjective attributes are defined by decision makers (DMs) using different linguistic scales, and the nature of these attributes is defined as qualitative [5].

The analytic hierarchy process (AHP) is a mostly preferred method to evaluate advanced manufacturing technology. Wabalickis [33] proposed a justification methodology based on AHP to evaluate FMS investment. Stam and Kuula [28] developed an AHP-based multi-objective programming model to choose an FMS. AHP is utilized firstly by combining objective criteria and subjective criteria, then the multi-objective mathematical programming is used to find the best option. Shang and Sueyoshi [26] introduced a decision making procedure in FMS employing AHP, simulation, and data envelopment analysis (DEA). Although AHP is mostly used for FMS selection problems owing to its simplicity, it has some drawbacks such as difficulties when the number of criteria/alternatives increase rank, reversal problems, and inappropriateness of the crisp ratio representation [2]. In Bayazit [1]'s study AHP is used for the decision by a tractor manufacturing plant to implement FMS. Yurdakul [38] utilized a combination of AHP and goal programming for selection of computer-integrated manufacturing technologies. Jain and Raj [7] also presented a method ranking of flexibility in FMS by using combined multiple attribute decision making methods, which are AHP and technique for order preference by similarity to ideal solution (TOPSIS). Tseng [32] applied a game theoretical model for a technology selection in FMS. Kulak and Kahraman [16] proposed axiomatic design principles for multiple attribute comparison of advanced manufacturing systems. Rao [22] proposed a decision making model for FMS selection using digraph and matrix methods. Liu [19] presented a DEA/AR (assurance region) approach for selection of FMS. Rao and Parnichkun [24] proposed a methodology based on a combinatorial mathematics-based decision making method for evaluation of alternative FMS. In another study of Rao [23], a procedure is based on a combined multiple attribute decision making method using TOPSIS and AHP methods together that is proposed to evaluate alternative FMS for a given industrial application. Chatterjee and Chakraborty [4] considered the application of six preference ranking methods for selecting the best FMS for a given manufacturing organization. Talebanpour and Javadi [29] presented a decision making model that includes the decision making trial and evaluation laboratory and simple additive weighting method in order to evaluate FMS.

Under ambiguous decision environment, fuzzy set theory is a helpful tool to incorporate subjective attributes like linguistic terms which are mostly based on imprecise judgments. Many researchers have utilized fuzzy multiple attribute decision making (FMADM) models for FMS selection problems. Liang and Wang [18] used the fuzzy set theory for robot selection problem. Karsak [12] considered an integrated framework that includes DEA and fuzzy robot selection algorithm. Perego and Rangone [20] proposed multi-criteria decision making (MCDM) that employs fuzzy set theory for determining advanced manufacturing technologies. Karsak and Tolga [15] proposed an FMADM procedure for evaluating advanced manufacturing system investments which consist of economic and strategic criteria. Karsak and Kuzgunkaya [14] presented a fuzzy multi-objective programming approach to evaluate FMS alternatives under economic and strategic aspects.

FMS has a multi-dimensional framework, and it is difficult to solve such complex systems under different aspects [25]. Most of the FMS evaluation problems include objective and subjective criteria together. Experts' evaluations of the subjective criteria result with imprecise and vague information to be considered. Fuzzy set theory is a very effective tool to make a decision considering imprecise judgment under different criteria in such complex models [14]. Additionally, the fuzzy set theory provides an acceptable way to evaluate qualitative and subjective criteria in unstable conditions like dynamic manufacturing market.

According to Kabak and Ervural [9], fuzzy set theory along with its extensions has been effectively used for solving real life group decision making problems, and especially the use of the hesitant and intuitionistic fuzzy sets is an open future study area. Although fuzzy set theory in general exhibits a satisfactory performance to deal with vague and incomplete data, ordinary fuzzy sets may be insufficient to achieve the expected results in different ambiguous situations. Hence, several extensions of fuzzy sets have been developed to overcome this shortcoming. Recently, a novel extension of fuzzy sets, hesitant fuzzy sets (HFS), has been introduced by Torra [30], which allows the membership degree of an element to a set to be represented by several possible values. Besides, HFS received growing interest among most scholars nowadays since it reflects the human's hesitancy more objectively [17].

Due to its complex nature, FMS evaluation problem also includes some hesitation in the judgments of the experts. Therefore; HFS could provide good results to handle vagueness caused by hesitation in FMS evaluation problem. In this study, we aim to deal with the assessment of the suitability of FMS technology by utilizing HFS owing to some of its advantages, as mentioned above. According to our best knowledge, there has been no work using HFS to solve this complex multi-dimensional FMS evaluation problem in the fuzzy environment.

The purpose of this paper is to implement hesitant fuzzy sets in the selection of the most appropriate alternative for an FMS in a MCDM framework. The rest of this paper is organized as follows. Section 2 presents hesitant fuzzy sets concept and a fuzzy MCDM approach for selecting a more suitable flexible manufacturing technology. Section 3 provides an illustrative example. The results of the study are discussed with comparative analysis and sensitivity analysis in Section 4, and finally, some concluding remarks and future research directions are given in Section 5.

2 An MAGDM Approach with Hesitant Fuzzy Information

Many group decision making problems often require the examination of objective criteria as well as subjective criteria [6]. Therefore, methods to solve these problems should be capable of integrating the objective criteria scores and subjective evaluations of experts. Moreover, the experts may hesitate between several numerical or linguistic values in assessing the criteria. Therefore, in this study, we propose a new FMADM methodology under hesitant fuzzy information capable of handling subjective and objective criteria for the evaluation and selection of FMS.

2.1 Preliminaries

Hesitant fuzzy set [30, 31], as an extension of fuzzy set, permits the membership degree of an element to a set to be represented by a set of possible values between 0 and 1. The hesitant fuzzy sets can better describe the situations where people have hesitancy in providing their preferences over alternatives in decision making process [35]. The main purpose of multiple attribute group decision making (MAGDM) problems is to select the best alternative among a set of feasible candidates, based on the preferences derived by a group of DMs. To combine individual preferences into a collective preference for each alternative, aggregation operators are mostly used [39]. In general, the weight vectors of the aggregation operators are assumed not reflecting the correlation of the aggregated opinions. To obtain the weight vector more objectively, Yager [37] developed the power average to provide an aggregation operator which allows preferences to support each other in the aggregation process, then Xu and Yager [36] presented a power geometric operator and its weighted form. In the studies of Zhang [39] and Xia et al. [35], the power aggregation operators to hesitant fuzzy environments are extended.

Definition ([30, 31]): Let X be a reference set; a hesitant fuzzy set A on X is defined in terms of a function $h_A(x)$ that when applied to X returns a subset of [0, 1], i.e.

$$A = \{\langle x, h_A(x)\rangle \mid x \in X\} \tag{1}$$

where $h_A(x)$ is a set of some different values in [0, 1], representing the possible membership degrees of the element $x \in X$ to A.

2.2 Problem Definition

The MAGDM under hesitant fuzzy information can be formulated as follows: Let $a = \{a_1, a_2, \ldots, a_m\}$ be a set of alternatives, $c^1 = \{c_1, c_2, \ldots, c_n\}$ the set of subjective criteria, $c^2 = \{c_{n+1}, c_{n+2}, \ldots, c_s\}$ the set of objective criteria, and $e=\{e_1, e_2, \ldots, e_l\}$ the set of DMs. Criteria are considered both objective and subjective criteria. The DM e_k evaluate the alternatives with respect to subjective criteria c_j and construct hesitant fuzzy decision matrices,

$D^{(k)}=(h_{ij}^{(k)})_{m\times n}$ $(k=1, 2,.., l)$, and also $D^0=(x_{ij})$, $(i=1, 2, ...m, j=n+1, n+2, \ldots, s)$ denotes decision matrix for objective criteria.

2.3 Proposed Method

To solve the above defined MAGDM problem, we present an approach for group decision making in a hesitant fuzzy environment based on weighted generalized hesitant fuzzy power average (WGHFPA) aggregation operator developed by Zhang [39]. The proposed approach involves the following steps:

Step 0 Normalization.
Firstly, decision matrix $D^{(k)}=(h_{ij}^{(k)})_{m\times n}$ transforms into a normalized matrix $R^{(k)}=(r_{ij}^{(k)})_{m\times n}$ as follows:

$$r_{ij}^{(k)}=\begin{cases} h_{ij}^{(k)}, \text{ for benefit criteria} \\ (h_{ij}^{(k)})^c, \text{ for cost criteria} \end{cases} \tag{2}$$

where $(h_{ij}^{(k)})^c$ is the complement of $h_{ij}^{(k)}$ $(j=1, 2,\ldots, n)$.

Secondly, the performance ratings for alternatives with respect to each objective criterion are normalized. The normalized values for benefit and cost objective criteria $(j=n+1, n+2,\ldots, s)$ are calculated as follows:

$$r_{ij}=\begin{cases} \dfrac{x_{ij}}{\sum_{i\in a}|x_{ij}|}, \text{ for benefit criteria} \\ \dfrac{1/x_{ij}}{\sum_{i\in a}|1/x_{ij}|}, \text{ for cost criteria} \end{cases} \tag{3}$$

Step 1 Calculate the weighted support for individual decision matrices.
Let $\mathrm{Sup}(r_{ij}^{(k)}, r_{ij}^{(t)})=1-d(r_{ij}^{(k)}, r_{ij}^{(t)})$, where $d(r_{ij}^{(k)}, r_{ij}^{(t)})$ is the hesitant normalized Hamming distance between $r_{ij}^{(k)}$ and $r_{ij}^{(t)}$. Then use the weights λ_k of the DMs e_k $(k=1, 2,\ldots, l)$ to calculate the weighted support $T(r_{ij}^{(k)})$ of hesitant fuzzy element (HFE) $r_{ij}^{(k)}$ by the other HFEs $r_{ij}^{(t)}$ $(i=1, 2,\ldots, m, j=1, 2,\ldots, n, t=1, 2,\ldots, l$ and $t\neq k)$:

$$T(r_{ij}^{(k)})=\sum_{\substack{t=1\\(t\neq k)}}^{l}\lambda_t \,\mathrm{Sup}(r_{ij}^{(k)}, r_{ij}^{(t)}). \tag{4}$$

Then, utilize the weights λ_k of the DMs e_k $(k=1, 2,\ldots, l)$ calculate the weights

$$\xi_{ij}^{(k)}=\frac{\lambda_k(1+T(r_{ij}^{(k)}))}{\sum_{k=1}^{l}\lambda_k(1+T(r_{ij}^{(k)}))}, \quad (k=1, 2, \ldots, l), \tag{5}$$

where $\xi_{ij}^{(k)}\geq 0, (k=1, 2,\ldots, l)$, and $\sum_{k=1}^{l}\xi_{ij}^{(k)}=1$.

Step 2 Calculate the collective decision matrix.
By the WGHFPA operator, aggregate all of the individual hesitant fuzzy decision matrices $R^{(k)}=(r_{ij}^{(k)})_{m\times n}$ $(k=1, 2,\ldots, l)$ into the collective hesitant fuzzy decision matrix $R=(r_{ij})_{m\times n}$.

$$r_{ij}=\mathrm{WGHFPA}(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)})=\bigcup_{\gamma_{ij}^{(1)}\in r_{ij}^{(1)}, \gamma_{ij}^{(2)}\in r_{ij}^{(2)}, \ldots, \gamma_{ij}^{(l)}\in r_{ij}^{(l)}}\left\{\left(1-\prod_{k=1}^{l}(1-(\gamma_{ij}^{(k)})^{\lambda})^{\xi_{ij}^{(k)}}\right)^{1/\lambda}\right\} \tag{6}$$

After the aggregation of individual hesitant fuzzy decision matrices under subjective criteria, the standardized performance ratings obtained by Eq. (3) under objective criteria are added to decision matrix R. Thus, we take overall decision matrix $R=(r_{ij})_{m\times s}$, for all criteria ($i=1, 2,\dots, m$, $j=1, 2,\dots, s$).

Step 3 Calculate the weighted support for collective decision matrix.
Let $\mathrm{Sup}(r_{ij}, r_{ip})=1-d(r_{ij}, r_{ip})$. Then use the weights w_j of criteria c_j ($i=1, 2,\dots, m$, j, $p=1, 2,\dots, s$) to calculate the weighted support $T(r_{ij})$ of HFE r_{ij} by the other HFEs r_{ip} ($p=1, 2,\dots, s$ and $p\neq j$):

$$T(r_{ij})=\sum_{\substack{p=1\\(p\neq j)}}^{s} w_p\ Sup(r_{ij},\ r_{ip}) \tag{7}$$

After that, use the weights w_j of criteria c_j to calculate the weights η_{ij} ($j=1, 2,\dots, s$) that are associated with HFE r_{ij} ($i=1, 2,\dots, m$, $j=1, 2,\dots, s$).

$$\eta_{ij}=\frac{w_j(1+T(r_{ij}))}{\sum_{j=1}^{s} w_j(1+T(r_{ij}))},\quad j=1,\ 2,\ \dots,\ s, \tag{8}$$

where $\eta_{ij}\geq 0$, ($j=1, 2,\dots, s$), and $\sum_{j=1}^{s}\eta_{ij}=1$.

Step 4 Obtain the collective overall preference.
Utilize WGHFPA operator to aggregate all preference values r_{ij} ($j=1, 2,\dots, s$) and then provide the collective overall preference value r_i ($i=1, 2,\dots, m$) of alternative a_i ($i=1, 2,\dots, m$).

$$r_i=\mathrm{WGHFPA}(r_{i1},\ r_{i2},\ \dots,\ r_{is})=\bigcup_{\gamma_{i1}\in r_{i1},\gamma_{i2}\in r_{i2},\dots,\gamma_{is}\in r_{is}}\left\{\left(1-\prod_{j=1}^{s}\ (1-(\gamma_{ij})^{\lambda})^{\eta_{ij}}\right)^{1/\lambda}\right\} \tag{9}$$

Step 5 Rank the alternatives.
Calculate the scores r_i, rank the alternatives according to $s(r_i)$ ($i=1, 2,\dots, m$), and then select the best alternative.

$$s(r_i)=\frac{\sum_{\gamma\in r}\gamma}{\delta(r)}, \tag{10}$$

where $\delta(r)$ is the number of elements in r.

The details of group decision making with hesitant fuzzy information methodology can be seen extensively in [35, 39].

3 An Illustrative Example

In this section, an FMS evaluation problem in a manufacturing company adapted from Karsak [13] and Chuu [5] is used to illustrate the proposed approach.

An expert group was formed to determine the best FMS among three FMS alternatives a_i($i=1, 2, 3$). Three experts e_k($k=1, 2, 3$) form a decision making committee with the weights vector $\lambda=(0.4, 0.3, 0.3)^{\mathrm{T}}$. In our problem, we consider both objective and subjective criteria. Six criteria are under consideration: (1) product flexibility (c_1); (2) product quality (c_2); (3) volume flexibility (c_3); (4) required floor space (m^2) (c_4); (5) investment cost (×\$100,000) (c_5); and (6) lead time (h) (c_6). Among the considered criteria, c_1, c_2, and c_3 are the benefit and subjective criteria, and c_4, c_5, and c_6 are the cost and objective criteria. Importance weight of criteria is c_j($j=1, 2,\dots, 6$), $w=(0.15, 0.20, 0.15, 0.20, 0.20, 0.10)^{\mathrm{T}}$. The experts evaluate the FMS alternatives with respect to subjective criteria c_j ($j=1, 2, 3$) and construct hesitant fuzzy decision matrices, $D^{(k)}=(h_{ij}^{(k)})_{m\times n}$ ($k=1, 2, 3$) as seen in Table 1, where $h_{ij}^{(k)}\in H$ is a HFE that indicates all of the possible values for the alternative with respect

Table 1: Decision Matrices $D^{(k)}$ Provided by e_k.

DMs	Subjective criteria	Performance rating		
		a_1	a_2	a_3
e_1	c_1	{0.5}	{0.8, 0.5}	{0.6}
	c_2	{0.4, 0.3}	{0.5}	{0.7, 0.5}
	c_3	{0.7}	{0.7, 0.5}	{0.5}
e_2	c_1	{0.5, 0.3}	{0.6}	{0.8}
	c_2	{0.6}	{0.8, 0.6}	{0.3}
	c_3	{0.6, 0.4}	{0.5}	{0.7, 0.5}
e_3	c_1	{0.7}	{0.5, 0.4}	{0.9, 0.5}
	c_2	{0.3, 0.1}	{0.4, 0.2}	{0.7}
	c_3	{0.3}	{0.7, 0.4}	{0.6, 0.4}

Table 2: The Performance Ratings for Three Alternatives under Each Objective Criterion.

Objective criteria	Performance rating		
	a_1	a_2	a_3
c_4	500	600	750
c_5	85	110	130
c_6	100	50	35

to criterion c_j. The performance ratings for three alternatives under each objective criterion (D^O) are shown in Table 2.

To select the best FMS alternative, the following steps are given:

Step 0 Normalization.

In this problem, since all of the subjective criteria are benefit criteria, decision matrix $D^{(k)}$ is equal to normalized matrix $R^{(k)}$. Then the normalized values for objective criteria are calculated using Eq. (3) as given in Table 3. For instance, for alternative 1 and criterion 4 normalized value is calculated as follows:

$$r_{1,4} = \frac{1/x_{1,4}}{\sum_{i=1}^{3} |1/x_{i,4}|} = \frac{1/500}{1/500 + 1/600 + 1/750} = 0.400$$

Step 1 Calculate the weighted support for individual decision matrices.

Calculate the supports $\mathrm{Sup}(r_{ij}^{(k)}, r_{ij}^{(t)}) = \mathrm{Sup}^{kt}$, which refers to supports between $R^{(k)}$ and $R^{(t)}$ ($i, j = 1, 2, 3, t, k = 1, 2, 3$and $t \neq k$) (Table 4).

For instance, supports between $R^{(2)}$ and $R^{(3)}$ for alternative 1 and criterion 3 is calculated as follows:

$$\mathrm{Sup}(r_{1,3}^{(2)}, r_{1,3}^{(3)}) = \mathrm{Sup}_{1,3}^{23} = 1 - d(r_{1,3}^{(2)}, r_{1,3}^{(3)}) = 1 - d(\{0.6,\ 0.4\},\ \{0.3\}) = 1 - \frac{|0.6 - 0.3| + |0.4 - 0.3|}{2} = 0.8$$

Table 3: The Normalized Performance Ratings under Each Objective Criterion.

Objective criteria	Performance rating		
	a_1	a_2	a_3
c_4	{0.400}	{0.333}	{0.267}
c_5	{0.412}	{0.318}	{0.269}
c_6	{0.171}	{0.341}	{0.488}

Table 4: Supports Between $R^{(k)}$ and $R^{(t)}$.

	Sup·1			Sup·2			Sup·3		
Sup1·			–	0.9000	0.7500	0.8000	0.8000	0.8500	0.6000
				0.8500	0.8000	0.9000	0.8000	0.8000	0.9500
				0.8000	0.7000	0.9000	0.8000	0.9000	0.9000
Sup2·	0.9000	0.7500	0.8000			–	0.7000	0.6000	0.8000
	0.8500	0.8000	0.9000				0.8500	0.6000	0.8500
	0.8000	0.7000	0.9000				0.8000	0.6000	0.9000
Sup3·	0.8000	0.8500	0.6000	0.7000	0.6000	0.8000			–
	0.8000	0.8000	0.9500	0.8500	0.6000	0.8500			
	0.8000	0.9000	0.9000	0.8000	0.6000	0.9000			

Then utilize Eq. (4) to calculate the weighted support $T(r_{ij}^{(k)})$ of the evaluated value $r_{ij}^{(k)}$ by the other evaluated values $r_{ij}^{(t)}$ ($t=1, 2, 3$ and $t \neq k$). T_k denotes $(T(r_{ij}^{(k)}))_{3\times 3}$ ($k=1, 2, 3$) in Table 5. For instance, for weighted support $T(r_{3,2}^{(2)})$ of the evaluated value $r_{3,2}^{(2)}$ by the other evaluated values $r_{ij}^{(t)}$ is calculated as follows:

$$T(r_{3,2}^{(2)})=\sum_{\substack{t=1\\(t\neq 2)}}^{3}\lambda_t \mathrm{Sup}(r_{3,2}^{(2)},\, r_{3,2}^{(t)})=0.4*0.7000+0.3*0.6000=0.4600$$

After that by Eq. (5), calculate the weights $\xi_{ij}^{(k)}$ ($i, j=1, 2, 3,\ k=1, 2, 3$). V_k denotes $(\xi_{ij}^{(k)})_{3\times 3}$ ($k=1, 2, 3$) in Table 6. For example, weight of expert 1 under alternative 2 and criterion 3 is calculated as follows:

$$\xi_{2,3}^{(1)}=\frac{\lambda_1(1+T(r_{2,3}^{(1)}))}{\sum_{k=1}^{3}\lambda_k(1+T(r_{2,3}^{(k)}))}=\frac{0.4*(1+0.55)}{0.4*(1+0.55)+0.3*(1+0.615)+0.3*(1+0.635)}=0.3895$$

Table 5: The Weighted Supports T_k.

T_1	0.5100	0.4800	0.4200
	0.4950	0.4800	0.5550
	0.4800	0.4800	0.5400
T_2	0.5700	0.4800	0.5600
	0.5950	0.5000	0.6150
	0.5600	0.4600	0.6300
T_3	0.5300	0.5200	0.4800
	0.5750	0.5000	0.6350
	0.5600	0.5400	0.6300

Table 6: The Weights Associated with the Evaluated Values.

V_1	0.3937	0.3968	0.3838
	0.3861	0.3968	0.3895
	0.3874	0.3968	0.3864
V_2	0.3070	0.2976	0.3162
	0.3089	0.3016	0.3034
	0.3063	0.2936	0.3068
V_3	0.2992	0.3056	0.3000
	0.3050	0.3016	0.3071
	0.3063	0.3097	0.3068

Step 2 Calculate the collective decision matrix.
By the WGHFPA operator given in Eq. (6), aggregate all individual hesitant fuzzy decision matrices $R^{(k)}=(r_{ij}^{(k)})_{m\times n}$ $(k=1, 2, 3)$ into the collective hesitant fuzzy decision matrix.

For instance, for alternative 1 and criterion 1 collective hesitant value $r_{1,1}$ is calculated as follows:

$$r_{1,1}=\text{WGHFPA}(r_{1,1}^{(1)}, r_{1,1}^{(2)})=\bigcup_{\gamma_{1,1}^{(1)}\epsilon r_{1,1}^{(1)},\gamma_{1,1}^{(2)}\epsilon r_{1,1}^{(2)}}\{(1-((1-0.5^2)^{0.3937})*((1-0.5^2)^{0.307})*((1-0.7^2)^{0.2992}))^{1/2},$$
$$(1-((1-0.5^2)^{0.3937})*((1-0.3^2)^{0.307})*((1-0.7^2)^{0.2992}))^{1/2}\}=\{0.5760,\ 0.5393\}$$

In this step, the standardized performance ratings obtained by Eq. (2) under each objective criterion are added to decision matrix $R=(r_{ij})m\times s$. The collective hesitant fuzzy decision matrix is shown in Table 7.

Step 3 Calculate the weighted support for collective decision matrix.
Calculate the supports $\text{Sup}(r_{ij}, r_{ip})=\text{Sup}_{jp}$, which refer to supports between rows of collective decision matrix R (Table 8). For instance, support between $R^{(1)}$ and $R^{(6)}$ for alternative 1 is calculated as follows:

$$\text{Sup}(r_{1,1}, r_{1,6})=\text{Sup}_{1,6}=1-\text{d}(r_{1,1}, r_{1,6})=1-\text{d}(\{0.5760,\ 0.5393\},\ \{0.171\})=1-\frac{|0.5760-0.171|+|0.5393-0.171|}{2}$$
$$=0.6131$$

Table 7: The Collective Hesitant Fuzzy Decision Matrix *R*.

	c_1	c_2	c_3	c_4	c_5	c_6
a_1	{0.5760, 0.5393}	{0.4540, 0.4306, 0.4248, 0.3989}	{0.5900, 0.5381}	{0.400}	{0.412}	{0.171}
a_2	{0.6798, 0.6657, 0.5347, 0.5106}	{0.6148, 0.5937, 0.5101, 0.4793}	{0.6532, 0.5760, 0.5777 0.4727}	{0.333}	{0.318}	{0.341}
a_3	{0.7938, 0.6608}	{06289, 0.5436}	{0.6044, 0.5569, 0.5344, 0.4727}	{0.267}	{0.269}	{0.488}

Table 8: Supports between Rows of *R*.

	Sup $_{.1}$	Sup $_{.2}$	Sup $_{.3}$	Sup $_{.4}$	Sup $_{.5}$	Sup $_{.6}$
Sup $_{1.}$	–	0.8786	0.9924	0.8424	0.8545	0.6131
		0.9518	0.9507	0.6606	0.6457	0.6687
		0.8590	0.8480	0.5394	0.5421	0.7605
Sup $_{2.}$	0.8786	–	0.8760	0.9724	0.9784	0.7437
	0.9518		0.9674	0.7838	0.7690	0.7920
	0.8590		0.9705	0.6804	0.6832	0.9015
Sup $_{3.}$	0.9924	0.8760	–	0.8360	0.8481	0.6067
	0.9507	0.9674		0.7634	0.7485	0.7716
	0.8480	0.9705		0.7246	0.7273	0.9381
Sup $_{4.}$	0.8424	0.9724	0.8360	–	0.9879	0.7707
	0.6606	0.7838	0.7634		0.9851	0.9919
	0.5394	0.6804	0.7246		0.9972	0.7789
Sup $_{5.}$	0.8545	0.9784	0.8481	0.9879	–	0.7586
	0.6457	0.7690	0.7485	0.9851		0.9770
	0.5421	0.6832	0.7273	0.9972		0.7816
Sup $_{6.}$	0.6131	0.7437	0.6067	0.7707	0.7586	–
	0.6687	0.7920	0.7716	0.9919	0.9770	
	0.7605	0.9015	0.9381	0.7789	0.7816	

Then utilize Eq. (7) to calculate the weighted support $T(r_{ij})$ of the evaluated value r_{ij} by the other evaluated values r_{ip} ($p=1, 2,\ldots, 6$ and $p \neq j$). T denotes $(T(r_{ij}))_{3\times 6}$ in Table 9. For instance, for weighted support $T_{(3,6)}$ of the evaluated value $r_{3,6}$ by the other evaluated values r_{ip} is calculated as follows:

$$T(r_{3,6}) = \sum_{\substack{p=1 \\ (p\neq 6)}}^{6} \mathrm{w}_p \ \mathrm{Sup}(r_{3,6}, r_{3,p}) = 0.15*0.7605+0.2*0.9015+0.15*0.9381+0.2*0.7789+0.2*0.7816=0.7472$$

After that by Eq. (8), calculate the weights η_{ij} ($i=1, 2, 3, j=1, 2,\ldots, 6$). V denotes $(\eta_{ij})_{3\times 6}$ in Table 10. For example, weight of alternative 3 and criterion 1 is calculated as follows:

$$\eta_{3,1} = \frac{\mathrm{w}_1(1+T(r_{3,1}))}{\sum_{j=1}^{6} \mathrm{w}_j(1+T(r_{3,j}))}$$

$$= \frac{0.15*(1+0.591)}{0.15*(1+0.591)+0.2*(1+0.637)+0.15*(1+0.706)+0.2*(1+0.603)+0.2*(1+0.605)+0.1*(1+0.747)}$$

$$=0.1457$$

Step 4 Obtain the collective overall preference.

Utilize the WGHFPA operator given in Eq. (9) to aggregate all of the preference values r_{ij} and then provide the collective overall preference value r_i of the alternatives a_i ($i=1, 2, 3$). It is shown in Table 11. For instance, for alternative 1 collective overall preference value r_1 is calculated as follows:

$$r_1 = \mathrm{WGHFPA}(r_{1,1}, \ \ldots, \ r_{1,s}) = \bigcup_{\gamma_{i1}\epsilon r_{i1}, \ldots, \gamma_{is}\epsilon r_{is}} \{(1-((1-0.576^2)^{0.1509})*((1-0.454^2)^{0.2014})$$

$$*((1-0.59^2)^{0.1505})*((1-0.4^2)^{0.2006})*((1-0.412^2)^{0.2011})*((1-0.171^2)^{0.0955}))^{1/2}, \ldots\} = \{0.468, \ \ldots\}$$

Table 9: The Weighted Supports T.

T	0.7252	0.7277	0.7215	0.7209	0.7245	0.6376
	0.6611	0.6776	0.7156	0.6666	0.6576	0.7682
	0.5914	0.6373	0.7055	0.6030	0.6047	0.7472

Table 10: The Weights of HFEs.

V	0.1509	0.2014	0.1505	0.2006	0.2011	0.0955
	0.1480	0.1993	0.1528	0.1980	0.1969	0.1050
	0.1457	0.1999	0.1562	0.1957	0.1959	0.1067

Table 11: The Collective Overall Preference Value.

a_1	{0.4678, 0.4634, 0.4624, 0.4602, 0.4579, 0.4568, 0.4557, 0.4547, 0.4522, 0.4512, 0.4501, 0.4489, 0.4466, 0.4443, 0.4432, 0.4384}
a_2	{0.5294, 0.5259, 0.5239, 0.5203, 0.5133, 0.513, 0.5096, 0.5093, 0.5075, 0.5072, 0.5049, 0.5037, 0.5034, 0.5011, 0.4994, 0.4989, 0.496, 0.4955, 0.495, 0.4933, 0.4921, 0.4899, 0.4894, 0.4874, 0.4871, 0.4859, 0.4834, 0.483, 0.4816, 0.4813, 0.4811, 0.4807, 0.4775, 0.4773, 0.4771, 0.477, 0.4752, 0.4748, 0.4722, 0.471, 0.4706, 0.4685, 0.468, 0.4655, 0.4642, 0.4623, 0.4618, 0.4612, 0.4584, 0.458, 0.4554, 0.4527, 0.4527, 0.4523, 0.451, 0.4478, 0.4456, 0.4452, 0.4409, 0.4405, 0.4314, 0.4265, 0.4242, 0.4191}
a_3	{0.559, 0.5509, 0.5474, 0.539, 0.5396, 0.5309, 0.5272, 0.5181, 0.5187, 0.5094, 0.5054, 0.4956, 0.4963, 0.4863, 0.482, 0.4714}

Step 5 Rank the alternatives.

Use Eq. (10) to calculate the score functions $s(r_i)$ of r_i, and rank the alternatives according to $s(r_i)$ ($i=1, 2, 3$) and then select the best alternative.

$$s(r_1)=0.453 \quad s(r_2)=0.478 \quad s(r_3)=0.517$$

For a group of DMs, based on score function, the ranking order of three alternatives is given as $a_3 > a_2 > a_1$.

The third alternative (a_3) appears to be the most appropriate FMS alternative as a result of the hesitant fuzzy multi-attribute decision procedure.

4 Comparison and Sensitivity Analysis

In this section, we compared the results of our methodology with Xia and Xu [34]'s method. And in order to show the robustness of the final decision, sensitivity analyses are conducted.

4.1 Comparison with an existing method

To validate the proposed MAGDM method, a comparative study is conducted with the method proposed by Xia and Xu [34]. Notice that in Xia and Xu's method there is no procedure for aggregating objective and subjective criteria. Before applying the steps of Xia and Xu's method, we first conducted a normalization as mentioned in Step 0 of our proposed approach. Then, we utilized the generalized hesitant fuzzy weighted averaging (GHFWA) operator in Xia and Xu [34] to find a final ranking for the illustrative example given in the previous section.

After processing the steps of Xia and Xu [34]'s method, the results given in Table 12 were obtained. It was observed that the ranking order of the alternatives obtained by the method of Xia and Xu is the same as the ranking obtained by our proposed approach. According to the results of our approach, as well as of the Xia and Xu method, a_3 would be suggested as the best option.

4.2 Sensitivity analysis

In order to examination the robustness of the proposed algorithm, it is necessary to estimate the rate of change in the output of a model caused by the changes in the model inputs. In the proposed model, the ranking of alternatives may depend on both criteria and DMs' weights. For this reason, sensitivity analysis was employed by changing the weights of the criteria and DMs separately.

First, a sensitivity analysis was performed by changing the criteria weights. In this analysis, Case 0 refers to the current situation. In each case, the weight of a criterion is decreased or increased by 0.10 and the weights of other criteria are updated proportionally. For instance, in Case 1, the weight of c_1 was decreased by 0.10 (from 0.15 to 0.05), and 0.10 weight is distributed to the other criteria proportionally. The other cases of the analysis are designed, and the related weights are calculated in the same way as given in Table 13.

Table 12: Comparisons of the Scores and Rankings.

	Xia and Xu's method		Proposed method	
	Score	**Rank**	**Score**	**Rank**
a_1	0.435	3rd	0.453	3rd
a_2	0.460	2nd	0.478	2nd
a_3	0.489	1st	0.517	1st

Table 13: Sensitivity Analysis for Criteria Weights.

		c_1	c_2	c_3	c_4	c_5	c_6
Case 0	Current situation	0.15	0.20	0.15	0.20	0.20	0.10
Case 1	c_1 decreased by 0.10	0.05	0.22	0.17	0.22	0.22	0.11
Case 2	c_1 increased by 0.10	0.25	0.18	0.13	0.18	0.18	0.09
Case 3	c_2 decreased by 0.10	0.17	0.10	0.17	0.23	0.23	0.11
Case 4	c_2 increased by 0.10	0.13	0.30	0.13	0.18	0.18	0.09
Case 5	c_3 decreased by 0.10	0.17	0.22	0.05	0.22	0.22	0.11
Case 6	c_3 increased by 0.10	0.13	0.18	0.25	0.18	0.18	0.09
Case 7	c_4 decreased by 0.10	0.17	0.23	0.17	0.10	0.23	0.11
Case 8	c_4 increased by 0.10	0.13	0.18	0.13	0.30	0.18	0.09
Case 9	c_5 decreased by 0.10	0.17	0.23	0.17	0.23	0.10	0.11
Case 10	c_5 increased by 0.10	0.13	0.18	0.13	0.18	0.30	0.09
Case 11	c_6 decreased by 0.10	0.17	0.22	0.17	0.22	0.22	0.00
Case 12	c_6 increased by 0.10	0.13	0.18	0.13	0.18	0.18	0.20

Twelve additional cases are used for analyzing, and the ranks of the alternatives are recalculated. The sensitivity results of ranking alternatives are shown in Figure 1. As a result of the analysis, although there were slight changes in the scores of the alternatives, the final rankings remained the same. These findings obtained by sensitivity analysis at different cases show that the result of the proposed approach in the illustrative example is robust to alterations in the weights of the criteria.

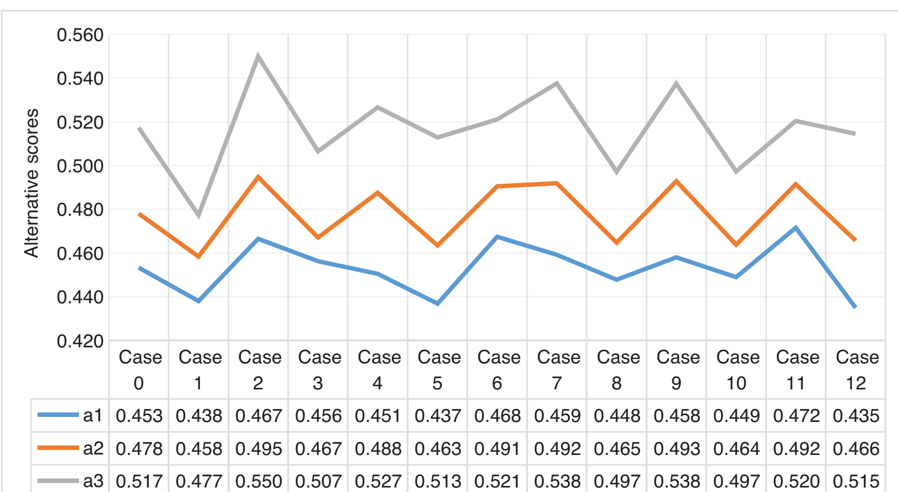
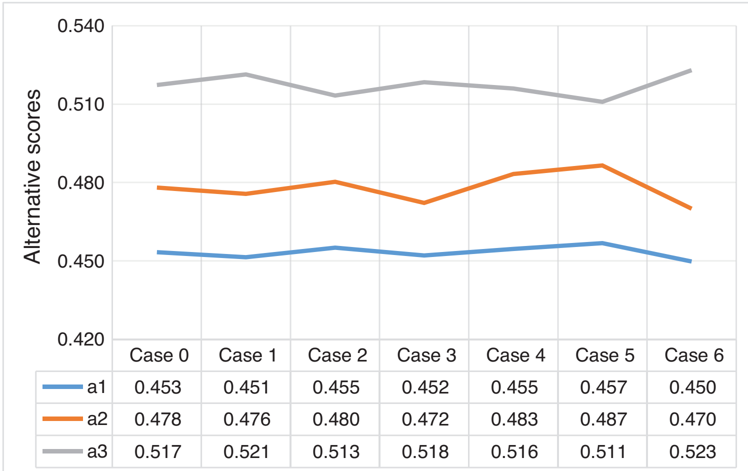


Figure 1: Sensitivity Analysis Results when the Weights of Criteria are Changed.

Table 14: Sensitivity Analysis for Experts' Weights.

		e_1	e_2	e_3
Case 0	Current situation	0.4	0.3	0.3
Case 1	e_1 decreased by 0.10	0.3	0.35	0.35
Case 2	e_1 increased by 0.10	0.5	0.25	0.25
Case 3	e_2 decreased by 0.10	0.46	0.2	0.34
Case 4	e_2 increased by 0.10	0.34	0.4	0.26
Case 5	e_3 decreased by 0.10	0.46	0.34	0.2
Case 6	e_3 increased by 0.10	0.34	0.26	0.4



	Case 0	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
a1	0.453	0.451	0.455	0.452	0.455	0.457	0.450
a2	0.478	0.476	0.480	0.472	0.483	0.487	0.470
a3	0.517	0.521	0.513	0.518	0.516	0.511	0.523

Figure 2: Sensitivity Analysis Results when the Weights of Experts are Changed.

The same process is applied for the experts' weights in order to study the effect on the results. In the analysis, expert weights were also increased and decreased by 0.10, which was proportionally reflected to the weight of the other experts. Table 14 shows the current situation and the corresponding weights of experts for the six cases. As can be seen in Figure 2, although there were minor changes in the scores of the alternatives, the final rankings remained the same.

The sensitivity analysis results presented in Figures 1 and 2 show that the model adopted for analysis is free from any biases and results are robust to changes in the weights of the criteria and experts.

5 Conclusion

One of the most important issues of manufacturing systems is to ensure sustainability and thus respond to customer demands according to expected quality with minimum cost and adapt to market conditions immediately. Under current conditions of competition, FMS has emerged as an important concept to satisfy all expected qualifications. Development of hesitant fuzzy sets is a recent approach to cope with vague and imprecise information in complex systems under the MCDM framework.

In this paper, we have analyzed an FMS in order to determine the most appropriate FMS alternative under the hesitant fuzzy environment. One of the important properties of the proposed method, which distinguishes it from traditional or hesitant versions of classical methods (e.g. AHP, TOPSIS, etc.), is that it can aggregate the objective criteria and subjective evaluations of the experts. The validity of the proposed methodology is presented using an illustrative example and a comparative study. The robustness of the results is tested through a sensitivity analysis. Consequently, the proposed methodology is shown to be effective to deal with problems including objective and subjective criteria that are presented as hesitant fuzzy sets. The results are proven to be robust to the changes of weights related to criteria as well as experts.

For further research, the proposed MAGDM method can be applied to different complex evaluation problems. For simplicity and ease of computation, a decision support software can be developed for the proposed method, and any regular aggregation operator can be used instead of the used power aggregation operator in this study. We also plan to use a method for aggregating expert evaluations given in any kind of format such as rating, interval, linguistic evaluations, fuzzy sets and their extensions like HFS and intuitionistic fuzzy sets, etc. Cumulative belief degree approach [10, 11] can be an appropriate method for this.

Bibliography

[1] O. Bayazit, Use of AHP in decision-making for flexible manufacturing systems, *J. Manuf. Technol. Manage.* **16** (2005), 808–819.

[2] T. O. Boucher, O. Gogus and E. M. Wicks, A comparison between two multiattribute decision methodologies used in capital investment decision analysis, *Eng. Economist*, **42** (1997), 179–202.

[3] J. Browne, D. Dubois, K. Rathmill, S. P. Sethi and K. E. Stecke, Classification of flexible manufacturing systems, *FMS Mag.* **2** (1984), 114–117.

[4] P. Chatterjee and S. Chakraborty, Flexible manufacturing system selection using preference ranking methods: a comparative study, *Int. J. Ind. Eng. Comput.* **5** (2014), 315–338.

[5] S.-J. Chuu, Group decision-making model using fuzzy multiple attributes analysis for the evaluation of advanced manufacturing technology, *Fuzzy Set. Syst.* **160** (2009), 586–602.

[6] B. Ervural and Ö. Kabak, A taxonomy for multiple attribute group decision making literature, in: *2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* pp. 1–8, 2015.

[7] V. Jain and T. Raj, Ranking of flexibility in flexible manufacturing system by using a combined multiple attribute decision making method, *Global J. Flex. Syst. Manage.* **14** (2013), 125–141.

[8] V. Jain and T. Raj, Modeling and analysis of FMS flexibility factors by TISM and fuzzy MICMAC, *Int. J. Syst. Assur. Eng. Manage.* **6** (2015), 350–371.

[9] Ö. Kabak and B. Ervural, Multiple attribute group decision making: a generic conceptual framework and a classification scheme, *Knowl.-Based Syst.* **123** (2017), 13–30.

[10] O. Kabak and D. Ruan, A comparison study of fuzzy MADM methods in nuclear safeguards evaluation, *J. Global Optim.* **51** (2011), 209–226.

[11] O. Kabak and D. Ruan, A cumulative belief degree-based approach for missing values in nuclear safeguards evaluation, *IEEE T. Knowl. Data Eng.* **23** (2011), 1441–1454.

[12] E. E. Karsak, A two-phase robot selection procedure, *Prod. Plan. Control* **9** (1998), 675–684.

[13] E. E. Karsak, Distance-based fuzzy MCDM approach for evaluating flexible manufacturing system alternatives, *Int. J. Prod. Res.* **40** (2002), 3167–3181.

[14] E. E. Karsak and O. Kuzgunkaya, A fuzzy multiple objective programming approach for the selection of a flexible manufacturing system, *Int. J. Prod. Econ.* **79** (2002), 101–111.

[15] E. E. Karsak and E. Tolga, Fuzzy multi-criteria decision-making procedure for evaluating advanced manufacturing system investments, *Int. J. Prod. Econ.* **69** (2001), 49–64.

[16] O. Kulak and C. Kahraman, Multi-attribute comparison of advanced manufacturing systems using fuzzy vs. crisp axiomatic design approach, *Int. J. Prod. Econ.* **95** (2005), 415–424.

[17] D. Li, W. Zeng and Y. Zhao, Note on distance measure of hesitant fuzzy sets, *Inform. Sciences* **321** (2015), 103–115.

[18] G.-S. Liang and M.-J. J. Wang, A fuzzy multi-criteria decision-making approach for robot selection, *Robot. Com.-Int. Manuf.* **10** (1993), 267–274.

[19] S.-T. Liu, A fuzzy DEA/AR approach to the selection of flexible manufacturing systems, *Comput. Ind. Eng.* **54** (2008), 66–76.

[20] A. Perego and A. Rangone, A reference framework for the application of MADM fuzzy techniques to selecting AMTS, *Int. J. Prod. Res.* **36** (1998), 437–458.

[21] P. Ragavan and M. Punniyamoorthy, A strategic decision model for the justification of technology selection, *Int. J. Adv. Manuf. Technol.* **21** (2003), 72–78.

[22] R. V. Rao, A decision-making framework model for evaluating flexible manufacturing systems using digraph and matrix methods, *Int. J. Adv. Manuf. Technol.* **30** (2006), 1101–1110.

[23] R. V. Rao, Evaluating flexible manufacturing systems using a combined multiple attribute decision making method, *Int. J. Prod. Res.* **46** (2008), 1975–1989.

[24] R. V. Rao and M. Parnichkun, Flexible manufacturing system selection using a combinatorial mathematics-based decision-making method, *Int. J. Prod. Res.* **47** (2009), 6981–6998.

[25] B. R. Sarker, S. Krishnamurthy and S. G. Kuthethur, A survey and critical review of flexibility measures in manufacturing systems, *Prod. Plan. Control* **5** (1994), 512–523.

[26] J. Shang and T. Sueyoshi, A unified framework for the selection of a flexible manufacturing system, *Eur. J. Oper. Res.* **85** (1995), 297–315.

[27] R. Slagmulder and W. Bruggeman, Investment justification of flexible manufacturing technologies: inferences from field research, *Int. J. Oper. Prod. Manage.* **12** (1992), 168–186.

[28] A. Stam and M. Kuula, Selecting a flexible manufacturing system using multiple criteria analysis, *Int. J. Prod. Res.* **29** (1991), 803–820.

[29] R. Talebanpour and M. Javadi, Decision-making for flexible manufacturing systems using DEMATEL and SAW, *Decision Sci. Lett.* **4** (2015), 363–372.

[30] V. Torra, Hesitant fuzzy sets, *Int. J. Intell. Syst.* **25** (2010), 529–39.

[31] V. Torra and Y. Narukawa, On hesitant fuzzy sets and decision, IEEE (2009), 1378–82.

[32] M.-C. Tseng, Strategic choice of flexible manufacturing technologies, *Int. J. Prod. Econ.* **91** (2004), 223–227.
[33] R. N. Wabalickis, Justification of FMS with the analytic hierarchy process, *J. Manuf. Syst.* **7** (1988), 175–182.
[34] M. Xia and Z. Xu, Hesitant fuzzy information aggregation in decision making, *Int. J. Approx. Reason.* **52** (2011), 395–407.
[35] M. Xia, Z. Xu and N. Chen, Some hesitant fuzzy aggregation operators with their application in group decision making, *Group Decis. Negot.* **22** (2013), 259–279.
[36] Z. Xu and R. R. Yager, Power-geometric operators and their use in group decision making, *IEEE T. Fuzzy Syst.* **18** (2010), 94–105.
[37] R. R. Yager, The power average operator. *IEEE T. Syst. Man Cy. A* **31** (2001), 724–731.
[38] M. Yurdakul, Selection of computer-integrated manufacturing technologies using a combined analytic hierarchy process and goal programming model, *Robot. Com.-Int. Manuf.* **20** (2004), 329–340.
[39] Z. Zhang, Hesitant fuzzy power aggregation operators and their application to multiple attribute group decision making, *Inform. Sciences* **234** (2013), 150–181.