PAPER

Critical Location of Communications Network with Power Grid Power Supply

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SUMMARY   When a disaster hits a network, network service disruptions can occur even if the network facilities have survived and battery and power generators are provided. This is because in the event of a disaster, the power supply will not be restarted within the lifetime of the battery or oil transportation will not be restarted before running out of oil and power will be running out. Therefore, taking a power grid into account is important. This paper proposes a polynomial-time algorithm to identify the critical location C_D^* of a communications network N_c when a disaster hits. Electrical power grid N_P supplies power to the nodes of N_c, and a link in N_c is disconnected when a node or a link in N_c or N_P fails. Here, the disaster area is modeled as co-centric disks and the failure probability is higher in the inner disk than the outer one. The location of the center of the disaster with the greatest expected number of disconnected links in N_c is taken as the critical location C_D^*.
key words: *disaster, critical location, network failure, cascading failure, power grid*

1. Introduction

Severe natural disasters, such as earthquakes and tsunamis, can take thousands of human lives and destroy network infrastructures [1]. For example, a severe earthquake in March 2011 off the northeast coast of Japan and its associated tsunami caused devastated damage. It killed many people and destroyed facilities including network infrastructure in cities and towns [2]. Earthquakes that cause similar damage occur every few years worldwide, such as the Shichuan earthquake in China in 2008 [3], [4]. The damage caused by an earthquake is huge and has a global impact.

Network infrastructure is becoming increasingly important in modern society, and the destruction of a network seriously impacts society. Therefore, many network operators make efforts to make their network robust against failures. Academia has also made an effort to improve network robustness against disasters through a huge number of papers (please see the references in [4]–[6]).

To discuss failures due to disasters, introducing a geographical disaster model or field data such as seismic centers is needed because such failures are limited to certain geographical regions and are geographically correlated. Examples of geographical disaster models used in literature are a disk [7]–[11], line (or line segment) [9], [12], [13], half-plane [14]–[16], polygon and elliptical [17], and convex set

[18]. This paper uses a co-centric disk as a disaster model.

Among such works, a number of them investigated a critical disaster location, which has the highest expected impact on a network [17], [19]. It was sometimes called the "worst-case cut" [9]. Finding the critical disaster location and evaluating the network performance deterioration due to the disaster at the critical location is the basis of network survivability. If the network performance deterioration is very serious, the network operator may need to take an action such as de-routing, where identifying the critical location is essential. The common objectives of these works were to provide efficient computational algorithms to identify the critical location in the network (the computation time of the algorithm proposed in [17] was proven to be polynomial). Their main differences were geographical disaster models.

Unfortunately, these existing works have not taken into account the impact of power grid failures on the communications network of which power is supplied by the power grid. Therefore, the critical location of the communications network and the power grid supplying power to it was not specified or the worst performance degradation was underestimated through the existing works. This paper addresses this issue and proposes a polynomial-time algorithm to identify the critical location and compute the worst performance deterioration.

In existing works, failure correlations between the communications network and the power grid have been discussed as a cascading failure in typical interdependent networks in modern society [20], [21]. They have focused on the mechanisms how a cascading failure occurs, but have not covered the critical location or the geographical structures of these networks.

To prevent cascading failures caused by blackouts, many communication network operators provide battery or electric power generators. Unfortunately, network service disruptions can occur when power runs out, even when battery and power generators are provided. This is because in the event of a disaster, the power supply will not be restarted within the lifetime of the battery or oil transportation will not be restarted before running out of oil. Therefore, taking a power grid into account is important. Because of such importance, a geographical communications network design using the geographical information of a power grid has recently been proposed in [16]. In [16], the proposed network design method determines an appropriate geographical route under the given geographical routes of the power grid when a new route of a communications network is built. This paper

also takes power grid geographical information into account and can identify the critical disaster location and evaluate the worst performance deterioration of the communications network. By using the result derived by the method proposed in this paper, we can specify how serious the network can be damaged and judge whether a number of actions must be taken.

The main contribution of this paper is the proposal of the polynomial-time algorithm that identifies the critical disaster location. Here, the critical location C_D^* means that the expected number of disconnected links in a communications network becomes the largest when the center of the disaster is at C_D^*. The algorithm has two unique and novel features. (i) The co-centric disk disaster model is introduced. The model can represent changes in failure probability as changes in distance from the disaster center. (ii) An electrical power grid is used as a part of the network. As a result, the impact of disasters on the network over the power grid can also be taken into account.

In the reminder of this paper, two co-centric disks are assumed as a disaster model. However, the extension to more than two disks is straightforward.

2. Preliminary Analysis

In the main body of this paper, we discuss a disaster hitting a network, and a co-centric disk pair is used as a disaster area model. The inner disk is a more devastating disaster area than the outer disk because it is closer to the disaster center. We need to evaluate whether the points of a network such as nodes are included in the outer or inner disk to evaluate the critical location. This section provides a basis of the main body of this paper.

$\{D_i\}_{i=1,2}$ are a co-centric disk pair of given radii and D_i includes a set of points $\tilde{\mathcal{P}}_i$ $(i = 1, 2)$, where $D_1 \supseteq D_2$. Define $\mathcal{P}_2 \equiv \tilde{\mathcal{P}}_2$ and $\mathcal{P}_1 \equiv \tilde{\mathcal{P}}_1 / \mathcal{P}_2$ where, for sets $\mathcal{S}_1, \mathcal{S}_2$, $\mathcal{S}_1 / \mathcal{S}_2$ is the intersection of $\mathcal{S}_1$ and the complementary of $\mathcal{S}_2$. Thus, $\mathcal{P}_1$ is a set of points on the outer disk outside of the inner disk. For D_i including a given set of points, there are an infinite number of possible locations of D_i, including that set of points. The following "finding boundary points" algorithm places D_i at a representative location among those locations. This representative location is specified by a pair of boundary points defined in the following, and the algorithm provides a boundary point pair for each $\{\mathcal{P}_i\}_{i=1,2}$[†]. The pair of boundary points are not unique, but there are $(|\tilde{\mathcal{P}}_1| - 1)|\tilde{\mathcal{P}}_1|/2$ choices. Here, for a set $\mathcal{S}$, $|\mathcal{S}|$ is the number of elements in $\mathcal{S}$. For each pair of boundary points, the representative location is not unique. However, we should note that the number of representative locations is finite.

["**Finding boundary points**" algorithm] Input data are (i) the location of the co-center, the radii of the inner and outer disks of a co-centric disk pair, (ii) a set $\mathcal{P}_1$ of points on the
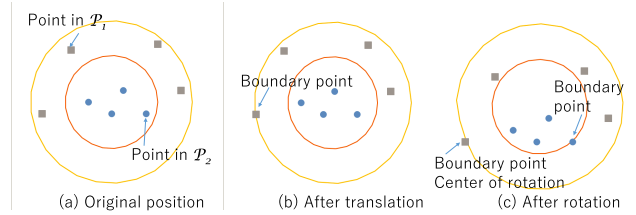

[†]Introducing more than two boundary points, potentially enables this algorithm to be extended to more generic disaster models [17].

**Fig. 1** "Finding boundary points" algorithm.

given outer disk outside of the inner disk, and (iii) a set $\mathcal{P}_2$ of points on the inner disk. If there is no point that lies on any boundary of these disks (Fig. 1(a)), translate the co-centric disk pair parallel to the x-axis until one point hits a boundary on one of the disks (Fig. 1(b)). Next, rotate the co-centric disk pair, either clockwise or counterclockwise, around the hit point on a boundary of one of the disks, until another point hits a boundary on one of the disks (Fig. 1(c)).

The location of (the center of) the co-centric disk pair after the translation and rotation is called a representative location. The two points hitting disk boundaries are called boundary points.

At the representative location, it is clear that the inner disk D_2 and outer disk outside of the inner disk D_1/D_2 include $\mathcal{P}_2$ and $\mathcal{P}_1$, respectively because no points from either set can move outside their respective disk without hitting their boundaries. Note that this algorithm shows the existence of a pair of boundary points and a representative location when a pair of sets of points $\mathcal{P}_1$ and $\mathcal{P}_2$ is given.

The following algorithm provides sets $\tilde{\mathcal{P}}_i$ $(i = 1, 2)$ of points corresponding to a pair of points $\mathbf{x}_1, \mathbf{x}_2$, where co-centric disks $\{D_i\}_{i=1,2}$ include $\{\tilde{\mathcal{P}}_i\}_{i=1,2}$. The pair of points in the following algorithm is a pair of boundary points for $\{D_i\}_{i=1,2}$.

["**Finding sets of points on co-centric disks**" algorithm] Input data are a set of points $\mathcal{P}_0$ and the radii r_1, r_2 of the outer and inner disks of a co-centric disk pair. Choose a pair of points $\mathbf{x}_1, \mathbf{x}_2$ among $\mathcal{P}_0$, and place two co-centric disks such that $\mathbf{x}_i$ is on a boundary of the inner or outer disk where $i = 1, 2$. There are four cases: both $\mathbf{x}_1$ and $\mathbf{x}_2$ are on a boundary of the inner disk, both are on that of the outer disk, and one of them is on that of the inner (outer) disk and the other is on that of the outer (inner) disk. Let $\tilde{r}_i$ be the radius of the disk of which boundary $\mathbf{x}_i$ is on. For each case, there are at most two possible locations for the center C_D of the co-centric disks. These possible locations can be obtained through the following: (i) Draw the circles of the radius (radii) $\tilde{r}_i$ around $\mathbf{x}_i$ $(i = 1, 2)$. (ii) Obtain (at most) two intersections of these circles. The possible locations of C_D are these intersections (Fig. 2). Obtain $\mathcal{H}(\mathbf{x}_1, \mathbf{x}_2)$, which is the set of these center locations for a given pair of points $\mathbf{x}_1, \mathbf{x}_2$. For the co-centric disks of which the center is at $\mathbf{h} \in \mathcal{H}(\mathbf{x}_1, \mathbf{x}_2)$, identify a set $\tilde{\mathcal{P}}_2(\mathbf{h})$ of points included in the inner disk and a set $\tilde{\mathcal{P}}_1(\mathbf{h})$ of points in the outer disk. Obtain $\mathcal{P}_1(\mathbf{h}) \equiv \tilde{\mathcal{P}}_1(\mathbf{h}) / \mathcal{P}_2(\mathbf{h})$ and $\mathcal{P}_2(\mathbf{h}) \equiv \tilde{\mathcal{P}}_2(\mathbf{h})$. Choose another pair of points $\mathbf{x}_1, \mathbf{x}_2$ among $\mathcal{P}_0$, repeat the aforementioned procedure until all the pairs of points in the given set of points
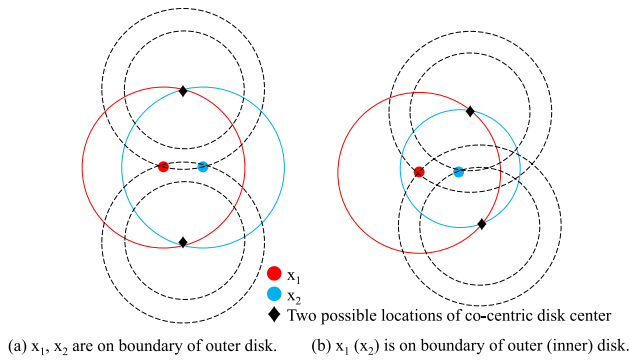
Fig. 2 Placement of co-disks.

Fig. 3 Network.

are covered, and obtain $\{\{\mathbf{h}\}_{\mathbf{h}\in\mathcal{H}(\mathbf{x}_1,\mathbf{x}_2)}, \mathcal{P}_1(\mathbf{h}), \mathcal{P}_2(\mathbf{h})\}_{\mathbf{x}_1,\mathbf{x}_2\in\mathcal{P}_0}$. Define $\mathcal{H}_{\mathcal{P}_0} \equiv \cup_{\mathbf{x}_1,\mathbf{x}_2\in\mathcal{P}_0}\mathcal{H}(\mathbf{x}_1,\mathbf{x}_2)$.

For each pair of points $\mathbf{x}_1, \mathbf{x}_2$, the aforementioned algorithm provides $\mathcal{H}(\mathbf{x}_1, \mathbf{x}_2)$, which includes at most eight center locations of co-centric disks. For each center location $\mathbf{h}$, it also provides (i) a set $\mathcal{P}_1(\mathbf{h})$ of points on the outer disk outside of the inner disk, and (ii) a set $\mathcal{P}_2(\mathbf{h})$ of points in the inner disk. The pair of points $\mathbf{x}_1, \mathbf{x}_2$ is a pair of boundary points. By adopting every point pair in $\mathcal{P}_0$ as $\mathbf{x}_1, \mathbf{x}_2$, all the possible classifications of $\mathcal{P}_0$ into $\mathcal{P}_1$ and $\mathcal{P}_2$ are covered where $\mathcal{P}_0 \supseteq \mathcal{P}_1 \cup \mathcal{P}_2$.

3. Model

This paper focuses on a communications network N_c of which power is supplied by an electric power grid N_p. The network $N_t \equiv N_c \cup N_p$ consists of nodes and links. Let $\mathcal{N}(x)$ be a set of the (switching/router) nodes in x where x is a part of N_t such as N_c, a route, and a link. Let $l(s,t)$ be the link between two consecutive nodes s and t, and $\mathcal{L}(x)$ be the set of links in x. Assume that the physical route of a link on a geographical map consists of several line segments. If an end point of a line segment in a link is not a node, it is called a corner (Fig. 3). (A corner is just a very small part of a link.) Let $C(x)$ be the set of all the corners in x. By introducing corners, the geographical route is converted to a finite number of points and the critical location problem becomes tractable. Let $r(s,t)$ be the geographical route consisting of links and nodes between $s, t \in N_c$ or $s, t \in N_p$. Note that a node in N_c needs to receive power delivered through N_p. Each node in N_c must have a working path in N_p to a special node called an electric power distribution station. Assume that there is a single route (path) between a node $n \in N_c$ and $e(n) \in N_p$. Here, $e(n)$ is the electric power distribution station offering power to n.

A disaster hits N_t, where the disaster area is modeled as co-centric disks. A disk is a good simple model to describe an explosion or an earthquake, because the energy caused by the disaster decays according to the distance from the center. Therefore, as indicated in Introduction, many papers use a disk as a disaster area model. Here, to describe the damage levels, co-centric disks are introduced. Let $\{D_i\}_{i=1,2}$ be co-centric disks of which the radii are r_i, where $r_1 > r_2$. To
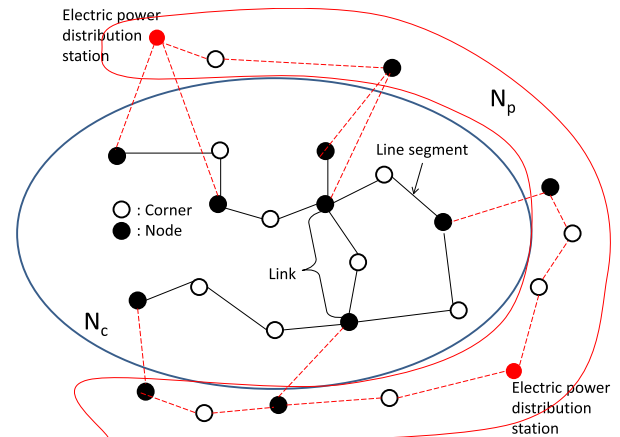
simplify notation, $D_3 \equiv \emptyset$. Assume that nodes and corners included in D_i/D_{i+1} fail with a certain probability, and that r_1 is larger than the minimum distance between consecutive nodes or corners.

Let $\tilde{p}_{i,c,k}$ be the failure probability of the communication function of node k in N_c in D_i/D_{i+1} and $\tilde{p}_{i,p,k}$ be that of the power management function of node k in N_c or that of node k in N_p in D_i/D_{i+1}. In addition, let $\tilde{q}_{i,c,k}$ ($\tilde{q}_{i,p,k}$) be the failure probability of corner k in N_c (N_p) in D_i/D_{i+1}. Failures independently occur with these probabilities when the location of the co-center of D_i is given. Define the working probabilities $p_{i,x,k} \equiv 1 - \tilde{p}_{i,x,k}$ and $q_{i,x,k} \equiv 1 - \tilde{q}_{i,x,k}$ where x is c or p. To work as a node in N_c, the communication and power management functions of the node must work. A failure of a corner is a model of a failure at a small portion of a link. (Specifically, there is a case such that D_i/D_{i+1} includes a part of a line segment without including its nodes or corners. However, when a link is geographically modeled in detail, the link segment length becomes short enough to make such a case negligible. Thus, this model indicates that the part of a link included in D_i/D_{i+1} fails with a certain probability.)

Divide $\mathcal{N}(x)$ ($C(x)$) into exclusive sets $\{\mathcal{N}_i(x)\}_{i\geq0}$ ($\{C_i(x)\}_{i\geq0}$), where $\mathcal{N}(x) = \cup_{i\geq0}\mathcal{N}_i(x)$ and $\mathcal{N}_i(x)\cap\mathcal{N}_j(x) = \emptyset$ ($C(x) = \cup_{i\geq0}C_i(x)$ and $C_i(x)\cap C_j(x) = \emptyset$) for any $i \neq j$. $\mathcal{N}_i(x)$ ($C_i(x)$) is the set of nodes (corners) in D_i/D_{i+1} among $\mathcal{N}(x)$ ($C(x)$) and $\mathcal{N}_0(x)$ ($C_0(x)$) is that outside D_1 among $\mathcal{N}(x)$ ($C(x)$). Define $\mathcal{V}(x) \equiv \{\mathcal{N}(x), C(x)\}$.

$E[L_c(C_D)]$, the expected number L_c of disconnected links in N_c, is used as a performance metric in this paper, where C_D is the location of the co-center of $\{D_i\}_{i=1,2}$. Disconnecting a link $l(s,t)$ in N_c occurs when (i) node s or t in N_c fails (in the communication or power management function), (ii) a corner in $l(s,t)$ fails, or (iii) the power distribution route in N_p offering power to node s or t fails. The proposed algorithm in this paper is a polynomial-time algorithm and identifies a location C_D^* that causes the largest $E[L_c(C_D)]$ when a disaster occurs at $C_D = C_D^*$. That is, the proposed algorithm can solve

Table 1 List of notations.

S_1/S_2	intersection of S_1 and complementary of S_2.
$\|S\|$	number of elements in discrete set S.
D_i	disaster area model. $D_1 \supseteq D_2$.
r_i	radius of D_i.
$\check{P}_i$	set of points included in D_i ($i = 1, 2$).
P_1, P_2	$P_1 \equiv \check{P}_1/P_2$. $P_2 \equiv \check{P}_2$.
P_0	set of points. $P_0 \supseteq P_1 \cup P_2$.
N_c	communications network.
N_p	electric power grid N_p.
N_t	$N_t \equiv N_c \cup N_p$.
$N(x)$	set of nodes in x.
$\mathcal{L}(x)$	set of links in x.
$l(s, t)$	link between nodes s and t.
$C(x)$	set of corners in x.
$r(s, t)$	geographical route between s and t.
$e(n)$	electric power distribution station offering power to n.
$N_i(x)$	set of nodes in D_i/D_{i+1} among $N(x)$.
$C_i(x)$	set of corners in D_i/D_{i+1} among $C(x)$.
$p_{i,c,k}$	working probabilities of node k in $N_i(N_c)$ in D_i/D_{i+1}.
$p_{i,p,k}$	working probabilities of node k in $N_i(N_p)$ in D_i/D_{i+1}.
$q_{i,c,k}$	working probabilities of corner k in $C_i(N_c)$ in D_i/D_{i+1}.
$q_{i,p,k}$	working probabilities of corner k in $C_i(N_p)$ in D_i/D_{i+1}.
$\mathcal{V}(x)$	$\mathcal{V}(x) \equiv \{N(x), C(x)\}$.
C_D	location of co-center of $\{D_i\}_{i=1,2}$.
L_c	number of disconnected links in N_c.

$$C_D^* = \text{argmax}_{C_D \in \mathcal{R}^2} E[L_c(C_D)]. \tag{1}$$

(Formally, the proposed algorithm identifies one of critical locations because C_D^* may not be unique.)

List of notations is provided in Table 1.

4. Analysis

This paper investigates the critical location of the disaster that causes the largest $E[L_c(C_D)]$. In the first subsection, the possible locations of C_D are analyzed to reduce the search space of C_D to identify the largest $E[L_c(C_D)]$. The second and third subsections analyze the probability of connecting a power distribution route offered to nodes in N_c and the working probability of a link in N_c. In the fourth subsection, by using the results of the first three subsections, an algorithm identifying C_D that achieves the largest $E[L_c(C_D)]$ is proposed.

4.1 Possible Locations of C_D

A link can fail when at least one node or a corner is in a disaster area and cannot fail when any node or corner is not. In particular, when $N(N_c) \cap D_i/D_{i+1}$, $N(N_p) \cap D_i/D_{i+1}$, $C(N_c) \cap D_i/D_{i+1}$, and $C(N_p) \cap D_i/D_{i+1}$ are given for all i, the probability of disconnecting $l(s, t)$ is fixed for all pairs $s, t \in N(N_t)$ regardless of C_D. Here, for a set S, $S \cap D_i/D_{i+1}$ is a subset of S included in D_i/D_{i+1}. In addition, when $P_i = \mathcal{V}(N_t) \cap D_i/D_{i+1}$ is given for all i, there exists a pair of boundary points and a representative location corresponding to P_1, P_2, because of the "finding boundary points" algorithm. Conversely, in accordance with the "finding sets of points on co-centric disks" algorithm, each pair of boundary points determines a set of center locations of a co-centric

disk pair that has a set of nodes and corners on D_i/D_{i+1} for $i = 1, 2$. Therefore, we can limit the search space of C_D to $\mathcal{H}_{\mathcal{V}(N_t)}$ from $\mathcal{R}^2$.

$$\max_{C_D \in \mathcal{R}^2} E[L_c(C_D)] = \max_{C_D \in \mathcal{H}_{\mathcal{V}(N_t)}} E[L_c(C_D)] \tag{2}$$

Note that $\mathcal{H}_{\mathcal{V}(N_t)}$ is a finite set of locations, and the number of elements in $\mathcal{H}_{\mathcal{V}(N_t)}$ is at most $4|\mathcal{V}(N_t)| \cdot (|\mathcal{V}(N_t)| - 1)$.

4.2 Probability of Connecting a Power Distribution Route

For $s, t \in N_c$, the probability of connecting a power distribution route (routes) $r(s, e(s)), r(t, e(t)) \subset N_p$ is investigated. Because of the assumption that the there is only a single route between s and $e(s)$ for any s, the event that both $r(s, e(s))$ and $r(t, e(t))$ are working requires all the nodes and corners in $r(s, e(s); t, e(t)) \equiv r(s, e(s)) \cup r(t, e(t))$ to be working.

The failure of the node k (corner k) independently occurs when the center of D_i/D_{i+1} is given, and its working probability is $p_{i,p,k}$ ($q_{i,p,k}$) when a node (corner) is in D_i/D_{i+1}. Thus, the probability that nodes in $r(s, e(s); t, e(t)) \subset N_p$ are working is

$$\alpha_N(s, t) = \prod_{k \in N_1(r(s,e(s);t,e(t)))} p_{1,p,k}$$
$$\prod_{m \in N_2(r(s,e(s);t,e(t)))} p_{2,p,m}, \tag{3}$$

where $N_i(x)$ is $N(x) \cap D_i/D_{i+1}$. Similarly, the probability that corners in $r(s, e(s); t, e(t)) \subset N_p$ are working is

$$\alpha_C(s, t) = \prod_{k \in C_1(r(s,e(s);t,e(t)))} q_{1,p,k}$$
$$\prod_{m \in C_2(r(s,e(s);t,e(t)))} q_{2,p,m}, \tag{4}$$

where $C_i(x)$ is $C(x) \cap D_i/D_{i+1}$. The failures of nodes and corners are independent when the center of D_i/D_{i+1} is given. Therefore, the probability $\alpha(s, t)$ of connecting $s, t \in N_c$ to their power distribution stations is given as follows.

$$\alpha(s, t) = \alpha_N(s, t)\alpha_C(s, t). \tag{5}$$

4.3 Probability of Connecting Link in N_c

A working link $l(s, t)$ between $s, t \in N(N_c)$ means that the following three items are working: (i) nodes s, t, (ii) a power distribution route between s and $e(s)$ and that between t and $e(t)$, and (iii) corners in $l(s, t)$. Similarly to the previous subsection, the probability $\beta(s, t)$ of connecting $l(s, t) \subset N_c$ is given as follows.

$$\beta(s, t) = \alpha(s, t)\beta_N(s, t)\beta_C(s, t). \tag{6}$$

Here, $\beta_N(s, t)$ is the working probability of the first item and is given as follows,

$$\beta_N(s,t) = \prod_{k \in \mathcal{N}_1(l(s,t))} p_{1,c,k} \prod_{m \in \mathcal{N}_2(l(s,t))} p_{2,c,m}, \qquad (7)$$

and $\beta_C(s,t)$ is the working probability of the third item and is given as follows,

$$\beta_C(s,t) = \prod_{k \in C_1(l(s,t))} q_{1,c,k} \prod_{m \in C_2(l(s,t))} q_{2,c,m}. \qquad (8)$$

4.4 Proposal of Algorithm

Because $E[L_c(C_D)] = E[\sum_{s,t \in \mathcal{N}(N_c)} \mathbf{1}(i \leftrightarrow j)|C_D] = \sum_{s,t \in \mathcal{N}(N_c)} E[\mathbf{1}(i \leftrightarrow j)|C_D]$, where $\mathbf{1}(i \leftrightarrow j)$ becomes 0 (1) if a link between $i \in N_c$ and $j \in N_c$ is connecting (disconnecting),

$$E[L_c(C_D)] = \sum_{s,t \in \mathcal{N}(N_c)} (1 - \beta(s,t)). \qquad (9)$$

Due to Eq. (2),

$$\begin{aligned} &\max_{C_D \in \mathcal{R}^2} E[L_c(C_D)] \\ &= \max_{C_D \in \mathcal{H}_{\mathcal{V}(N_t)}} \sum_{s,t \in \mathcal{N}(N_c)} (1 - \beta(s,t)). \end{aligned} \qquad (10)$$

Therefore, the following algorithm can evaluate Eq. (10) and derive the critical disaster location that causes the largest expected number of disconnected links in N_c.
[**"Finding critical location" algorithm**] $\mathcal{V}(N_t)$, $\mathcal{L}(N_t)$, $\{r_i, \{p_{i,x,k}\}_{x=c,p;k}, \{q_{i,x,k}\}_{x=c,p;k}\}_{i=1,2}$ are given as input. Apply the "finding sets of points on co-centric disks" algorithm to $\mathcal{V}(N_t)$ and $\{r_i\}_{i=1,2}$, and obtain $\{\{\mathbf{h}\}_{\mathbf{h} \in \mathcal{H}(\mathbf{x}_1, \mathbf{x}_2)}, \mathcal{P}_1(\mathbf{h}), \mathcal{P}_2(\mathbf{h})\}_{\mathbf{x}_1, \mathbf{x}_2 \in \mathcal{V}(N_t)}$, where $\mathbf{h}$ is a possible location of C_D. By using $\mathcal{L}(N_t), \{\{p_{i,x,k}\}_{x=c,p;k}, \{q_{i,x,k}\}_{x=c,p;k}\}_{i=1,2}$ and $\mathcal{P}_1(\mathbf{h}), \mathcal{P}_2(\mathbf{h})$, calculate $\beta(s,t)$ through Eq. (6) for $\forall s, t \in \mathcal{N}(N_c)$ and derive $\sum_{s,t \in \mathcal{N}(N_c)} (1 - \beta(s,t))$ for each $\mathbf{h}$. Repeat this for all $\mathbf{h} \in \mathcal{H}_{\mathcal{V}(N_t)}$. As a result, C_D^*, which is $\mathbf{h}$ making $\sum_{s,t \in \mathcal{N}(N_c)} (1 - \beta(s,t))$ the largest, is obtained. C_D^* is the output.

5. Complexity of Proposed Algorithm

This section evaluates the time-complexity of the proposed algorithm.

In the "finding sets of points on co-centric disks" algorithm, the number of choices of the boundary point pairs is $O(|\mathcal{V}(N_t)|^2)$. For each boundary point pair $\mathbf{x}_1, \mathbf{x}_2$, the time-complexity obtaining $\mathcal{H}(\mathbf{x}_1, \mathbf{x}_2)$ is $O(1)$ and that deriving $\mathcal{P}_1, \mathcal{P}_2$ is $O(|\mathcal{V}(N_t)|)$. To calculate Eq. (5), $r(s, e(s); t, e(t))$ must be obtained. Assume that a table of links of N_p has the upward-links of each link, which is its consecutive link toward the electric power distribution station. (Note that the upward-link is unique for each link of N_p.) For each link of N_c, obtaining $r(s, e(s); t, e(t))$ takes $O(|\mathcal{L}(N_p)|)$. For all links of N_c, it takes $O(|\mathcal{L}(N_p)| \cdot |\mathcal{L}(N_c)|)$. To calculate Eq. (10), use a list of nodes and corners in N_c with the link-IDs to which they belong. Then, calculating Eq. (10) requires to determine which nodes and corners in N_t are in D_i/D_{i+1}. Its complexity is $O(|\mathcal{V}(N_t)|)$ for each boundary point pair
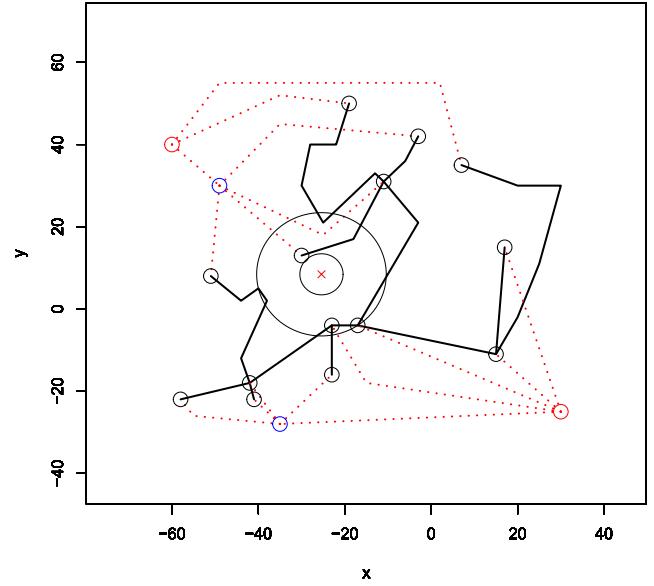

Fig. 4 Result of proposed algorithm for tree network model under default parameter values.
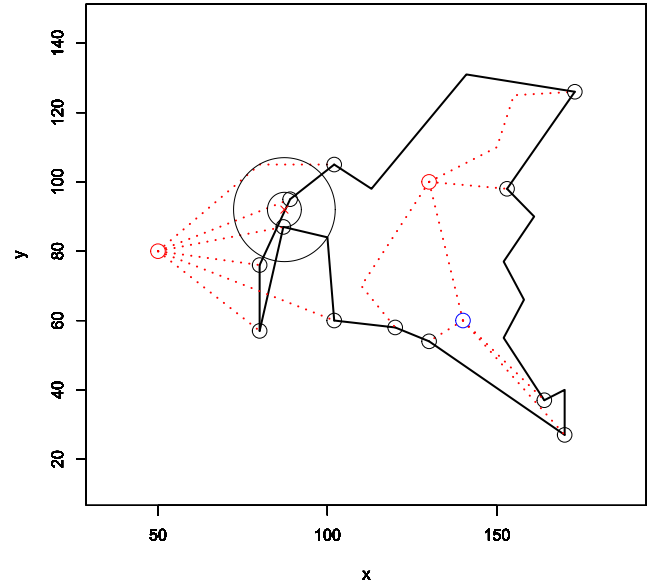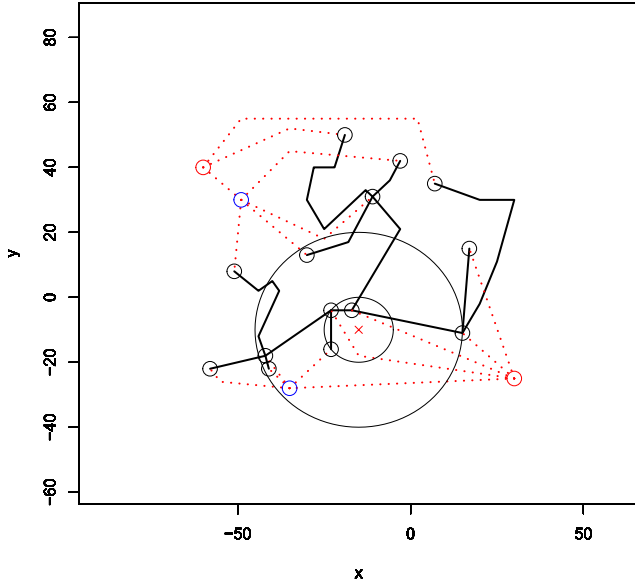

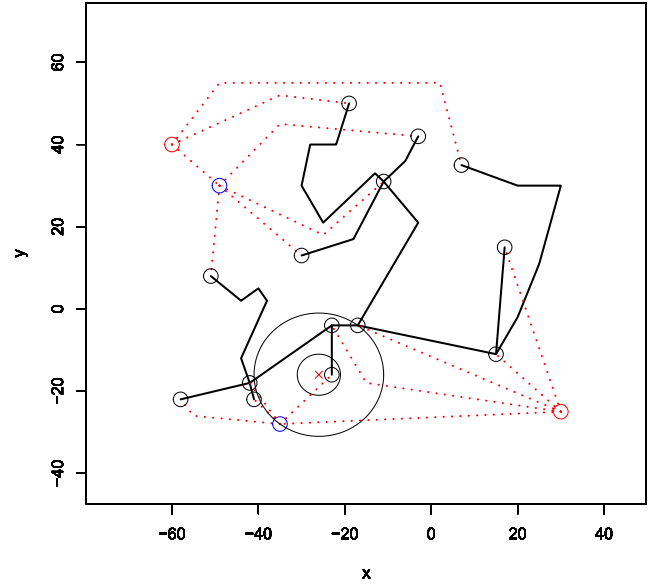**Fig. 5** Result of proposed algorithm for ring network model under default parameter values.

$\mathbf{x}_1, \mathbf{x}_2$. For all boundary point pairs, it takes $O(|\mathcal{V}(N_t)|^3)$. As a total, the time-complexity of the proposed algorithm is $O(|\mathcal{V}(N_t)|^3 + |\mathcal{L}(N_p)| \cdot |\mathcal{L}(N_c)|)$.

6. Numerical Examples

In this section, numerical examples are provided to demonstrate the effectiveness of the proposed method. On the basis of two network models, tree and ring, used in [15], N_ps are added to them. Default parameter values are as follows: $r_1 = 15$, $r_2 = 5$, $p_{1,c,k} = p_{1,p,k} = 0.9$, $q_{1,c,k} = q_{1,p,k} = 0.7$, $p_{2,c,k} = p_{2,p,k} = 0.8$, $q_{2,c,k} = q_{2,p,k} = 0.5$ for all k. In the

Fig. 6 Result of proposed algorithm for tree network model with large disaster area.

Fig. 8 Result of proposed algorithm for tree network model with fragile N_p.
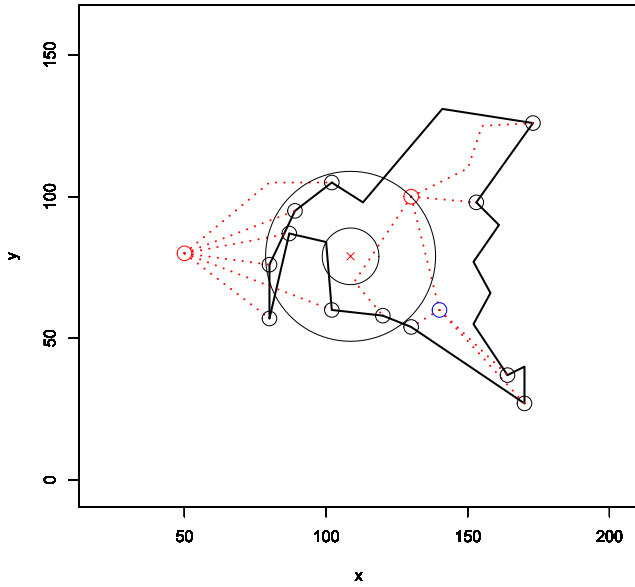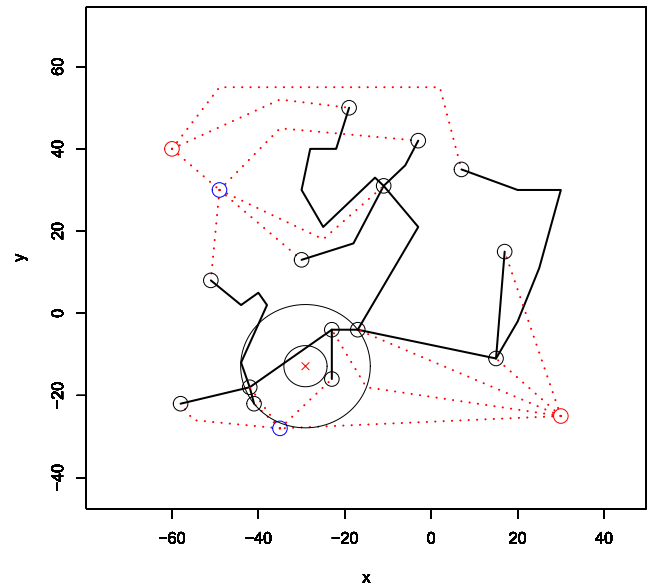

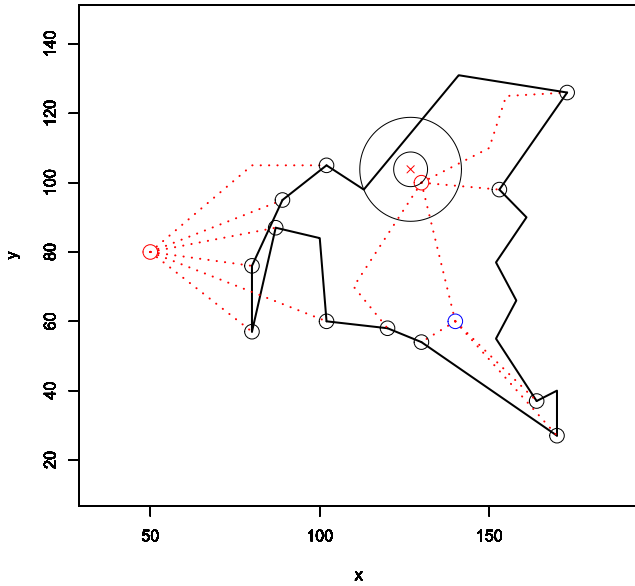**Fig. 7** Result of proposed algorithm for ring network model with large disaster area.

Fig. 9 Result of proposed algorithm for tree network model with fragile N_c.

figures in this section, N_c is drawn in black solid lines, N_p is drawn in red dotted lines, nodes in N_c are black circles, electric power distribution stations are red circles, and nodes in N_p are blue circles. The results of the proposed algorithm are illustrated in Figs. 4–11.
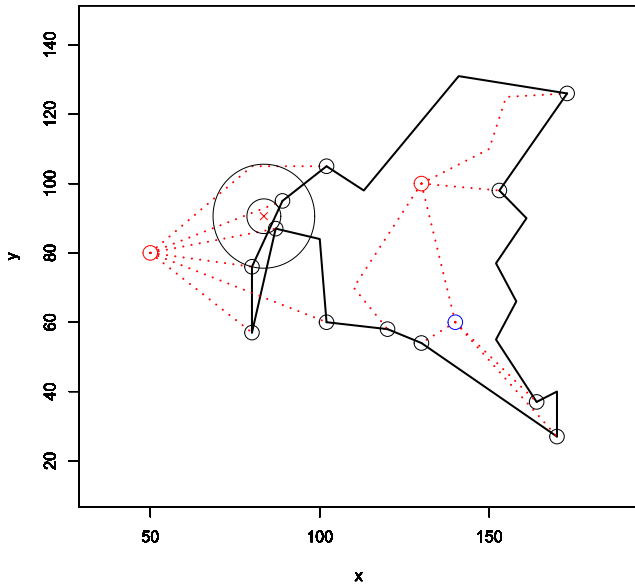
Under the set of default parameter values, the critical location of the disaster center is plotted for each of the network models (Figs. 4 and 5). For a larger disaster ($r_1 = 30$, $r_2 = 10$), it is also plotted (Figs. 6 and 7). In addition, the results for fragile N_p ($p_{1,p,k} = 0.7$, $q_{1,p,k} = 0.5$, $p_{2,p,k} = 0.6$, $q_{2,p,k} = 0.3$ for all k) and N_c ($p_{1,c,k} = 0.7$, $q_{1,c,k} = 0.5$, $p_{2,c,k} = 0.6$, $q_{2,c,k} = 0.3$ for all k) are plotted for the tree

network (Figs. 8 and 9) and ring network (Figs. 10 and 11).

For both the tree and ring networks, under the default parameter values, the disaster was critical when it was in a region where nodes and links are densely populated (Figs. 4 and 5). For a large disaster, it was critical when it was around the center of the whole network because it can cover nearly the whole network. It was clearly seen for the ring network (Figs. 5 and 7). The critical locations for fragile N_p and fragile N_c were very similar for the tree network (Figs. 8 and 9). However, the reasons of causing disconnections are different. The former mainly disconnects N_p, but the latter mainly disconnects N_c. For the ring network, the

Fig. 10 Result of proposed algorithm for ring network model with fragile N_P.

Fig. 11 Result of proposed algorithm for ring network model with fragile N_c.

critical location under the default parameter values and that for fragile N_c were similar (Figs. 5 and 11). The location covered a fairly large number of links in N_c. However, the critical location for fragile N_p was different from them. It covered an electric power distributing station. Thus, the location caused a large number of disconnected links in N_c by disconnecting many power distribution routes.

7. Conclusion

This paper proposed a polynomial-time algorithm that identifies the critical disaster location that causes the largest ex-

pected number of disconnected links in N_c. This work can evaluate the worst case scenario of future disasters and can contribute to the improvement of network survivability. In particular, by taking an electric power grid into account, the effect of disaster through power grid failures can be evaluated. More generic disaster models remain as future work.

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