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Analysis on Asymptotic Optimality of Round-Robin Scheduling for Minimizing Age of Information with HARQ

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SUMMARY In a heterogeneous unreliable multiaccess network, wherein terminals share a common wireless channel with distinct error probabilities, existing works have shown that a persistent round-robin (RR-P) scheduling policy can be arbitrarily worse than the optimum in terms of Age of Information (AoI) under standard Automatic Repeat reQuest (ARQ). In this paper, practical Hybrid ARQ (HARQ) schemes which are widely-used in today's wireless networks are considered. We show that RR-P is very close to optimum with asymptotically many terminals in this case, by explicitly deriving tight, closed-form AoI gaps between optimum and achievable AoI by RR-P. In particular, it is rigorously proved that for RR-P, under HARQ models concerning fading channels (resp. finite-blocklength regime), the relative AoI gap compared with the optimum is within a constant of 6.4% (resp. 6.2% with error exponential decay rate of 0.5). In addition, RR-P enjoys the distinctive advantage of implementation simplicity with channel-unaware and easy-to-decentralize operations, making it favorable in practice. A further investigation considering constraint imposed on the number of retransmissions is presented. The performance gap is indicated through numerical simulations.

key words: *Age of Information, Persistent round-robin scheduling policy, Hybrid Automatic Repeat reQuest, asymptotic optimality, wireless networks*

1. Introduction

In recent years, with the increasing deployment of various real-time applications in wireless networks, stricter delay and reliability requirements have aroused more attention to the research of Age of Information (AoI) [1]. As a performance metric used to evaluate the freshness of data from receivers' perspective [2], AoI represents the time elapsed since the generation of the newest received packet. Compared with conventional end-to-end delay which only focuses on a single packet, AoI pays attention to the latency of the whole process and captures the timeliness of critical status information, making it very relevant to time-sensitive network control systems, such as autonomous vehicles and sensor networks. Therefore, there is a growing and strong motivation of optimizing AoI in wireless networks, as the future wireless systems are more and more concerned with machine-type applications.

We consider one of the most prevalent and representative wireless communication settings, which can be modeled as unreliable multiaccess networks, where terminals share a common wireless channel (error-prone due to noise and channel fading) to communicate with a master node, e.g., a central controller or base station. In this regard, most existing research is focused on independently identically distributed (i.i.d.) channels and standard Automatic Repeat reQuest (ARQ). Kadota et al. [2] considered a network with terminal-dependent error probabilities and active sources (i.e., sources generate a fresh status whenever scheduled), in which case AoI is in fact identical with the definition of time-since-last-service in [3]. It is shown that, intuitively, the optimal policy that minimizes the time-average AoI should serve the terminals both timely and regularly. The timeliness requirement is related to the first-order metric which is known as peak-age (in this case, identical with inter-delivery time [4]); and the regularity requirement is related to the second-order moment of peak-age. A stationary randomized policy with optimized access probabilities can minimize the peak-age, but only 2-optimal which means its AoI is within twice the optimum, due to its non-regularity. A Persistent Round-Robin (RR-P) policy (i.e., greedy policy in [2] since it selects the terminal with highest current AoI) meets regularity, but falls short for timeliness and is shown to not have a constant multiplicative optimality guarantee. Due to the fact that, under standard ARQ scheme, a terminal with error probability of one would jam the system forever, Hybrid ARQ (HARQ) is taken into account in this paper.

HARQ is a widely-used technique in modern wireless systems, which differs with standard ARQ in the way they treat previous transmissions. HARQ can usually combine historical transmissions of the same packet whereas standard ARQ discards previous ones. At the expense of affordable additional complexity, the transmission reliability is significantly improved and hence HARQ is adopted in almost every wireless system in today's networks. The consideration of HARQ in the context of AoI presents distinct challenges. Unlike conventional packet transmissions wherein only reliability matters, optimizing AoI forces the HARQ mechanism to consider whether it is worthwhile to repeat an old packet — the tradeoff lies in that repetition is definitely more likely to succeed but sacrificing timeliness.

In this paper, we are particularly interested in the RR-P policy. RR-P selects terminals according to the descending order of AoI, and the scheduled terminal transmits packets uninterruptedly until a packet is delivered. It was shown

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in [5] that round-robin scheduling is asymptotically optimal when stochastic arrivals and reliable channels are considered. In addition, RR-P has several practically desirable merits which are listed below.

Channel-unaware: Unlike Whittle's index policy which needs to be aware of the channel conditions of all terminals, effectively entailing a pilot overhead which is often ignored in existing works, RR-P does not need any channel knowledge.

Easy-to-decentralize: A round-robin-type scheduling is friendly to decentralized access. A token ring passing channel access scheme, e.g., in IEEE 802.5 [8], can be utilized to realize round-robin scheduling. Distributed terminals only need to know their local AoI (by a simple acknowledgment feedback). In contrast, index-based policies need to compare all terminals' states, hindering decentralized implementation.

However, as mentioned previously, RR-P in scenarios with heterogeneous terminals channels renders arbitrarily worse performance compared with optimum. Our main goal in this paper is to find out how RR-P behaves in minimizing AoI under HARQ system, and surprisingly, it will be shown that its performance is in fact very close to optimum. Moreover, motivated by the fact that in the real-world network systems, sources (usually composed of sensors) tend to have certain energy or power constraints and cannot afford to send packets to the destination node without restrictions, we investigate the performance of RR-P when the number of retransmissions is limited. In this case, we consider transmitting a fresh status update when the number of retransmissions of one packet reaches a preset value, so as to prevent the increase of AoI due to some packet that cannot be transmitted for a long time.

1.1 Contributions

The main contribution of this paper is the analysis of RR-P policy under HARQ system and the corresponding error model. We first prove that RR-P with HARQ is very close to optimum with a large number of terminals. Specifically, the theoretical results considering practical HARQ models show that the asymptotic AoI loss, in terms of relative AoI increase compared with optimum with many terminals, is within a constant of 6.4%. Then, the RR-P performance under a constraint imposed on the transmitter is studied, in which case we set the maximum number of retransmissions of the same packet. Finally, we provide more intuitive numerical results based on computer simulation to show the real AoI loss of RR-P in practice.

1.2 Related Work

AoI has been extensively studied in recent years. In [1], a queueing theoretical method was developed to analyze AoI in M/M/1, M/D/1, and D/M/1 first-come-first-served (FCFS) queueing networks, the authors find the optimal server utilization for the above queueing models and the lower bound on achievable age for any service time distribution.

In [2], [6], [9], the problem of minimizing AoI in wireless networks is considered. The authors formulate a discrete-time decision problem and provide the development and analysis of four low-complexity scheduling policies: a Greedy policy, a randomized policy, a Max-Weight policy and a Whittle's Index policy. An AoI lower bound was obtained which is conjectured to be asymptotically tight. In the literature [7], [9]–[11], it is found that a Whittle's index policy is practically and asymptotically optimal with many terminals since it schedules a terminal based on a scaled age, i.e., approximately $\sqrt{p_i}h_i$ where p_i and h_i are success probability and AoI of terminal-i respectively, which jointly accounts for timeliness and regularity in an optimal way (the Max-Weight policy [2] is effectively observing the same rule). Although several works proposed optimization techniques, the explicit AoI analysis is relatively scarcely treated.

As for the optimization of AoI under HARQ scheme, status updates through M/G/1/1/ queues with HARQ is studied in [12]. The authors investigate the average age and optimal arrival rate of two possible transmission policies: preempting the current update or discarding the new one, and apply the results on two practical HARQ scenarios. It is found in [13] that the optimal policy to minimize AoI with HARQ follows a threshold-based manner, for a single-link scenario. Resource constraints are considered by limiting the average number of transmissions, and Lagrangian relaxation is applied to problem formulation in their work.

Energy constraint has also been considered in [14] and [15]. In [14], the authors analyze the tradeoff between energy cost, service price and utility of information. A aging control policy is devised, based on the aging tolerance of the applications and and the future availability of wireless access points. In [15], the continuous time problem of optimizing the average AoI under constraints on the number of updates that may be sent by a given time is addressed. The uniqueness of our work is to start with the HARQ mechanism and analyze the performance of the RR-P policy in discrete-time model by imposing an upper limit on the number of retransmissions for a single packet.

The remainder of this paper is organized as follows. In Sect. 2, we will introduce the system model under consideration, including HARQ models and AoI evolution. The main results are presented in Sect. 3, wherein we derive theoretical AoI lower bound and achievable AoI by RR-P, and further show that they are close. In Sect. 4, we obtain the performance of AoI with retransmission restrictions, then compare and analyze the causing AoI loss. Simulation results to numerically exhibit the performance is given in Sect. 5. Several proof details are presented in the Appendix.

2. System Model

We consider a one-hop wireless network wherein a central node communicates with N distributed terminals. The terminals share the wireless channel based on a scheduling policy denoted by $\boldsymbol{\pi}$. A time-slotted status update system is considered. The status packet generation is assumed to be generate-at-will, i.e., a fresh status for terminal-n is generated whenever it is scheduled. AoI of a single terminal increases linearly in time when there is no reception of a status update packet and decreases to a certain value after a successful delivery. We are interested in average AoI. Concretely, the T-horizon time-average AoI of the system is defined by

$$\Delta_{\boldsymbol{\pi}}^{(T)} \triangleq \frac{1}{TN}\sum_{t=1}^{T}\sum_{n=1}^{N}\mathbb{E}[h_{n,\boldsymbol{\pi}}(t)], \tag{1}$$

where T is the time horizon length, and $h_{n,\boldsymbol{\pi}}(t)$ denotes the AoI of terminal-n at the t-th time slot under policy $\boldsymbol{\pi}$. The long-time average AoI is defined by

$$\bar{\Delta}_{\boldsymbol{\pi}} \triangleq \limsup_{T\to\infty} \Delta_{\boldsymbol{\pi}}^{(T)}. \tag{2}$$

2.1 Status Updates with HARQ

We assume a perfect (i.e., error- and delay-free) one-bit feedback channel from the status update destination to the source node. In case of a successful reception of a status update packet, the destination feeds back an ACK; otherwise a NACK is fed back to indicate a transmission failure. In principle, retransmissions based on the feedback have the potential to improve the performance. Therefore, HARQ is considered in this paper. There are many different HARQ schemes in the literature. As a convention, they are categorized into two types. First, the type-I HARQ schemes, by which the destination node discards previous transmitted packets and treats each (re)transmissions as new—this is similar with standard ARQ except for the naming convention. Secondly, the type-II HARQ schemes combine (re)transmissions of the same packet for lower packet error performance, at the expense of more complicated buffer and algorithm design. Furthermore, there two widely-used type-II HARQ schemes:

- Chase Combining HARQ (CC-HARQ): The receiver uses Maximum Ratio Combining (MRC) to achieve a signal power gain, and all (re)transmissions carry the same coded bits. The MRC is implemented on the symbol level before the channel decoder.
- Incremental Redundancy HARQ (IR-HARQ): The information bits are coded with incremental redundant bits for error correction, each increment is carried in a retransmission. The receiver combines the coded bits of (re)transmissions and feeds them into the channel decoder.

One distinct tradeoff for type-II HARQ in status update is between the transmission success probability and the status freshness, in light of the fact that retransmissions carry the same old information dated back to the original transmission. Whereas type-I HARQ discards old packets anyway, it can always transmit fresh information. Without going into much details about HARQ which is out of the scope of this paper, we consider two models of packet error probability, i.e.,

$$\mathrm{g}_{n,1}(r) = \frac{p_{n,0}}{r+1},\ \mathrm{g}_{n,2}(r) = p_{n,0}\lambda^{r}, \tag{3}$$

where $\mathrm{g}_{n,i}(r)$ denotes the packet error probability after the r-th (re)transmissions, $r \in \{0, 1, 2, ...\}$. The packet error probability of the first transmission (or type-I HARQ retransmissions) for terminal-n is denoted by $p_{n,0} \in [0, 1]$, which can be different among terminals, and $\lambda \in (0, 1)$ is a parameter related to HARQ protocol and channel conditions. It is noted that $\mathrm{g}_{n,1}(r)$ is suited for i.i.d. fading scenarios with sufficient coding blocklength [16], whereas $\mathrm{g}_{n,2}(r)$ is more appropriate to model finite blocklength effects in quasi-static channels [13]. A detailed justification is presented in Appendix A. We further assume that the packet lengths and transmit power of (re)transmissions are the same. Each packet transmission is an independent Bernoulli trail with failure probability given above. We first consider the case when the maximum number of retransmissions is unlimited. The following lemma is useful in our analysis, regarding the average consecutive transmission attempts for a successful delivery.

Lemma 1. *The number of consecutive transmission attempts for a successful delivery of terminal-n is denoted by $K_{i,n}$ and i represents one of the HARQ model in Eq.* (3). *With the error probability give above, the first and second moments of the number of consecutive transmission attempts for a successful delivery satisfy*

$$\begin{aligned}
\mathbb{E}[K_1] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1}\mathrm{g}_1(i)\,(1-\mathrm{g}_1(r))\,(r+1)\right] = e^{p_0},\\
\mathbb{E}\left[K_1^2\right] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1}\mathrm{g}_1(i)\,(1-\mathrm{g}_1(r))\,(r+1)^2\right]\\
&= (1+2p_0)\,e^{p_0},\\
\mathbb{E}[K_2] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1}\mathrm{g}_2(i)\,(1-\mathrm{g}_2(r))\,(r+1)\right]\\
&\leq 1+\left(1+\sqrt{\frac{2\pi}{-\log\lambda}}\right)p_0,\\
\mathbb{E}\left[K_2^2\right] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1}\mathrm{g}_2(i)\,(1-\mathrm{g}_2(r))\,(r+1)^2\right]\\
&\leq \frac{2\log p_0-2}{\log\lambda}-1+\left(2-\frac{2\log p_0}{\log\lambda}\right)\mathbb{E}[K_2],
\end{aligned} \tag{4}$$

respectively, where the terminal index is omitted, and we prescribe $\mathrm{g}_i(-1) = 1$, $i = 1, 2$.

Proof. See Appendix B. □

Then extending to the situation when the limitation of the number of retransmissions is considered, if a packet carrying the same information cannot be delivered successfully within a certain number of retransmissions, then after the number of retransmissions reaches the maximum value, resample and transmit a fresh status update packet. In this case, the result in Lemma 2 cannot be applied. The specific analysis will be given in the Sect. 4.

2.2 State, Action and Problem Formulation

At each time slot, the state of terminal-n is defined as $s_n(t) \triangleq (h_n(t), r_n(t))$, wherein $r_n(t)$ denotes the number of previous (re)transmissions of the same packet. Note that a reasonable policy would not re-send an older packet, since the policy has decided to transmit a new packet in previous time slots.

The scheduling action includes deciding which terminal to be scheduled, and whether it should re-transmit, if any, an old packet, or transmit a new one. Formally, the action space is denoted by $\mathcal{A} \triangleq \{n_{\mathsf{x}} | n \in \{1, ..., N\}, \mathsf{x} \in \{\mathsf{n}, \mathsf{o}\}\}$, wherein $\mathsf{x} = \mathsf{n}$ and $\mathsf{x} = \mathsf{o}$ denote transmitting a new packet and re-transmitting an old one, respectively. The state transition probability is hence written as

$$\begin{aligned}
&\Pr\{h_n + 1, 1 \mid h_n, r_n, n_{\mathsf{n}}\} = \mathrm{g}(0);\\
&\Pr\{1, 0 \mid h_n, r_n, n_{\mathsf{n}}\} = 1 - \mathrm{g}(0);\\
&\Pr\{h_n + 1, r_n + 1 \mid h_n, r_n n_{\circ}\} = \mathrm{g}(r);\\
&\Pr\{r_n + 1, 0 \mid h_n, r_n, n_{\circ}\} = 1 - \mathrm{g}(r);
\end{aligned} \tag{5}$$

and when terminal-n is not scheduled,

$$\Pr\{h_n + 1, r_n | h_n, r_n, i_{\mathsf{x}}, i \neq n\} = 1, \tag{6}$$

and other transition probabilities equal zero.

We assume that in each time slot, only one terminal can be scheduled. The objective is to find a policy $\boldsymbol{\pi}$ that minimizes the long-term average AoI in (2), and to analyze its performance. In most parts of the paper, we consider a large number of terminals, i.e., $N \to \infty$.

3. Optimality of RR-P with Type-II HARQ

In this section, the asymptotic optimality of RR-P when the number of terminals N is large is shown. The method is based on first finding an AoI lower bound which leverages a similar method in [2], and then deriving an achievable AoI analytical results (upper bound) by the RR-P. By showing that the gap in between is vanishing, it can be concluded that RR-P is asymptotically optimal.

3.1 AoI Lower Bound with HARQ

The AoI is shown to have the following property [3, Lemma 1], [2, Theorem 6].

Lemma 2. *For a scheduling policy that schedules every terminal infinitely often, i.e., ergodic, the long-time average AoI satisfies*

$$\bar{\Delta}_{\boldsymbol{\pi}} \geq \frac{1}{2N} \sum_{n=1}^{N} \frac{\bar{\mathbb{M}}[\delta_n^2]}{\bar{\mathbb{M}}[\delta_n]} + \frac{1}{2}, \tag{7}$$

where $\bar{\mathbb{M}}(\cdot)$ denotes the sample mean, and δ_n is the inter-delivery time of terminal-n, i.e., number of time slots between consecutive successful deliveries. In addition, if the policy is also renewal, i.e., the successful delivery interval of each terminal is i.i.d. non-negative random variables, using the generalization of the elementary renewal theorem for renewal-reward processes yields

$$\bar{\Delta}_{\boldsymbol{\pi}_{\mathsf{R}}} \geq \frac{1}{2N} \sum_{n=1}^{N} \frac{\mathbb{E}[\delta_n^2]}{\mathbb{E}[\delta_n]} + \frac{1}{2}, \tag{8}$$

where $\mathbb{E}(\cdot)$ denotes the expectation.

Proof. The proposition is satisfied with equality in previous works without considering HARQ, i.e., the corresponding AoI is reset to 1 after each successful delivery. Therefore, the AoI with HARQ is lower bounded by the expressions in the lemma, considering that the AoI would reduce to the time duration since the first transmission of the current packet, which is larger or equal to one (in case this is the first attempt, the AoI would return to one). □

This Lemma clearly shows the relationship between inter-delivery time and AoI, and is leveraged in the rest of the paper. Note that the condition of ergodicity is not restrictive, since a policy that starves any terminal is apparently sub-optimal. Also note that a renewal policy is defined as one that results in i.i.d. inter-delivery time. Hence a stationary policy that schedules any terminal based on a time-invariant probability is included; the RR-type policy is also included based on the definition.

Lemma 3 (Lower Bound)**.** *The long time-average AoI is lower bounded by*

$$\bar{\Delta}_{\boldsymbol{\pi}} \geq \frac{1}{2N} \left(\sum_{n=1}^{N} \sqrt{\mathbb{E}[K_{i,n}]} \right)^2 + \frac{1}{2} \tag{9}$$

where $K_{i,n}$ denotes the number of consecutive transmission attempts for a successful delivery of terminal-n and i represents one of the HARQ error models in Eq. (3) *as defined previously.*

Proof. Denote the number of successful deliveries of terminal-n up to the L-th time slot as $D_n(L)$, and the number of transmission attempts of terminal-n up to the L-th time slot as $A_n(L)$, then

$$\begin{aligned}
\bar{\Delta}_{\boldsymbol{\pi}} &\overset{(a)}{\geq} \frac{1}{2N} \sum_{n=1}^{N} \bar{\mathbb{M}}[\delta_n] + \frac{1}{2}\\
&\overset{(b)}{=} \frac{1}{2N} \sum_{n=1}^{N} \lim_{L\to\infty} \frac{L}{D_n(L)} + \frac{1}{2}\\
&\overset{(c)}{\geq} \frac{1}{2N} \lim_{L\to\infty} \sum_{n=1}^{N} A_n(L) \sum_{m=1}^{N} \frac{1}{D_m(L)} + \frac{1}{2}
\end{aligned}$$

$$
\begin{aligned}
&\overset{(d)}{\geq} \frac{1}{2N}\left(\sum_{n=1}^{N}\sqrt{\lim_{L\to\infty}\frac{A_n(L)}{D_n(L)}}\right)^2+\frac{1}{2}\\
&\overset{(e)}{\geq} \frac{1}{2N}\left(\sum_{n=1}^{N}\sqrt{\mathbb{E}[K_{i,n}]}\right)^2+\frac{1}{2}\\
&= \frac{N}{2}\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]^2+\frac{1}{2},
\end{aligned}
\tag{10}
$$

which concludes the proof. Denote by $\bar{\mathbb{M}}\left[\sqrt{\mathbb{E}[K_{i,n}]}\right] \triangleq \frac{\sum\sqrt{\mathbb{E}[K_{i,n}]}}{N}$ as the sample mean among the terminals, and $g(\omega_{i,n}) \triangleq \mathbb{E}[K_{i,n}]$ is a function of channel parameters, i.e., $\omega_{1,n}=[p_{n,0}]$ and $\omega_{2,n}=[p_{n,0},\lambda]$. The inequality (a) follows from $\bar{\mathbb{M}}[\delta_n^2]\geq\bar{\mathbb{M}}[\delta_n]^2$, wherein the equality holds when the variance is zero. The equality (b) is obtained by definition. The inequality (c) is because $L\geq\sum_{n=1}^{N}A_n(L)$ since there are altogether L time slots. The Cauchy-Schwarz inequality gives (d), and the last inequality (e) follows from the fact that the minimum average transmission attempts required to reach a successful delivery by HARQ is obtained by successive repetitive transmissions of old packets since the transmission error probability of the same old packet will decrease as the number of retransmissions increases under the HARQ models. The average is given by Lemma 1. □

Remark 1. *When type-I HARQ is considered, i.e., equivalent with standard ARQ in this context, the lower bound results in [2] is a special case of this lemma, wherein $\frac{A_n(L)}{D_n(L)}$ tends to the inverse of the transmission error probability.*

Denote the AoI lower bound in Lemma 3 as $\bar{\Delta}_{\mathsf{LB}}$, then the following corollary follows straightforwardly.

Corollary 1.

$$
\bar{\Delta}_{\mathsf{LB}}\leq\frac{N}{2}\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]+\frac{1}{2}
\tag{11}
$$

Proof. The inequality follows from $\bar{\mathbb{M}}[x]^2\leq\bar{\mathbb{M}}[x^2]$. In the subsequent section, we will see that this corollary reflects the gap between the lower bound and the achievable AoI by the RR-P policy, as RR-P achieves (approximately) the RHS of (11). □

3.2 Achievable AoI by RR-P

Definition 1. *The RR-P scheduling policy schedules the terminals in a round-robin manner. When scheduled, the terminal will transmit and re-transmit the same packet until successful delivery.*

The achievable AoI by RR-P is shown by the following theorem.

Theorem 1. *Under the HARQ models in* (3)*, the long time-average AoI achieved by RR-P is*

$$
\bar{\Delta}_{\mathsf{RR\_P},i}=\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]+\frac{1}{2N}\frac{\mathbb{E}\left[\left(\sum_{n=1}^{N}[K_{i,n}]\right)^2\right]}{\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]}-\frac{1}{2},
\tag{12}
$$

which satisfies

$$
\bar{\Delta}_{\mathsf{RR\_P},i}\leq\frac{N+1}{2}\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]+\frac{1}{2}\frac{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}^2]\right]}{\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]}-\frac{1}{2}.
\tag{13}
$$

Furthermore,

$$
\frac{N}{2}\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]-\frac{1}{2}\leq\bar{\Delta}_{\mathsf{RR\_P},i}\leq\frac{N}{2}\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]+c,
\tag{14}
$$

where c is a constant irrelevant with N. This inequality gives the asymptotic scaling factor when the number of terminals is large, i.e.,

$$
\lim_{N\to\infty}\frac{\bar{\Delta}_{\mathsf{RR\_P},i}}{N}=\frac{\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]}{2}
\tag{15}
$$

Proof. See Appendix C. □

Armed with this theorem, in particular the asymptotic results, the relative AoI gap between RR-P and the optimum (i.e., $(1+\gamma)$-optimality where γ denotes the relative gap) with a large number of terminals can be studied. In the following subsection, explicit and tight results will be presented.

3.3 Asymptotic $(1+\gamma)$-Optimality of RR-P

We investigate the asymptotic order-optimality of RR-P, that is, the relative AoI gap compared with optimum when the number of terminals is large. Define the asymptotic relative AoI gap of RR-P as

$$
\gamma_i\triangleq\lim_{N\to\infty}\frac{\bar{\Delta}_{\mathsf{RR\_P},i}-\bar{\Delta}_{\mathsf{opt}}}{\bar{\Delta}_{\mathsf{opt}}}.
\tag{16}
$$

Based on Lemma 3 and Theorem 1, the gap is smaller or equal to

$$
\begin{aligned}
\gamma_i&\leq\lim_{N\to\infty}\frac{\bar{\Delta}_{\mathsf{RR\_P},i}-\bar{\Delta}_{\mathsf{LB}}}{\bar{\Delta}_{\mathsf{LB}}}\\
&=\frac{\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]-\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]^2}{\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]^2}.
\end{aligned}
\tag{17}
$$

The following theorem explicitly bound the gap.

Theorem 2. *Under the HARQ models in* (3) *with relative gaps γ_i, $i\in\{1,2\}$, RR-P is within $(1+\gamma_i)$-optimality with $N\to\infty$, and*

$$
\begin{aligned}
\gamma_1&\leq\frac{(\sqrt{e}-1)^2}{4\sqrt{e}}\cong 6.4\%,\\
\gamma_2&\leq\frac{\left(\sqrt{2+\sqrt{\frac{2\pi}{-\log\lambda}}}-1\right)^2}{4\sqrt{2+\sqrt{\frac{2\pi}{-\log\lambda}}}}.
\end{aligned}
\tag{18}
$$

Proof. Following (17),

$$\begin{aligned}\gamma_i &\leq \frac{\bar{\mathbb{V}}\left[\sqrt{g(\omega_{i,n})}\right]}{\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]^2} \\ &\overset{(a)}{\leq} \frac{1}{\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]^2}\left(\sqrt{g_{\max,i}} - \bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right]\right) \\ &\quad \times\left(\bar{\mathbb{M}}\left[\sqrt{g(\omega_{i,n})}\right] - \sqrt{g_{\min,i}}\right) \\ &\leq \frac{\left(\sqrt{g_{\max,i}} - \sqrt{g_{\min,i}}\right)^2}{4\sqrt{g_{\max,i}g_{\min,i}}},\end{aligned} \tag{19}$$

wherein $\bar{\mathbb{V}}[\cdot]$ denotes the variance operator, and the inequality (a) stems from [17]. Based on Lemma 1, we select

$$g_{\min,i} = 1,\ i = 1, 2. \tag{20}$$

$$g_{\max,1} = e,\ g_{\max,2} = 2 + \sqrt{\frac{2\pi}{-\log\lambda}}, \tag{21}$$

and hence the conclusion follows immediately. □

Remark 2. *Based on Theorem 2, it is shown that when the number of terminals is large, the relative AoI increase by RR-P compared with optimum is within* 6.4 *percents with the first HARQ model; for a practical value of* $\lambda = 0.5$, *the gap is within* 17.1 *percents with the second HARQ model. Note that this does not mean the performance loss with the second model is larger—this is mainly due to the fact that we can only obtain an upper bound with* K_2, *which is often loose. In fact, applying a better bound by Corollary 2 with* $R = 4$, *we can show the* $\gamma_2 \leq 6.2\%$ *with* $\lambda = 0.5$.

In contrast, based on [2, Theorem 8], no constant γ *can be found for RR-P with type-I HARQ, or equivalently standard ARQ in this context. In other words, RR-P can be arbitrarily worse than optimum with standard ARQ (transmitting a new packet at each opportunity). In fact, as far as we know, the best proven bound for standard ARQ with unequal error probabilities is* $\gamma = 1$ *(i.e., 2-optimal), using a stationary randomized policy with optimized transmission probabilities. Policies with simulated better performance, e.g., Whittle index policy, can only be proven with very loose bounds which render meaningless due to their the non-renewal nature, resulting in difficulties in age analysis. The bounds in this paper are much tighter, in scenarios with HARQ which essentially favors (re)transmissions and hence makes the analysis more tractable.*

4. AoI Analysis under a Transmission Constraint

In this section, we analyze the performance of AoI with limited number of retransmissions. For each packet delivered by a terminal, we denote the maximum number of retransmissions by R, i.e., one terminal can only re-transmit the same packet for R times. If a packet has not been successfully delivered after the R-th retransmissions, then re-sample and transmit a fresh one. Therefore, in this case, a successful delivery of a terminal may include the transmission of multiple new packets.

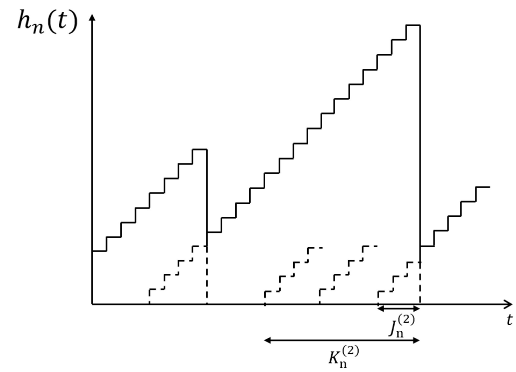

Fig. 1 AoI evolution of terminal-n under retransmission constraints.

For terminal-n, the new sample path of the AoI evolution under a transmission constraint is shown in Fig. 1. Let J_n denote the number of transmission attempts of the packet which is successfully delivered in the current round, and $J_n \in [1, 1+R]$. Let K_n denote the number of total transmission attempts for a successful delivery, which numerically equal to the time duration since the terminal transmits its first packet when scheduled. Then, in a certain round-j, we have

$$K_n^{(j)} = (R+1)C_n^{(j)} + J_n^{(j)} \tag{22}$$

where C_n denotes the number of newly sampled packets transmitted before the successful one.

Theorem 3. *Under the limitation of the maximum number of retransmissions, the achievable long time-average AoI is*

$$\bar{\Delta}_{\mathsf{R},i} = \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}\left[J_{i,n}\right] + \frac{1}{2}\frac{\mathbb{E}\left[\left(\sum_{n=1}^{N}K_{i,n}\right)^2\right]}{\sum_{n=1}^{N}\mathbb{E}\left[K_{i,n}\right]} - \frac{1}{2}, \tag{23}$$

which satisfies

$$\begin{aligned}\bar{\Delta}_{\mathsf{R},i} \leq{}& \bar{\mathbb{M}}\left[\mathbb{E}[J_{i,n}]\right] + \frac{N-1}{2}\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}]\right] \\ &+ \frac{1}{2}\frac{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}^2]\right]}{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}]\right]} - \frac{1}{2}.\end{aligned} \tag{24}$$

When the number of terminals goes to infinity, the asymptotic scaling factor is

$$\lim_{N\to\infty}\frac{\bar{\Delta}_{\mathsf{R},i}}{N} = \frac{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}]\right]}{2}. \tag{25}$$

Proof. See Appendix D □

Similarly, based on Theorem 2 and Theorem 3, the relative AoI gap between unconstrained and constrained situations follows that

$$\begin{aligned}\delta_i &\triangleq \lim_{N\to\infty}\frac{\bar{\Delta}_{\mathsf{R},i} - \bar{\Delta}_{\mathsf{RR\_P},i}}{\bar{\Delta}_{\mathsf{RR\_P},i}} \\ &= \frac{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}]\right] - \bar{\mathbb{M}}\left[g(\omega_{i,n})\right]}{\bar{\mathbb{M}}\left[g(\omega_{i,n})\right]}.\end{aligned} \tag{26}$$

Theorem 4. *The relative AoI gap δ_i decreases monotonically with the maximum number of retransmissions R, where $i \in \{1, 2\}$ corresponding to two HARQ models, and*

$$\delta_1 \leq \frac{1}{e}\frac{\sum_{r=0}^{R}\frac{1}{r!}}{1-\frac{1}{(R+1)!}} - 1,$$
$$\delta_2 \leq \frac{1}{g_{\mathsf{max},2}}\frac{\sum_{r=0}^{R}\lambda^{\frac{r(r-1)}{2}}}{1-\lambda^{\frac{R(R+1)}{2}}} - 1. \tag{27}$$

Proof. Following (26),

$$\begin{aligned}\delta_i &= \frac{\bar{\mathbb{M}}\left[\mathbb{E}[K_{i,n}]\right]}{\bar{\mathbb{M}}\left[g(\boldsymbol{\omega}_{i,n})\right]} - 1\\ &\overset{(a)}{\leq} \max\frac{\mathbb{E}[K_{i,n}]}{g(\boldsymbol{\omega}_{i,n})} - 1\\ &\overset{(b)}{=} \frac{\max\mathbb{E}[K_{i,n}]}{\max g(\boldsymbol{\omega}_{i,n})} - 1.\end{aligned} \tag{28}$$

Inequality (a) is based on the fact that both $\mathbb{E}[K_{i,n}]$ and $g(\boldsymbol{\omega}_{i,n})$ increase monotonically with p_0, and equality (b) follows from the fact that the quotient of $\mathbb{E}[K_{i,n}]$ and $g(\boldsymbol{\omega}_{i,n})$ also increases monotonically with p_0. Hence, $\mathbb{E}[K_{i,n}]$ takes the maximum value when $p_0 = 1$. Together with the result obtained in (21), wherein $g_{\mathsf{max},2}$ uses the upper bound in Corollary 2 with $R = 7$ and $\lambda = 0.5$, then the conclusion is drawn. □

Remark 3. *It is shown in Theorem 4 that with large number of terminals, the relative performance loss under constraints narrows as the retransmission limit R becomes larger. Subsitituting specific value of R, more intuitive results can be obtained as follows. When R takes 1, 2, 3, 4 in turn, the performance gap compared with unconstrained AoI is* 47.2%, 10.4%, 2.4%, 0.5% *(resp.* 51.4%, 8.2%, 1.0%, 0.1%*).*

5. Simulation Results

We run computer simulations for RR-P for 10^6 time slots and take the time-average AoI. The AoI lower bound in Lemma 3 and achievable AoI upper bound by RR-P in Theorem 1 are calculated and compared in the Fig. 2. The number of terminals N varies from 3 to 100. The corresponding initial transmission error probabilities $p_{n,0}, n = 1, \cdots, N$ are set to $[1/N, 2/N, \cdots, 1]$, respectively. The AoI is normalized by the lower bound derived by Lemma 3. It is observed that, with both HARQ models, the relative AoI increase by RR-P approaches very small with the number of terminals greater than, e.g., 20. The upper bound that is used to prove the main result follows this trend closely, and moreover, the actual RR-P performance obtained by Monte-Carlo simulations is even closer to the lower bound, indicating that RR-P can perform even better than the proved theoretical results in practice.

Figure 3 shows the simulated AoI performance under the first HARQ model when the maximum number of retransmissions R takes 1, 2, 3, 4, respectively. The initial AoI refers to the simulated AoI without a limit on the number of retransmissions. We can see that as the maximum number of retransmissions increases sequentially, the curve with retransmission constraints gradually approaches the curve of initial AoI, and the performance in terms of AoI under limitation is basically the same as that under unconstrained conditions when $R = 4$, which confirms the theoretical analysis results in Sect. 4.

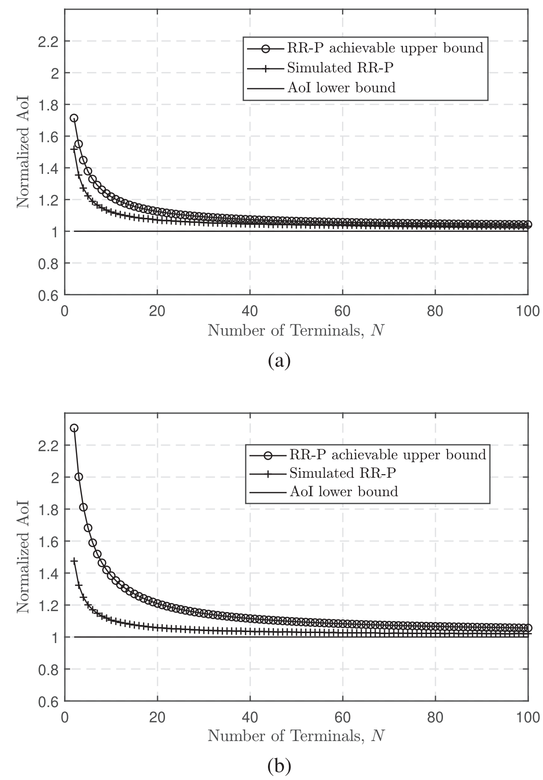

Fig. 2 AoI normalized by the lower bound in Lemma 3, achieved by the RR-P through Monte-Carlo simulations and its upper bound in Theorem 1. HARQ models in (3) are both shown in subfigure (a) and (b), respectively. $\lambda = 0.5$ in subfigure (b), and the upper bound is calculated based on Corollary 2 with $R = 4$.

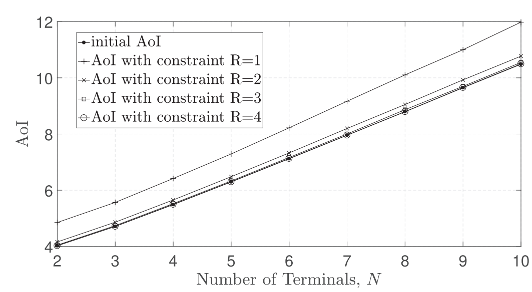

Fig. 3 AoI achieved by the RR-P through Monte-Carlo simulations when there is no retransmission limit, and there is a retransmission limit with the maximum number of retransmissions of a packet R taking 1, 2, 3, 4.

6. Conclusions

This paper has shown that RR-P is provably near-optimal for AoI optimizations with HARQ in heterogeneous unreliable multiaccess channels wherein terminals have distinctive transmission error probabilities and the number of terminals is large. Concretely, it is proved that the relative AoI gap by RR-P compared with the optimum is within a constant of $(\sqrt{e}-1)^2/4\sqrt{e} \cong 6.4\%$ (resp. 6.2% with error exponential decay rate of 0.5) with fading channels (resp. finite blocklength scenarios) asymptotically. In reality, the gap becomes even smaller than the theoretical bounds, which is shown by computer simulations. The simulation results also reveal that the number of terminals required for the asymptotic results to hold is approximately 20. Moreover, the gap increases with the terminals' transmission error heterogeneity, i.e., the variance of terminals' transmission error probabilities.

Besides, under the consideration of the limit on the number of retransmissions for the same packet, performance loss of AoI is analyzed and the results show that when the upper limit of retransmission is set to 4, the performance is almost the same as when there is no constraint. This result conveys to us that in the case of limited resources, setting a suitable maximum number of retransmissions can also achieve performance close to the previous unrestricted situation.

The results in this paper rely crucially on the renewal structure of RR-P. It is still difficult to obtain closed-form AoI analysis for non-renewal policies, as evidenced by several studies in the literature [2], [11]. More advanced mathematical tools are needed to address this issue in future works.

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Appendix A: Justification of HARQ Models of (3)

Without loss of generality, let us consider one representative terminal. Assuming a block-fading Rayleigh channel based on which the complex baseband channel stays constant during each HARQ transmission round and changes to another value based on an i.i.d. complex Gaussian distribution. The block error probability in each round is approximated by the information outage probability which is defined to be the probability that instantaneous spectral efficiency given by Shannon formula is smaller than the target spectral efficiency. Furthermore, assume that the transmission power stays the same and CC-HARQ is adopted. Then the block error rate in the r-th round can be well approximated by the first model in (3) based on [16, Theorem 1]. More precisely, the probability that the first l transmissions all fail is approximately

$$p_{\text{out},l} \cong \frac{p_{\text{out},1}^{l}}{l!} + O(p_{\text{out},1}^{l+1}), \tag{A·1}$$

wherein the factorial term represents the power gain by CC-HARQ.

On the other hand, consider the finite blocklength regime and a non-fading AWGN channel, wherein the block error, instead of fading, is mainly caused by insufficient channel coding bits and white noise. For ease of exposition, consider a Binary Erasure Channel (BEC) for each bit with erasure rate of δ. Consider IR-HARQ, a message can only be correctly decoded when the total number of successful bits is more than C, and the total number of transmitted bits is lB where B is the blocklength of one transmission and l is total transmission rounds. Therefore, it follows that the error probability follows the cumulative distribution function of binomial distribution, i.e.,

$$p_{\mathsf{e},l} = \sum_{c=0}^{C-1} \binom{lB}{c} (1-\delta)^c \delta^{lB-c} \tag{A$\cdot$2}$$

It is well-known that when lB is large enough compared with C,[†] a reasonable approximation of the binomial distribution is Gaussian distribution of $\mathcal{N}(lB\delta, lB\delta(1-\delta))$, i.e., the probability mass function can be approximated by

$$f_{\mathsf{e},l,c} \cong \frac{1}{\sqrt{2\pi lB\delta(1-\delta)}} e^{\frac{-(c-lB\delta)^2}{2lB\delta(1-\delta)}}. \tag{A$\cdot$3}$$

Therefore, the success probability after l rounds is approximated by

$$\begin{aligned} p_{\mathsf{e},l} &\cong \mathrm{Q}\left(\frac{|C-lB\delta|}{\sqrt{lB\delta(1-\delta)}}\right) \lessapprox e^{-\frac{(C-lB\delta)^2}{2lB\delta(1-\delta)}} \\ &\overset{lB\gg C}{\cong} e^{-\frac{lB\delta}{2(1-\delta)}}, \end{aligned} \tag{A$\cdot$4}$$

which coincides with the second model in (3) whereby the error probability scales down exponentially with the number of (re)transmission attempts. Note that by definition, $l = r+1$. A similar, in fact much stronger arguments can be made based on [19], wheres the present paper provides a more intuitive explanation.

It can be observed that the two models in (3) suit i.i.d. Rayleigh fading with CC-HARQ and finite blocklength with IR-HARQ methods, respectively.

Appendix B: Proof of Lemma 1

Considering K_1, we obtain

$$\begin{aligned} \mathbb{E}[K_1] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1} \mathrm{g}_1(i)(1-\mathrm{g}_1(r))(r+1)\right] \\ &= \sum_{r=0}^{+\infty} \frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)(r+1) \\ &= \sum_{r=0}^{+\infty} \frac{p_0^r}{r!}(r+1) - \sum_{r=0}^{+\infty} \frac{p_0^r}{r!} r \\ &= \sum_{r=0}^{+\infty} \frac{p_0^r}{r!} = e^{p_0}. \end{aligned} \tag{A$\cdot$5}$$

$$\begin{aligned} \mathbb{E}[K_1^2] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1} \mathrm{g}_1(i)(1-\mathrm{g}_1(r))(r+1)^2\right] \\ &= \sum_{r=0}^{+\infty} \frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)(r+1)^2 \\ &= \sum_{r=0}^{+\infty} \frac{p_0^r}{r!}(2r+1) \\ &= (1+2p_0)e^{p_0}. \end{aligned} \tag{A$\cdot$6}$$

Similarly, with K_2,

$$\begin{aligned} \mathbb{E}[K_2] &\triangleq \sum_{r=0}^{+\infty}\left[\prod_{i=0}^{r-1} \mathrm{g}_2(i)(1-\mathrm{g}_2(r))(r+1)\right] \\ &= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} (1-p_0\lambda^r)(r+1) \\ &= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}}(r+1) - \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} r \\ &= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} = 1 + p_0 + \sum_{r=2}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} \\ &\overset{(a)}{\le} 1 + p_0 + \int_1^{+\infty} p_0^x \lambda^{\frac{x(x-1)}{2}} \mathsf{d}x, \end{aligned} \tag{A$\cdot$7}$$

where inequality (a) is due to the fact that for a monotonically decreasing function $f(x) = p_0^x \lambda^{\frac{x(x-1)}{2}}$, $x \in [2, +\infty)$,

$$\sum_{r=2}^{+\infty} f(r) \le \int_1^{+\infty} f(x)\mathsf{d}x. \tag{A$\cdot$8}$$

Denote $\alpha \triangleq -\frac{\log\lambda}{2}$ and $\beta \triangleq -\log p_0$, then following (A$\cdot$7),

$$\begin{aligned} \mathbb{E}[K_2] &\le 1 + p_0 + e^{\frac{(\alpha-\beta)^2}{4\alpha}} \int_{\frac{1}{2}+\frac{\beta}{2\alpha}}^{+\infty} e^{-\alpha x^2}\mathsf{d}x, \\ &= 1 + p_0 + e^{\frac{(\alpha-\beta)^2}{4\alpha}} \sqrt{\frac{\pi}{\alpha}}\,\mathrm{Q}\left(\frac{\alpha+\beta}{\sqrt{2\alpha}}\right) \\ &\overset{(a)}{\le} 1 + p_0 + \sqrt{\frac{\pi}{\alpha}} e^{-\beta} \\ &= 1 + \left(1 + \sqrt{\frac{2\pi}{-\log\lambda}}\right) p_0, \end{aligned} \tag{A$\cdot$9}$$

where $\mathrm{Q}(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-t^2/2}\mathsf{d}t$ is the Q-function, and inequality (a) follows from the Chernoff bound $\mathrm{Q}(x) \le e^{-x^2/2}$. The following corollary gives a tighter bound.

Corollary 2.

$$\mathbb{E}[K_2] \le \sum_{r=0}^{R-1} p_0^r \lambda^{\frac{r(r-1)}{2}} + \left(1 + \sqrt{\frac{2\pi}{-\log\lambda}}\right) p_0^R \lambda^{\frac{R(R-1)}{2}}, \tag{A$\cdot$10}$$

[†] A common condition is that [18] $\frac{|1-2\delta|}{\sqrt{lB\delta(1-\delta)}} < \frac{1}{3}$.

wherein $R \in \mathbb{N}^{+}$.

Proof.

$$\begin{aligned}
\mathbb{E}[K_2] &\leq \sum_{r=0}^{R} p_0^r \lambda^{\frac{r(r-1)}{2}} + \sum_{r=R+1}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} \\
&\leq \sum_{r=0}^{R} p_0^r \lambda^{\frac{r(r-1)}{2}} + \int_{R}^{+\infty} p_0^x \lambda^{\frac{x(x-1)}{2}} \mathsf{d}x, \\
&= \sum_{r=0}^{R} p_0^r \lambda^{\frac{r(r-1)}{2}} \\
&\quad + e^{\frac{(\alpha-\beta)^2}{4a}} \sqrt{\frac{\pi}{\alpha}}\, \mathrm{Q}\left(\frac{(2R-1)\alpha+\beta}{\sqrt{2\alpha}}\right) \\
&\leq \sum_{r=0}^{R} p_0^r \lambda^{\frac{r(r-1)}{2}} + e^{\frac{(\alpha-\beta)^2-((2R-1)\alpha+\beta)^2}{4a}} \sqrt{\frac{\pi}{\alpha}} \\
&= \sum_{r=0}^{R} p_0^r \lambda^{\frac{r(r-1)}{2}} + e^{-(R(R-1)\alpha+R\beta)} \sqrt{\frac{\pi}{\alpha}},
\end{aligned} \tag{A· 11}$$

which concludes the proof. □

$$\begin{aligned}
\mathbb{E}[K_2^2] &\triangleq \sum_{r=0}^{+\infty} \left[\prod_{i=0}^{r-1} \mathrm{g}_2(i)(1-\mathrm{g}_2(r))(r+1)^2\right] \\
&= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} (1-p_0\lambda^r)(r+1)^2 \\
&= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} (r+1)^2 - \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} r^2 \\
&= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} (2r+1) \\
&= \sum_{r=0}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} 2\left(r-\frac{\alpha-\beta}{2\alpha}\right) + \left(2-\frac{\beta}{\alpha}\right)\mathbb{E}[K_2] \\
&= \sum_{r=1}^{+\infty} p_0^r \lambda^{\frac{r(r-1)}{2}} 2\left(r-\frac{\alpha-\beta}{2\alpha}\right) \\
&\quad + \left(2-\frac{\beta}{\alpha}\right)\mathbb{E}[K_2] + \frac{\beta}{\alpha} - 1 \\
&\leq e^{\frac{(\alpha-\beta)^2}{4\alpha}} \int_0^{+\infty} e^{-\alpha\left(x-\frac{\alpha-\beta}{2\alpha}\right)^2} \left(x-\frac{\alpha-\beta}{2\alpha}\right) \mathsf{d}x \\
&\quad + \left(2-\frac{\beta}{\alpha}\right)\mathbb{E}[K_2] + \frac{\beta}{\alpha} - 1 \\
&= \left(2-\frac{\beta}{\alpha}\right)\mathbb{E}[K_2] + \frac{\beta+1}{\alpha} - 1,
\end{aligned} \tag{A· 12}$$

which concludes the proof.

Appendix C: Proof of Theorem 1

Note that RR-P is a renewal policy that for each terminal, the j-th successful delivery interval is

$$S_{n,\mathsf{RR\_P}}^{(j)} \triangleq \sum_{m=1}^{N} K_m^{(j)},\ \forall n \in \{1, ..., N\}, \tag{A· 13}$$

and $\{K_m^{(j)} | j = 1, 2, ...\}$ are i.i.d. This is because based on RR-P, the successful delivery interval of every terminal is the total time that all terminals delivers an update. Therefore, without loss of generality, a sample path of the AoI evolution of terminal-n is shown in Fig. A· 1. The time between the $i-1$-th and i-th deliveries is denoted by the i-th round, during which, the moment that the terminal is scheduled and transmits its first packet is denoted by $a_n^{(i)}$ in Fig. A· 1. The retransmissions continue until a successful delivery based on RR-P, and hence the age $h_n(t)$ drops to $K_n^{(i)}$ upon that—the time of which is denoted by $s_n^{(i)}$.

Following the same arguments in, e.g., [12], the time-average AoI can be readily calculated by the sum of the geometric areas $Q_n^{(i)}$ in Fig. A· 1:

$$\mathbb{E}[h_n(t)] = \lim_{T\to\infty} \frac{D}{T}\frac{1}{D}\sum_{i=1}^{I} Q_n^{(i)} = \frac{\mathbb{E}[Q_{k,n}]}{\eta}, \tag{A· 14}$$

where D denotes the number of successful deliveries until time T, and

$$\eta \triangleq \mathbb{E}\left[S_{n,\mathsf{RR\_P}}^{(j)}\right] = \sum_{m=1}^{N} \mathbb{E}\left[K_m^{(i)}\right]. \tag{A· 15}$$

When T goes to infinity, D also goes to infinity. The last equality is based on the elementary renewal theorem [20]. It then follows that

$$\begin{aligned}
&\mathbb{E}[h_n(t)] \\
&= \frac{1}{\eta}\mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)} K_n^{(i-1)} + \left(S_{n,\mathsf{RR_P}}^{(i)} - 1\right)\frac{S_{n,\mathsf{RR_P}}^{(i)}}{2}\right] \\
&\overset{(a)}{=} \frac{1}{\eta}\left(\mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)}\right]\mathbb{E}\left[K_n^{(i-1)}\right]\right. \\
&\quad \left. + \frac{1}{2}\left(\mathbb{E}\left[\left(S_{n,\mathsf{RR_P}}^{(i)}\right)^2\right] - \mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)}\right]\right)\right) \\
&= \mathbb{E}\left[K_n^{(i-1)}\right] + \frac{1}{2\eta}\mathbb{E}\left[\left(\sum_{m=1}^{N} K_m^{(i)}\right)^2\right] - \frac{1}{2} \\
&\overset{(b)}{\leq} \mathbb{E}\left[K_n\right] + \frac{1}{2\eta}\left(\sum_{m=1}^{N}\mathbb{E}\left[K_m^2\right] + \frac{N-1}{N}\eta^2\right) - \frac{1}{2}
\end{aligned}$$

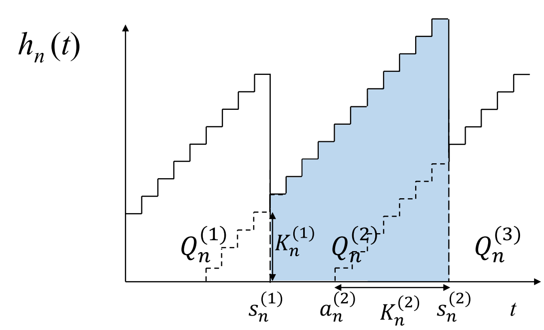

Fig. A· 1 AoI evolution of terminal-n under HARQ.

$$= \mathbb{E}[K_n] - \frac{\eta}{2N} + \frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_m^2\right] + \frac{\eta-1}{2} \qquad \text{(A· 16)}$$

where the equality (a) is based on the fact that the number of transmission attempts during the $(i-1)$-th round is independent with the ones in the i-th round, and the inequality (b) follows from the fact that for independent random variables $x_1, ..., x_N$,

$$\begin{aligned}
&\mathbb{E}\left[\left(\sum_{i=1}^{N} x_i\right)^2\right] \\
&= \sum_{i=1}^{N}\mathbb{E}\left[x_i^2\right] + \sum_{i<j}^{N} 2\mathbb{E}[x_i]\,\mathbb{E}\left[x_j\right] \\
&= \sum_{i=1}^{N}\mathbb{E}\left[x_i^2\right] + \left(\frac{1}{N}+1-\frac{1}{N}\right)\sum_{i<j}^{N} 2\mathbb{E}[x_i]\,\mathbb{E}\left[x_j\right] \\
&\leq \sum_{i=1}^{N}\mathbb{E}\left[x_i^2\right] + \frac{N-1}{N}\sum_{i=1}^{N}\mathbb{E}[x_i]^2 \\
&\quad + \frac{N-1}{N}\sum_{i<j}^{N} 2\mathbb{E}[x_i]\,\mathbb{E}\left[x_j\right] \\
&= \sum_{i=1}^{N}\mathbb{E}\left[x_i^2\right] + \frac{N-1}{N}\left(\sum_{i=1}^{N}\mathbb{E}[x_i]\right)^2. \qquad \text{(A· 17)}
\end{aligned}$$

Note that after inequality (b), we ignore the index of round for brevity. Now averaging over all terminals, we obtain

$$\begin{aligned}
\bar{\Delta}_{\mathsf{RR_P}} &= \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[h_n(t)] \\
&\overset{(a)}{=} \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}\left[K_n^{(i-1)}\right] + \frac{1}{2\eta}\mathbb{E}\left[\left(\sum_{m=1}^{N} K_m^{(i)}\right)^2\right] - \frac{1}{2} \\
&\leq \frac{\eta}{2N} + \frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_m^2\right] + \frac{\eta-1}{2}, \qquad \text{(A· 18)}
\end{aligned}$$

wherein the equality (a) gives (12) directly, and (13) readily follows given $\eta = N\bar{\mathbb{M}}[g(\omega_{i,n})]$. For the asymptotic results of (14), considering the HARQ models in (3) and Lemma 1, we obtain respectively for two models,

$$\begin{aligned}
\bar{\Delta}_{\mathsf{RR_P,1}} &\leq \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{1,n})] + \frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_{1,m}^2\right] - \frac{1}{2} \\
&\leq \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{1,n})] + \frac{\sum_{m=1}^{N} p_{m,0}e^{p_{m,0}}}{\sum_{m=1}^{N} e^{p_{m,0}}} \\
&\leq \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{1,n})] + 1. \qquad \text{(A· 19)}
\end{aligned}$$

Given that

$$\bar{\mathbb{M}}[g(\omega_{1,n})] = \frac{1}{N}\sum_{m=1}^{N}\mathbb{E}[K_{1,m}] \leq e, \qquad \text{(A· 20)}$$

and that the left inequality of (14) is straightforward, we can conclude the asymptotic results immediately. Similarly,

$$\begin{aligned}
&\bar{\Delta}_{\mathsf{RR_P,2}} \\
&\leq \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{2,n})] + \frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_{2,m}^2\right] - \frac{1}{2} \\
&\leq \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{2,n})] \\
&\quad + \frac{\sum_{m=1}^{N}\left[-\frac{\beta_m}{\alpha}\mathbb{E}[K_{2,m}] + \frac{\beta_m+1}{\alpha} - 1\right]}{2\sum_{m=1}^{N}\mathbb{E}[K_{2,m}]} \\
&\overset{(a)}{\leq} \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{2,n})] \\
&\quad + \frac{\sum_{m=1}^{N}\left[-\frac{\beta_m}{\alpha}\left(1+\sqrt{\frac{\pi}{\alpha}}\right)p_{m,0} + \frac{1}{\alpha} - 1\right]}{2\sum_{m=1}^{N}\mathbb{E}[K_{2,m}]} \\
&\overset{(b)}{\leq} \frac{N+1}{2}\bar{\mathbb{M}}[g(\omega_{2,n})] + \frac{1}{2\alpha}, \qquad \text{(A· 21)}
\end{aligned}$$

wherein $\beta_m = -\log p_{m,0}$, $\alpha = -\frac{1}{2}\log\lambda$, the inequality (a) follows from Lemma 1 and (b) is obtained by noting $\min_m[\mathbb{E}[K_{2,m}]] \geq 1$. With

$$\bar{\mathbb{M}}[g(\omega_{2,n})] = \frac{1}{N}\sum_{m=1}^{N}\mathbb{E}[K_{2,m}] \leq 2 + \sqrt{\frac{\pi}{\alpha}}, \qquad \text{(A· 22)}$$

which is irrelevant with N, the conclusion follows immediately.

Appendix D: Proof of Theorem 3

From the Fig. 1, we can intuitively find that the calculation of the time average AoI of terminal-n is the same as before. Therefore, it follows that

$$\begin{aligned}
&\mathbb{E}[h_n(t)] \\
&= \frac{1}{\eta}\mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)}J_n^{(i-1)} + \left(S_{n,\mathsf{RR_P}}^{(i)} - 1\right)\frac{S_{n,\mathsf{RR_P}}^{(i)}}{2}\right] \\
&= \frac{1}{\eta}\left(\mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)}\right]\mathbb{E}\left[J_n^{(i-1)}\right]\right. \\
&\quad \left.+ \frac{1}{2}\left(\mathbb{E}\left[\left(S_{n,\mathsf{RR_P}}^{(i)}\right)^2\right] - \mathbb{E}\left[S_{n,\mathsf{RR_P}}^{(i)}\right]\right)\right) \\
&= \mathbb{E}\left[J_n^{(i-1)}\right] + \frac{1}{2\eta}\mathbb{E}\left[\left(\sum_{m=1}^{N} K_m^{(i)}\right)^2\right] - \frac{1}{2} \\
&\leq \mathbb{E}[J_n] - \frac{\eta}{2N} + \frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_m^2\right] + \frac{\eta-1}{2}. \qquad \text{(A· 23)}
\end{aligned}$$

The relationship among $S_{\mathsf{RR\_P}}$, K and η is also consistent with Eqs. (A· 13) and (A· 15). Taking all terminals into account, we obtain

$$\bar{\Delta}_{\mathsf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[h_n(t)]$$

$$\begin{aligned}&\overset{(a)}{=}\frac{1}{N}\sum_{n=1}^{N}\mathbb{E}\left[J_n\right]+\frac{1}{2\eta}\mathbb{E}\left[\left(\sum_{m=1}^{N}K_m\right)^2\right]-\frac{1}{2}\\&\le\frac{1}{N}\sum_{n=1}^{N}\mathbb{E}\left[J_n\right]-\frac{\eta}{2N}+\frac{1}{2\eta}\sum_{m=1}^{N}\mathbb{E}\left[K_m^2\right]+\frac{\eta-1}{2},\end{aligned}\tag{A$\cdot$24}$$

wherein the equality (a) gives (23) directly. Considering random variables in Eq. (22) are independent of each other,then it holds that

$$\mathbb{E}[K_n]=(R+1)\mathbb{E}[C_n]+\mathbb{E}[J_n],\tag{A$\cdot$25}$$

$$\begin{aligned}\mathbb{E}[K_n^2]=&(R+1)^2\mathbb{E}[C_n^2]+\mathbb{E}[J_n^2]\\&+2(R+1)\mathbb{E}[C_n]\mathbb{E}[J_n].\end{aligned}\tag{A$\cdot$26}$$

Since the transmission of each new packet is independent, C_n is distributed as a geometric with success probability $1-P_e$, wherein $P_e=\prod_{i=0}^{R}\mathrm{g}(i)$ and $1-P_e$ indicates the probability of a packet being delivered successfully within R attempts. For an individual terminal, we omit n in the discussion of C_n and J_n. Hence,

$$\begin{aligned}\mathbb{E}[C_1]&\triangleq\frac{1}{1-P_{1,e}}-1\\&=\frac{1}{1-\prod_{i=0}^{R}\mathrm{g}_1(i)}-1\\&=\frac{1}{1-\frac{p_0^{R+1}}{(R+1)!}}-1,\end{aligned}\tag{A$\cdot$27}$$

$$\begin{aligned}\mathbb{E}[C_1^2]&\triangleq\frac{P_{1,e}\left(1+P_{1,e}\right)}{\left(1-P_{1,e}\right)^2}\\&=\frac{\prod_{i=0}^{R}\mathrm{g}_1(i)\left(1+\prod_{i=0}^{R}\mathrm{g}_1(i)\right)}{\left(1-\prod_{i=0}^{R}\mathrm{g}_1(i)\right)^2}\\&=\frac{\frac{p_0^{R+1}}{(R+1)!}\left(1+\frac{p_0^{R+1}}{(R+1)!}\right)}{\left(1-\frac{p_0^{R+1}}{(R+1)!}\right)^2},\end{aligned}\tag{A$\cdot$28}$$

$$\begin{aligned}\mathbb{E}[C_2]&\triangleq\frac{1}{1-P_{2,e}}-1\\&=\frac{1}{1-\prod_{i=0}^{R}\mathrm{g}_2(i)}-1\\&=\frac{1}{1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}-1,\end{aligned}\tag{A$\cdot$29}$$

$$\begin{aligned}\mathbb{E}[C_2^2]&\triangleq\frac{P_{2,e}\left(1+P_{2,e}\right)}{\left(1-P_{2,e}\right)^2}\\&=\frac{\prod_{i=0}^{R}\mathrm{g}_2(i)\left(1+\prod_{i=0}^{R}\mathrm{g}_2(i)\right)}{\left(1-\prod_{i=0}^{R}\mathrm{g}_2(i)\right)^2}\\&=\frac{p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\left(1+p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\right)}{\left(1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\right)^2}.\end{aligned}\tag{A$\cdot$30}$$

Based on Appendix B, we obtain

$$\begin{aligned}\mathbb{E}[J_1]&\triangleq\sum_{r=0}^{R}\left[\frac{\prod_{i=0}^{r-1}\mathrm{g}_1(i)(1-\mathrm{g}_1(r))}{\sum_{r=0}^{R}\left[\prod_{i=0}^{r-1}\mathrm{g}_1(i)(1-\mathrm{g}_1(r))\right]}(r+1)\right]\\&=\sum_{r=0}^{R}\frac{\frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)}{\sum_{r=0}^{R}\frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)}(r+1)\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}(r+1)-\sum_{r=0}^{R}\frac{p_0^r}{r!}p_0}{\sum_{r=0}^{R}\frac{p_0^r}{r!}-\sum_{r=0}^{R}\frac{p_0^{r+1}}{(r+1)!}}\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}(r+1)-\sum_{r=0}^{R}\frac{p_0^{r+1}}{(r+1)!}(r+1)}{\sum_{r=0}^{R}\frac{p_0^r}{r!}-\sum_{r=1}^{R+1}\frac{p_0^r}{r!}}\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}(r+1)-\sum_{r=1}^{R+1}\frac{p_0^r}{r!}r}{1-\frac{p_0^{R+1}}{(R+1)!}}\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}(r+1)-\sum_{r=0}^{R}\frac{p_0^r}{r!}r-(R+1)\frac{p_0^{R+1}}{(R+1)!}}{1-\frac{p_0^{R+1}}{(R+1)!}}\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}-(R+1)\frac{p_0^{R+1}}{(R+1)!}}{1-\frac{p_0^{R+1}}{(R+1)!}}.\end{aligned}\tag{A$\cdot$31}$$

$$\begin{aligned}\mathbb{E}[J_1^2]&\triangleq\sum_{r=0}^{R}\left[\frac{\prod_{i=0}^{r-1}\mathrm{g}_1(i)(1-\mathrm{g}_1(r))}{\sum_{r=0}^{R}\left[\prod_{i=0}^{r-1}\mathrm{g}_1(i)(1-\mathrm{g}_1(r))\right]}(r+1)^2\right]\\&=\sum_{r=0}^{R}\frac{\frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)}{\sum_{r=0}^{R}\frac{p_0^r}{r!}\left(1-\frac{p_0}{r+1}\right)}(r+1)^2\\&=\frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}(2r+1)-(R+1)^2\frac{p_0^{R+1}}{(R+1)!}}{1-\frac{p_0^{R+1}}{(R+1)!}}.\end{aligned}\tag{A$\cdot$32}$$

$$\begin{aligned}\mathbb{E}[J_2]&\triangleq\sum_{r=0}^{R}\left[\frac{\prod_{i=0}^{r-1}\mathrm{g}_2(i)(1-\mathrm{g}_2(r))}{\sum_{r=0}^{R}\left[\prod_{i=0}^{r-1}\mathrm{g}_2(i)(1-\mathrm{g}_2(r))\right]}(r+1)\right]\\&=\sum_{r=0}^{R}\frac{p_0^r\lambda^{\frac{r(r-1)}{2}}\left(1-p_0\lambda^r\right)}{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}\left(1-p_0\lambda^r\right)}(r+1)\\&=\frac{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}(r+1)}{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}-\sum_{r=0}^{R}p_0^{r+1}\lambda^{\frac{r(r+1)}{2}}}\\&\quad-\frac{\sum_{r=0}^{R}p_0^{r+1}\lambda^{\frac{r(r+1)}{2}}(r+1)}{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}-\sum_{r=0}^{R}p_0^{r+1}\lambda^{\frac{r(r+1)}{2}}}\\&=\frac{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}(r+1)-\sum_{r=1}^{R+1}p_0^r\lambda^{\frac{r(r-1)}{2}}r}{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}-\sum_{r=1}^{R+1}p_0^r\lambda^{\frac{r(r-1)}{2}}}\\&=\frac{\sum_{r=0}^{R}p_0^r\lambda^{\frac{r(r-1)}{2}}-(R+1)p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}{1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}.\end{aligned}\tag{A$\cdot$33}$$

$$\begin{aligned}\mathbb{E}[J_2^2] &\triangleq \sum_{r=0}^{R}\left[\frac{\prod_{i=0}^{r-1} g_2(i)(1-g_2(r))}{\sum_{r=0}^{R}\left[\prod_{i=0}^{r-1} g_2(i)(1-g_2(r))\right]}(r+1)\right] \\ &= \sum_{r=0}^{R}\frac{\lambda^{\frac{r(r-1)}{2}}(1-p_0\lambda^r)}{\sum_{r=0}^{R} p_0^r\lambda^{\frac{r(r-1)}{2}}(1-p_0\lambda^r)}(r+1)^2 \\ &= \frac{\sum_{r=0}^{R} p_0^r\lambda^{\frac{r(r-1)}{2}}(2r+1)}{1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}} \\ &\quad - \frac{(R+1)^2 p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}{1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}. \end{aligned} \tag{A$\cdot$34}$$

By substituting (A$\cdot$27)–(A$\cdot$34) to (A$\cdot$25) and (A$\cdot$26), we obtain the first and second moments of K_n, i.e.,

$$\mathbb{E}[K_1] = \frac{\sum_{r=0}^{R}\frac{p_0^r}{r!}}{1-\frac{p_0^{R+1}}{(R+1)!}}, \tag{A$\cdot$35}$$

$$\begin{aligned}\mathbb{E}[K_1^2] &= \frac{\left(1-\frac{p_0^{R+1}}{(R+1)!}\right)\sum_{r=0}^{R}\frac{p_0^r}{r!}(2r+1)}{\left(1-\frac{p_0^{R+1}}{(R+1)!}\right)^2} \\ &\quad + \frac{2(R+1)\frac{p_0^{R+1}}{(R+1)!}\sum_{r=0}^{R}\frac{p_0^r}{r!}}{\left(1-\frac{p_0^{R+1}}{(R+1)!}\right)^2}. \end{aligned} \tag{A$\cdot$36}$$

$$\mathbb{E}[K_2] = \frac{\sum_{r=0}^{R} p_0^r\lambda^{\frac{r(r-1)}{2}}}{1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}}. \tag{A$\cdot$37}$$

$$\begin{aligned}\mathbb{E}[K_2^2] &= \frac{\left(1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\right)\sum_{r=0}^{R} p_0^r\lambda^{\frac{r(r-1)}{2}}(2r+1)}{\left(1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\right)^2} \\ &\quad + \frac{2(R+1)p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\sum_{r=0}^{R} p_0^r\lambda^{\frac{r(r-1)}{2}}}{\left(1-p_0^{R+1}\lambda^{\frac{R(R+1)}{2}}\right)^2}. \end{aligned} \tag{A$\cdot$38}$$

Further more, the achievable AoI for the considered HARQ models satisfies

$$\begin{aligned}\bar{\Delta}_{\mathsf{R},1} &\le \frac{N-1}{2}\bar{\mathbb{M}}\left[\mathbb{E}[K_{1,n}]\right] + \bar{\mathbb{M}}\left[\mathbb{E}[J_{1,n}]\right] \\ &\quad + \frac{1}{2}\frac{\sum_{m=1}^{N}\mathbb{E}\left[K_{1,m}^2\right]}{\sum_{m=1}^{N}\mathbb{E}\left[K_{1,m}\right]} - \frac{1}{2} \\ &\le \frac{N}{2}\bar{\mathbb{M}}\left[\mathbb{E}[K_{1,n}]\right] + \frac{5}{2}, \end{aligned} \tag{A$\cdot$39}$$

wherein

$$\begin{aligned}\bar{\mathbb{M}}\left[\mathbb{E}[J_{1,n}]\right] &\le \sum_{r=0}^{R}\frac{1}{r!}, \\ \bar{\mathbb{M}}\left[\mathbb{E}[K_{1,n}]\right] &\le \frac{\sum_{r=0}^{R}\frac{1}{r!}}{1-\frac{1}{(R+1)!}}. \end{aligned} \tag{A$\cdot$40}$$

The similar results can be obtained under the second model, hence it can be summarized that

$$\bar{\Delta}_{\mathsf{R},i} \le \frac{N}{2}\bar{\mathbb{M}}[\mathbb{E}[K_{i,n}]] + c, \tag{A$\cdot$41}$$

where c is a constant irrelevant with N.

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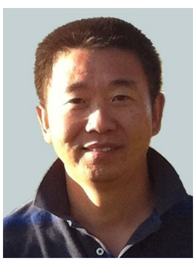

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