PAPER

Overloaded MIMO Bi-Directional Communication with Physical Layer Network Coding in Heterogeneous Multihop Networks

Satoshi DENNO[†a)], *Senior Member*, **Tomoya TANIKAWA**[††], *Nonmember, and* **Yafei HOU**[†], *Senior Member*

SUMMARY   This paper proposes overloaded multiple input multiple output (MIMO) bi-directional communication with physical layer network coding (PLNC) to enhance the transmission speed in heterogeneous wireless multihop networks where the number of antennas on the relay is less than that on the terminals. The proposed overloaded MIMO communication system applies precoding and relay filtering to reduce computational complexity in spite of the transmission speed. An eigenvector-based filter is proposed for the relay filter. Furthermore, we propose a technique to select the best filter among candidates eigenvector-based filters. The performance of the proposed overloaded MIMO bi-directional communication is evaluated by computer simulation in a heterogeneous wireless 2-hop network. The proposed filter selection technique attains a gain of about $1.5\,\mathrm{dB}$ at the BER of 10^{-5} in a 2-hop network where 2 antennas and 4 antennas are placed on the relay and the terminal, respectively. This paper shows that 6 stream spatial multiplexing is made possible in the system with 2 antennas on the relay.
key words: wireless multihop network, bi-directional communication, network coding, overloaded MIMO, MMSE precoding

1.   Introduction

A lot of devices have been scattered to make a cyber-physical system (CPS) come reality [1]. Some of those devices are connected to the internet via wireless multihop networks as is done in wireless smart utility network (WiSUN) in order to guarantee the connection in spite of places where those devices are placed [2]. Frequency bands tend to be raised for wider bandwidth, which is crucial to provide users with high speed wireless communication links. As frequency bands are going higher, wireless multihop networks get more indispensable because of higher radio signal attenuation in channels. While time division multiplexing (TDD) or frequency division multiplexing (FDD) is applied to multihop networks to avoid harmful interference, the use of those multiplexing reduces the frequency utilization efficiency. Bi-directional communication further reduces the frequency utilization efficiency in wireless multihop networks. To improve the efficiency, physical layer network coding (PLNC) has been considered [3]–[7]. XOR-PLNC has been shown to achieve superior transmission performance as well as high frequency utilization efficiency. Since the increase in the frequency utilization efficiency is one of the most important issues, on the other hand, many techniques have been proposed to increase the efficiency. For instance, resource allocation, scheduling, distributed antennas, and multiple antenna techniques have been intensively investigated. Multiple antenna techniques, so called multiple input multiple output (MIMO) [8], have been applied to the commercial wireless communication systems, such as the fifth generation cellular system and the IEEE802.11 wireless local area networks (WLAN) [9]–[12]. For further frequency utilization efficiency enhancement, overloaded MIMO has been investigated [13], [14]. Whereas non-linear detectors have been proposed for superior transmission performance [15]–[20], linear detectors have been also proposed to reduce the signal detection complexity [21]. MIMO has been introduced to PLNCs for increasing the frequency utilization efficiency [22], [23]. Precoding for MIMO-PLNCs further have been proposed to reduce detection complexity at relays [24]–[26]. However, when precoding is applied to PLNCs for lower computational complexity, the number of spatial multiplexed signal streams in MIMO is limited to the smallest number of the antennas on the terminals or the relays.

This paper proposes overload MIMO bi-directional communication with PLNC that performs high speed signal transmission with high frequency utilization efficiency in wireless heterogeneous multihop networks[*]. The proposed overloaded MIMO bi-directional communication applies non-linear precoding to reduce the detection complexity. A relay filter is proposed to raise the number of spatial multiplexed signal streams equal to that of the antennas on the terminal, even when the number of antennas on a relay is smaller than that on terminals. We propose a technique to select the best relay filter among candidate filters for the performance improvement. The performance of the proposed overloaded MIMO bi-directional communication is evaluated by computer simulations.

Throughout the paper, $(\mathbf{A})^{-1}$, superscript T, and superscript H denote an inverse matrix of a matrix $\mathbf{A}$, transpose, and Hermitian transpose of a matrix or a vector, respectively. $\mathrm{tr}\,[\mathbf{A}]$ indicates trace of a diagonal matrix A. j denotes the imaginary unit. $\mathrm{E}\,[\zeta]$, $\mathfrak{R}[\alpha]$, and $\mathfrak{I}[\alpha]$ represent the ensemble average of a variable ζ, a real part, and an imaginary part of a complex number α. Let $a \in \mathbb{R}$ and $b \in \mathbb{R}$ represent

[*]Heterogeneous multihop networks is meant to comprise relays and terminals, in which different number of antennas are installed.

real numbers, $\mathrm{mod}\,[a, b]$ indicates the modulo function defined as $\mathrm{mod}\,[a, b] = a - \mathrm{int}\left[\frac{a}{b}\right] \cdot b$ where $\mathrm{int}\,[\bullet]$ indicates a function where an input is rounded down to the nearest integer of an input signal.

2. System Model

We assume that two terminals with N_T antennas exchange their packets via a relay with N_R antennas. The two terminals are called "Terminal A" and "Terminal B" in this paper. For the packet exchange, we apply the time division duplex (TDD) in order to avoid the harmful interference. For improving the spectrum efficiency, the terminals simultaneously send their packets for the relay in the first time slot. The packets received at the relay are forwarded to the terminals after some signal processing in the next time slot. We apply spatial multiplexing for the signal transmission in the two slots for enhancing the throughput between the terminals. In realistic situations, the number of the terminal antennas N_T is not always equal to that of the relay antennas N_R. In this paper, we assume that the number of the terminal antennas N_T is more than that of the relay antennas N_R.

Information bit stream sent from the terminal A is fed to a quadrature phase shift keying (QPSK) modulator and the modulator outputs modulation signals $d_\mathrm{A}(n)$ where n denotes a stream number. Let $\mathbf{D}_\mathrm{A} \in \mathbb{C}^{N_\mathrm{T}}$ represent a modulation signal vector defined as $\mathbf{D}_\mathrm{A} = (d_\mathrm{A}(1) \cdots d_\mathrm{A}(N_\mathrm{T}))^\mathrm{T}$ The modulation signal vector $\mathbf{D}_\mathrm{A}$ is precoded in a manner described in the following section, and sent for the relay. When a precoded signal vector is denoted as $\mathbf{X}_\mathrm{A} \in \mathbb{C}^{N_\mathrm{T}}$, the vector is defined as $\mathbf{X}_\mathrm{A} = [x_\mathrm{A}(1) \cdots x_\mathrm{A}(N_\mathrm{T})]^\mathrm{T}$ where $x_\mathrm{A}(k) \in \mathbb{C}$ represents a transmission signal sent from the kth antenna. When the terminal A and B simultaneously send their packets containing the precoded signals, those packets are received at the relay with N_R antennas in the first slot. Let $\mathbf{Y}_\mathrm{R} \in \mathbb{C}^{N_\mathrm{R}}$ denote a received signal vector, the received signal vector is written as,

$$\mathbf{Y}_\mathrm{R} = \mathbf{H}_\mathrm{A}\mathbf{X}_\mathrm{A} + \mathbf{H}_\mathrm{B}\mathbf{X}_\mathrm{B} + \mathbf{N}. \tag{1}$$

In (1), $\mathbf{X}_\mathrm{B} \in \mathbb{C}^{N_\mathrm{T}}$, $\mathbf{H}_\Omega \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}}$ $\Omega = \mathrm{A}$ or B, and $\mathbf{N} \in \mathbb{C}^{N_\mathrm{R}}$ represent a transmission signal vector sent from the terminal B, a channel matrix between the relay and the terminal A or B, and the additive white Gaussian noise (AWGN) vector.

Rate-one coding is introduced to the received signal vector at the relay and the coded signals are forwarded for the terminals in the second slot, which is essential in the physical layer network coding to double the frequency utilization efficiency in bi-directional communications [4]. Let $\oplus$ indicate rate-one encoding[†], the rate-one encoded signal $d_\mathrm{R}(k) \in \mathbb{C}$ is defined as $d_\mathrm{R}(k) = d_\mathrm{A}(k) \oplus d_\mathrm{B}(k)$. Although the

---

[†]So called Exclusive OR is used as rate-one encoding in this paper. Let input signals and an output signal for the rate-one encoding be denoted as $a \in \mathbb{C}$, $b \in \mathbb{C}$, and $c \in \mathbb{C}$, the rate-one encoding is actually defined as $c = 2 \;\mathrm{mod}\; \left[\frac{\Re[a] + \Re[b]}{2} + \frac{E_\mathrm{d}}{2}, E_\mathrm{d}\right] - \frac{E_\mathrm{d}}{2} + \mathrm{j} \cdot \left(2 \;\mathrm{mod}\; \left[\frac{\Im[a] + \Im[b]}{2} + \frac{E_\mathrm{d}}{2}, E_\mathrm{d}\right] - \frac{E_\mathrm{d}}{2}\right)$ where $E_\mathrm{d} \in \mathbb{R}$ denotes the minimum Euclidean distance in the QPSK modulation.

transmission signals $d_\mathrm{A}(k)$ and $d_\mathrm{B}(k)$ look to be needed for estimating the encoded signal, actually, the encoded signal can be directly estimated from one of the received signals by employing precoding at the terminals as follows [25], [26].

$$\overline{d_\mathrm{R}}(k) = \underset{d_\mathrm{R}(k)}{\arg\max}\, p\left(d_\mathrm{R}(k)|\dot{\mathbf{Y}}_\mathrm{R}\right)$$

$$= \underset{d_\mathrm{R}(k) = d_\mathrm{A}(k) \oplus d_\mathrm{B}(k)}{\arg\max}\, p\left(d_\mathrm{A}(k) \oplus d_\mathrm{B}(k)|\dot{y}_\mathrm{R}(k)\right) \tag{2}$$

In (2), $\overline{d_\mathrm{R}}(k)$ and $p(a|b)$ denote an estimate of the rate-one encoded signal and a conditional probability that an event a happens when an event b occurred. In addition, $\dot{\mathbf{Y}}_\mathrm{R}$ and $\dot{y}_\mathrm{R}(k)$ represent a received signal vector transformed with some signal processing proposed in the following, and the kth element of the vector $\dot{\mathbf{Y}}_\mathrm{R}$, i.e., $\dot{\mathbf{Y}}_\mathrm{R} = (\dot{y}_\mathrm{R}(1) \cdots \dot{y}_\mathrm{R}(N_\mathrm{T}))$. The encoded signals are precoded at the relay and are broadcast for the terminals in the second slot. The transmitted signals are received at the terminals after going through fading channels. Let $\mathbf{X}_\mathrm{R} \in \mathbb{C}^{N_\mathrm{R}}$ denote a precoded signal transmitted from the relay, a signal vector received at the terminal Ω (=A or B) is expressed as,

$$\mathbf{Y}_\Omega = \mathbf{H}_\Omega^\mathrm{H}\mathbf{X}_\mathrm{R} + \mathbf{N}_\Omega. \tag{3}$$

$\mathbf{Y}_\Omega \in \mathbb{C}^{N_\mathrm{T}}$ and $\mathbf{N}_\Omega \in \mathbb{C}^{N_\mathrm{T}}$ $\Omega = \mathrm{A}$ or B denote a received signal vector and the AWGN vector at the terminal Ω. Because the system model shown in (3) is regarded as a usual non-overloaded channel, any conventional MIMO detectors can be applied such as a minimum mean square (MMSE) filter. If the encoded signals $d_\mathrm{R}(k)$ is detected with one of those detectors, the transmission signals sent from the other terminals can be obtained with the rate-one encoding at the terminal Ω, i.e., $\overline{d_{\tilde{\Omega}}}(k) = \overline{d_\mathrm{R}}(k) \oplus d_\Omega(k)$, where $\overline{d_{\tilde{\Omega}}}(k)$ represents an estimate of the signal sent from the other terminal $\tilde{\Omega}$.

When the number of the relay antennas is less than that of the terminal antennas as described above, the number of signal streams is reduced to that of the relay antennas if the precoding such as the MMSE precoding and the zero forcing (ZF) precoding is applied at the terminals for detection complexity reduction of the relay [25], [26]. We propose a technique to enhance the throughput in the next section[††].

3. Overloaded MIMO Signal Transmission with Physical Layer Network Coding

As is described above, the number of the terminal antennas is

---

[††]While the technique proposed in the following section is described only for systems with the QPSK modulation, the proposed technique can be applied to systems with any quadrature amplitude modulations (QAMs), if the rate-one coding is adjusted for the modulation schemes [27].

[††]Although the number of the antennas on the terminals is same in the above system model, this is not mandatory. The proposed technique can be applied to systems where the number of the antennas on a terminal is different from that on the other. However, we are afraid that the imbalance causes the proposed technique difficult to understand. Hence, we would like to explain the proposed technique in the system where the two terminals have the same number of antennas, for simplicity.

more than that of the relay antennas. If the terminals send the signal streams without any precoding in the first slot, the relay will receive so many independent signals. In a word, the system will become a very heavy overloaded MIMO system, which causes signal detection at the relay to be very difficult and the transmission performance to be degraded. For the sake of resolving the difficulty, we apply precoding at the terminal as described in the previous section. When precoding is applied at the terminal for the signal transmission in the first slot, however, the number of the spatial multiplexed signal streams is reduced to that of the relay antennas, even though the transmitter has more antennas than the relay.

This section proposes a technique to increase the number of the spatial multiplexed signal streams to that of the terminal antennas for enhancing the frequency utilization efficiency.

3.1 Spatial Filtering at Relay in First Slot

For the sake of increasing the number of the spatial multiplexing signal streams, we introduce a spatial filter $\mathbf{G} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{R}}$ that is applied to the relay as follows.

$$\dot{\mathbf{Y}}_\mathrm{R}^{(\mathbf{G})} = \mathbf{G}\mathbf{Y}_\mathrm{R} = \mathbf{G}\mathbf{H}_\mathrm{A}\mathbf{X}_\mathrm{A}^{(\mathbf{G})} + \mathbf{G}\mathbf{H}_\mathrm{B}\mathbf{X}_\mathrm{B}^{(\mathbf{G})} + \mathbf{G}\mathbf{N} \qquad (4)$$

$\dot{\mathbf{Y}}_\mathrm{R}^{(\mathbf{G})} \in \mathbb{C}^{N_\mathrm{T}}$ and $\mathbf{X}_\Omega^{(\mathbf{G})} \in \mathbb{C}^{N_\mathrm{T}}$ represent a filtered received signal vector and a precoded transmission signal vector for a filtered channel $\mathbf{G}\mathbf{H}_\Omega{}^\dagger$. The MMSE filter is applied for the precoding in this paper. The precoded signal $\mathbf{X}_\Omega^{(\mathbf{G})}$ for the filtered channel is written as [28],

$$\mathbf{X}_\Omega^{(\mathbf{G})} = g_\Omega \left(\mathbf{G}\mathbf{H}_\Omega\right)^\mathrm{H} \left(\left(\mathbf{G}\mathbf{H}_\Omega\right)\left(\mathbf{G}\mathbf{H}_\Omega\right)^\mathrm{H} + \gamma\mathbf{I}_{N_\mathrm{T}}\right)^{-1} \dot{\mathbf{D}}_\Omega. \tag{5}$$

$g_\Omega \in \mathbb{R}$, $\mathbf{I}_{N_\mathrm{T}} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$, and $\dot{\mathbf{D}}_\Omega \in \mathbb{C}^{N_\mathrm{T}}$ denote a normalization factor, the N_T-dimensional identity matrix, and a modulation signal vector with Gaussian integer multiples of a modulus M_Ω which is afterwards defined through nonlinear filtering. In addition, $\gamma \in \mathbb{R}$ is a constant defined as $\gamma = \frac{2\sigma^2 \mathrm{tr}[\mathbf{G}^\mathrm{H}\mathbf{G}]}{P_0}$, where P_0 represents the transmission power of the precoded signal vector, i.e., $P_0 = \mathrm{E}\left[\left(\mathbf{X}_\Omega^{(\mathbf{G})}\right)^\mathrm{H} \mathbf{X}_\Omega^{(\mathbf{G})}\right]$. Even if the spatial filter $\mathbf{G}$ is introduced in the system, the channel in the first slot is regarded as one of overloaded MIMO channels. To improve the transmission performance in the overloaded channel, the lattice reduction is applied to the precoding. We employ the Lenstra–Lenstra–Lovász (LLL) algorithm [29] that implements the lattice reduction as follows [21], [30], [31].

$$\begin{pmatrix} (\mathbf{G}\mathbf{H}_\Omega)^\mathrm{H} \\ \sqrt{\gamma}\mathbf{I}_{N_\mathrm{T}} \end{pmatrix} \mathbf{T}_\Omega = \mathbf{Q}_\Omega \mathbf{\Gamma}_\Omega \mathbf{R}_\Omega$$

$$= \begin{pmatrix} \mathbf{Q}_{1,\Omega} \\ \mathbf{Q}_{2,\Omega} \end{pmatrix} \mathbf{\Gamma}_\Omega \mathbf{R}_\Omega \tag{6}$$

In (6), $\mathbf{Q}_\Omega \in \mathbb{C}^{2N_\mathrm{T} \times N_\mathrm{T}}$, $\mathbf{\Gamma}_\Omega \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$, $\mathbf{R}_\Omega \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$, and $\mathbf{T}_\Omega \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$ represent an orthogonal matrix, a diagonal matrix, an upper triangular matrix with all diagonal elements equal to 1, and a unimodular matrix that has Gaussian integers as its elements. In addition, $\mathbf{Q}_{1,\Omega} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$ and $\mathbf{Q}_{2,\Omega} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$ denote an upper part and a lower part of the orthogonal matrix $\mathbf{Q}_\Omega$. By substituting the equation in (6) for that in (5), the precoded signal can be simplified as,

$$\mathbf{X}_\Omega^{(\mathbf{G})} = g_\Omega \mathbf{Q}_{1,\Omega} \mathbf{\Gamma}_\Omega^{-1} \mathbf{V}_\Omega. \tag{7}$$

$\mathbf{V}_\Omega \in \mathbb{C}^{N_\mathrm{T}}$ represents a non-linear filter output vector defined below. Let the non-linear filter output vector be expressed as $\mathbf{V}_\Omega = [v_\Omega(1) \cdots v_\Omega(N_\mathrm{T})]^\mathrm{T}$, the non-linear filtering is expressed as,

$$\begin{aligned} v_\Omega(k) &= \mathrm{MOD}\left[\mathbf{T}_\Omega(k)^\mathrm{H}\mathbf{D}_\Omega - \mathbf{B}_\Omega(k)\mathbf{V}_\Omega, M_\Omega\right] \\ &= \mathbf{T}_\Omega(k)^\mathrm{H}\mathbf{D}_\Omega - \mathbf{B}_\Omega(k)\mathbf{V}_\Omega + \varphi_\Omega(k)M_\Omega. \end{aligned} \tag{8}$$

$\varphi_\Omega(k) \in \mathbb{C}$, $\mathbf{T}_\Omega(k) \in \mathbb{C}^{N_\mathrm{T}}$, and $\mathbf{B}_\Omega(k) \in \mathbb{C}^{1 \times N_\mathrm{T}}$ represent a Gaussian integer, the kth column of the unimodular matrix $\mathbf{T}_\Omega$, and the kth row vector of a lower triangular matrix $\mathbf{B}_\Omega \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$ defined as $\mathbf{B}_\Omega = \mathbf{R}_\Omega^\mathrm{H} - \mathbf{I}_{N_\mathrm{T}}$. Let $\mathbf{C} \in \mathbb{C}^L$ denote a vector defined as $\mathbf{C} = (c(1) \cdots c(L))^\mathrm{T}$ where $c(k) \in \mathbb{C}$ denotes kth entry of the vector $\mathbf{C}$, in addition, $\mathrm{MOD}[\mathbf{C}, M] \in \mathbb{C}^L$ represents a modulo function defined as $\mathrm{MOD}[\mathbf{C}, M] = (\bmod[\Re[c(1)], M] + \mathrm{j} \cdot \bmod[\Im[c(1)], M] \cdots$ $\bmod[\Re[c(L)], M] + \mathrm{j} \cdot \bmod[\Im[c(L)], M])^\mathrm{T}$. If the above non-linear filter is expressed in a vector form, the non-linear filter output vector $\mathbf{V}_\Omega$ can be written as,

$$\begin{aligned} \mathbf{V}_\Omega &= \mathbf{T}_\Omega^\mathrm{H}\mathbf{D}_\Omega - \mathbf{B}_\Omega\mathbf{V}_\Omega + \mathbf{\Phi}_\Omega M_\Omega \\ &= \mathbf{T}_\Omega^\mathrm{H}\dot{\mathbf{D}}_\Omega - \mathbf{B}_\Omega\mathbf{V}_\Omega, \end{aligned} \tag{9}$$

where

$$\dot{\mathbf{D}}_\Omega = \mathbf{D}_\Omega + \mathbf{T}_\Omega^{-\mathrm{H}}\mathbf{\Phi}_\Omega M_\Omega. \tag{10}$$

$\mathbf{\Phi}_\Omega \in \mathbb{C}^{N_\mathrm{T}}$ represents a vector containing the Gaussian integers $\varphi_\Omega(k)$, i.e., $\mathbf{\Phi}_\Omega = [\varphi_\Omega(1) \cdots \varphi_\Omega(N_\mathrm{T})]^\mathrm{T}$. As is shown above, the modulation signal vector $\dot{\mathbf{D}}_\Omega$ is output by the feedback filter where the Gaussian-integers $\varphi_\Omega(n)$ are added to the input signal vectors $d_\mathrm{A}(n)$ to limit the output signal amplitude less than $\frac{M_\Omega}{2}$. The unimodular matrix $\mathbf{T}_\Omega$ helps the feedback filter to find better Gaussian-integers.

On the other hand, the normalization factor g_Ω can be defined as follows.

$$g_\Omega = \sqrt{\frac{P_0}{\mathrm{tr}\left[\mathbf{Q}_{1,\Omega}^\mathrm{H}\mathbf{Q}_{1,\Omega}\mathbf{\Gamma}_\Omega^{-1}\mathbf{\Psi}_\Omega\mathbf{\Gamma}_\Omega^{-1}\right]}} \tag{11}$$

In (11), $\mathbf{\Psi}_\Omega \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{T}}$ represents a correlation matrix of the non-linear filter output vector $\mathbf{V}_\Omega$, which is approximated as [28][††],

---

†The vector $\dot{\mathbf{Y}}_\mathrm{R}^{(\mathbf{G})}$ is equal to the received signal vector $\dot{\mathbf{Y}}_\mathrm{R}$ written in (2).

††As the distribution of the non-linear filter is close to the uniform distribution from $-\frac{M_\Omega}{2}$ to $\frac{M_\Omega}{2}$, the approximation becomes precise.

$$\Psi_\Omega = E\left[\mathbf{V}_\Omega \mathbf{V}_\Omega^H\right] \approx \frac{1}{6} M_\Omega^2 \mathbf{I}_{N_T}. \tag{12}$$

In (12), $\mathbf{I}_{N_T} \in \mathbb{C}^{N_T \times N_T}$ denote the identity matrix. Because the LLL algorithm improves system performances, the characteristics of the LLL algorithm has been reviewed in literature [30], [31]. Such literature implies that the LLL algorithm enlarges the minimum diagonal element in the diagonal matrix $\mathbf{\Gamma}_\Omega$ with the unimodular matrix $\mathbf{T}_\Omega$. This leads to increase the normalization factor, because the reciprocal of the eigenvalues are included in the denominator of the Eq. (11). The increase in the normalization factor results in the improvement of the signal power to noise power ratio (SNR).

As is shown in the above, the non-linear filter keeps the power of the output signals constant despite of channel conditions and the number of transmit signals, i.e., overloading ratio with the MOD function[†]. This improves the SNR at the relay.

3.1.1 Modulus Setting and Signal Reception

After the relay receives the signal vector $\mathbf{Y}_R$ with the spatial filter $\mathbf{G}$, the relay carries out signal detection defined in (2). However, since the non-linear filter is used at the terminal, the detection can be implemented with region detection, which is carried out after the modulo operation as follows.

$$\mathbf{D}_R = \mathrm{DET}\left[\mathrm{mod}\left[\dot{\mathbf{Y}}_R^{(\mathbf{G})}, M_R\right], g_A, g_B\right] \tag{13}$$

$\mathbf{D}_R \in \mathbb{C}^{N_T}$ in (13) denotes a rate-one encoded signal vector defined as $\mathbf{D}_R = [d_R(1) \cdots d_R(N_T)]^T$. In addition, M_R denotes a modulus defined as [26],

$$M_R = 2(g_A + g_B). \tag{14}$$

The modulus for the terminals is also derived from the modulus above.

$$M_A = 2\left(1 + \frac{g_B}{g_A}\right), \quad M_B = 2\left(1 + \frac{g_A}{g_B}\right) \tag{15}$$

The function DET[●] carries out region detection for every input signal, independently. This means the same region detection is performed for every input signal of the input signal vector. In a word, the function DET[●] can be defined for vectors with any dimension and even for scalars without change of the definition, because of the independency of the signal processing. Let $a \in \mathbb{C}$ be a complex number, hence, the function is decomposed as $\mathrm{DET}[a, g_A, g_B] = \det[\Re[a], g_A, g_B] + \mathrm{j} \cdot \det[\Im[a], g_A, g_B]$. Let $x \in \mathbb{R}$ be a real number, the function $\det[x]$ is defined as [26],

$$\det[x, g_A, g_B] = \begin{cases} \frac{1}{2}E_d & |x| \le \max[g_A, g_B] \\ -\frac{1}{2}E_d & |x| > \max[g_A, g_B] \end{cases}, \tag{16}$$

where E_d represents the minimum Euclidean distance of the QPSK constellation. The detector only needs amplifiers and the rate-one encoders. Since the rate-one encoders are implemented with the region detectors defined in (13) and (16), the complexity of the detector is negligible small compared with that of the spatial filter. In other words, the detection complexity is quite small because the detector does not need any complex multiplications, even though the detector carries out the demodulation of more than NR signals.

As is described above, the rate-one encoded vector $\mathbf{D}_R$ is broadcast for the terminals in a manner proposed in the following section.

3.2 Signal Transmission in Second Slot

Because N_T streams are transmitted in the first slot, N_T rate-one encoded signal steams are obtained as shown in the previous section. This means that N_T signal streams have to be transmitted from the relay even if only N_R ($< N_T$) antennas are placed on the relay. For the signal transmission, we apply the spatial filter $\mathbf{G}$ even in the second slot as,

$$\mathbf{X}_R = g_R \mathbf{G}^H \mathbf{D}_R, \tag{17}$$

where $g_R \in \mathbb{R}$ represents a normalization factor defined in the following.

$$g_R = \sqrt{\frac{P_0}{\sigma_D^2 \mathrm{tr}\left[\mathbf{G}^H \mathbf{G}\right]}}, \tag{18}$$

where σ_D^2 denotes the power of the modulation signal, i.e., $\sigma_D^2 = E\left[|d_R(k)|^2\right]$. Although the number of the antennas on the relay is different from that on the terminal, the same transmission power is set to the relay, because the number of the signal streams sent from the terminals is the same to that from the relay.

We explained our proposed technique in the two hop network, because this network is the simplest in multihop networks. The proposed technique can be applied to more general heterogeneous multihop networks, in principle, although we might have some difficulties to solve in such general networks. To solve such difficulties is one of our future works.

3.3 Spatial Filter

As is shown above, the spatial filter $\mathbf{G}$ plays an important role in the proposed system. The section introduces a configuration for the spatial filter. First of all, the channel matrix $\mathbf{H}_\Omega$ is decomposed with the singular value decomposition (SVD) as,

$$\mathbf{H}_\Omega = \mathbf{S}_\Omega \mathbf{\Lambda}_\Omega \mathbf{U}_\Omega^H. \tag{19}$$

In (19), $\mathbf{S}_\Omega \in \mathbb{C}^{N_R \times N_R}$, $\mathbf{\Lambda}_\Omega \in \mathbb{C}^{N_R \times N_R}$, and $\mathbf{U}_\Omega \in \mathbb{C}^{N_T \times N_R}$ represent a unitary matrix, a diagonal matrix with singular values in the diagonal positions, and an orthogonal matrix, i.e., $\mathbf{U}_\Omega^H \mathbf{U}_\Omega = \mathbf{I}_{N_R}$. With the unitary matrix $\mathbf{S}_\Omega$ and the

---

[†]If the MOD[●] function is removed, the filter will be reduced to a linear filter. The power of the output signals from the linear filter is fluctuated, depending on channel conditions and overloading ratio.

orthogonal matrix $\mathbf{U}_\Omega$, we propose a configuration of the spatial filter.

$$\mathbf{G}^{(\Omega)} = \mathbf{U}_\Omega \mathbf{S}_\Omega^{\mathrm{H}} \quad \Omega = \text{A or B} \tag{20}$$

This configuration can be seen reasonable because of the reason described in Appendix. Furthermore, we propose a technique to select the best spatial filter in the two candidates, i.e., $\mathbf{G}^{(A)}$ and $\mathbf{G}^{(B)}$, in the next section.

4. Filter Selection

4.1 Noise Based Selection

The selection of the spatial filter affects the transmission performance of the proposed system. The performance is dependent on the SNR. When precoding based on the MMSE criterion is used, the transmission performance is proportional to the reciprocal of the noise in the received signal at the relay. The noise can be defined as follows.

$$\begin{aligned}
\bar{N}_{\mathrm{R}}^{(\mathbf{G})} &= \mathbf{G}\mathbf{Y}_{\mathrm{R}}^{(\mathbf{G})} - (g_{\mathrm{A}}\mathbf{D}_{\mathrm{A}} + g_{\mathrm{B}}\mathbf{D}_{\mathrm{B}}) \\
&= \left(\mathbf{G}\mathbf{H}_{\mathrm{A}}\mathbf{X}_{\mathrm{A}}^{(\mathbf{G})} - g_{\mathrm{A}}\mathbf{D}_{\mathrm{A}}\right) + \left(\mathbf{G}\mathbf{H}_{\mathrm{B}}\mathbf{X}_{\mathrm{B}}^{(\mathbf{G})} - g_{\mathrm{B}}\mathbf{D}_{\mathrm{B}}\right) + \mathbf{G}\mathbf{N}
\end{aligned} \tag{21}$$

$\bar{N}_{\mathrm{R}}^{(\mathbf{G})} \in \mathbb{C}^{N_{\mathrm{T}}}$ in (21) denotes a noise vector in the vector received at the relay. If we assume that the Gaussian integer multiples of the modulus $\boldsymbol{\Phi}_{\mathrm{A}} M_{\mathrm{A}} + \boldsymbol{\Phi}_{\mathrm{B}} M_{\mathrm{B}}$ are ideally removed by the modulo function in (13), the terms in the parenthesis in the right hand side of (21) can be rewritten by substituting the equation in (6) for (5) as follows.

$$\mathbf{G}\mathbf{H}_\Omega \mathbf{X}_\Omega^{(\mathbf{G})} = g_\Omega \left(\mathbf{D}_\Omega - \gamma \mathbf{T}_\Omega \mathbf{R}_\Omega^{-1} |\mathbf{\Lambda}_\Omega|^{-2} \mathbf{V}_\Omega\right) \tag{22}$$

By using the above equation, the power of the noise can be derived as,

$$\begin{aligned}
\epsilon^{(\mathbf{G})} &= \mathrm{E}\left[\left(\bar{N}_{\mathrm{R}}^{(\mathbf{G})}\right)^{\mathrm{H}} \bar{N}_{\mathrm{R}}^{(\mathbf{G})}\right] \\
&= \mathrm{E}\left[\Big| -g_{\mathrm{A}} \left(\gamma \mathbf{T}_{\mathrm{A}} \mathbf{R}_{\mathrm{A}}^{-1} |\mathbf{\Lambda}_{\mathrm{A}}|^{-2} \mathbf{V}_{\mathrm{A}}\right)\right. \\
&\qquad\qquad \left.-g_{\mathrm{A}} \left(\gamma \mathbf{T}_{\mathrm{A}} \mathbf{R}_{\mathrm{A}}^{-1} |\mathbf{\Lambda}_{\mathrm{A}}|^{-2} \mathbf{V}_{\mathrm{A}}\right) + \mathbf{G}\mathbf{N}\Big|^2\right] \\
&= \gamma^2 \mathrm{tr}\left[\gamma^2 \left(g_{\mathrm{A}}^2 \mathbf{R}_{\mathrm{A}}^{-\mathrm{H}} |\mathbf{T}_{\mathrm{A}}|^2 \mathbf{R}_{\mathrm{A}}^{-1} |\mathbf{\Lambda}_{\mathrm{A}}|^{-4} \boldsymbol{\Phi}_{\mathrm{A}}\right.\right. \\
&\qquad \left.\left.+g_{\mathrm{B}}^2 \mathbf{R}_{\mathrm{B}}^{-\mathrm{H}} |\mathbf{T}_{\mathrm{B}}|^2 \mathbf{R}_{\mathrm{B}}^{-1} |\mathbf{\Lambda}_{\mathrm{B}}|^{-4} \boldsymbol{\Phi}_{\mathrm{B}}\right) + 2\sigma^2 \mathbf{G}\mathbf{G}^{\mathrm{H}}\right].
\end{aligned} \tag{23}$$

$\epsilon^{(\mathbf{G})} \in \mathbb{R}$ represents the noise power when the spatial filter $\mathbf{G}$ is used in the proposed communication system.

We select one of the spatial filters proposed in the previous section based on the following criterion.

$$\mathbf{G} = \begin{cases} \mathbf{G}^{(A)} & \epsilon^{(\mathbf{G}^{(A)})} \leq \epsilon^{(\mathbf{G}^{(B)})} \\ \mathbf{G}^{(B)} & \epsilon^{(\mathbf{G}^{(A)})} > \epsilon^{(\mathbf{G}^{(B)})} \end{cases} \tag{24}$$

In a word, the proposed technique selects the spatial filter to improve the SNR performance, which is expected to lead to

the improvement in the BER performance.

4.2 Eigenvalues Based Selection

Motivated by a technique proposed in [32], we consider eigenvalues of channels for the spatial filter selection. Let the diagonal matrix $\mathbf{\Lambda}_\Omega$ be defined as $\mathbf{\Lambda}_\Omega = \mathrm{diag}\left[\lambda_\Omega(1) \cdots \lambda_\Omega(N_{\mathrm{R}})\right]$ where $\lambda_\Omega(i)$ represents ith eigenvalue in the channel, the product of all the eigenvalues in the channel can be written as,

$$\Xi_\Omega = \prod_{i=1}^{N_{\mathrm{R}}} \lambda_\Omega(i) \quad \Omega = \text{A or B}. \tag{25}$$

$\Xi_\Omega \in \mathbb{R}$ denotes a product of the eigenvalues.

The product of all the eigenvalues in the channels is applied for the selection as follows.

$$\mathbf{G} = \begin{cases} \mathbf{G}^{(A)} & \Xi_{\mathrm{A}} \geq \Xi_{\mathrm{B}} \\ \mathbf{G}^{(B)} & \Xi_{\mathrm{A}} < \Xi_{\mathrm{B}} \end{cases} \tag{26}$$

If the product Ξ_{A} is bigger than Ξ_{B}, we speculate that the minimum eigenvalue in $\mathbf{\Lambda}_{\mathrm{A}}$ is greater than that in $\mathbf{\Lambda}_{\mathrm{B}}$. When this speculation is applied for the selection, the spatial filter can be selected as follows.

$$\mathbf{G} = \begin{cases} \mathbf{G}^{(A)} & \min_{k=1\cdots N_{\mathrm{T}}} [\lambda_{\mathrm{A}}(k)] \geq \min_{k=1\cdots N_{\mathrm{R}}} [\lambda_{\mathrm{B}}(k)] \\ \mathbf{G}^{(B)} & \min_{k=1\cdots N_{\mathrm{T}}} [\lambda_{\mathrm{A}}(k)] < \min_{k=1\cdots N_{\mathrm{R}}} [\lambda_{\mathrm{B}}(k)] \end{cases} \tag{27}$$

5. Simulation

The performance of the proposed communication is evaluated by computer simulation. While the number of the antennas on the terminals is more than that on the relay as described in Sect. 2, the number of the spatial multiplexed signal streams is the same to that of the antennas on the terminal. The modulation scheme is quaternary phase shift keying (QPSK), and no error correction coding is applied. Rayleigh fading based on the Jakes' model [33] is applied to the channels between the relay and the terminals. Simulation parameters are listed in Table 1.

5.1 Performance at Relay

The BER performance in the first slot is shown in Fig. 1 for the comparison of the proposed selection techniques. In the figure, a technique to use the spatial filter without selection

Table 1 Simulation parameters.

Modulation	QPSK/Single Carrier
No. of hops	2
Channel model	Rayleigh fading
Channel estimation	perfect
No. of antennas on relay	2
No. of antennas on terminals	2, 3, 4, 6
Error correction coding	N.A.

Fig. 1 Selection technique comparison.

Fig. 2 BER performance in the fist slot.
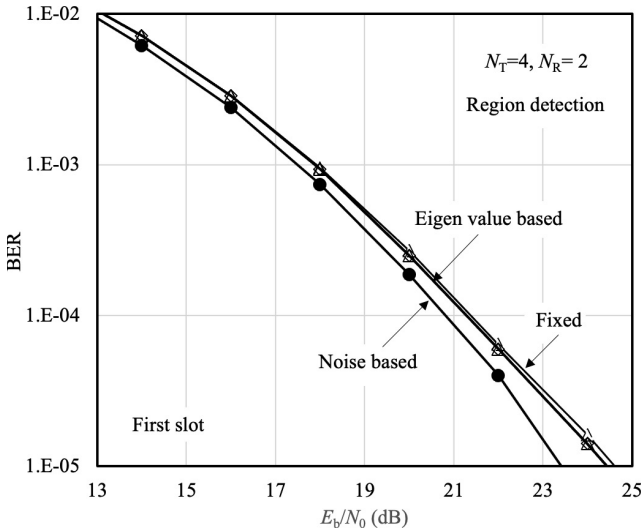
is added as a reference, which is named "fixed" in the figure. In this paper, actually, the "fixed" technique applies the spatial filter made from the channel matrix between the relay and the terminal A. The number of the antennas on the terminal N_T is 4 and that on the relay N_R is 2 in the figure. The two eigenvalue based selection techniques attain the same performance. The proposed noise based selection achieves the best transmission performance among all the techniques shown in the figure. The proposed noise based selection achieves about 1.5 dB better BER performance than the other techniques at the BER of 10^{-5}. The fixed technique is the worst in all the techniques, though the performance gap between the fixed technique and the eigenvalue based technique is small. In principle, the transmission performance is affected by eigenvalue distributions in MIMO channels. Since the number of eigenvalues is smaller than that of the spatial multiplexing streams in overloaded MIMO systems, the transmission performance is greatly dependent on the number of the eigenvalues. Even though we apply the spatial filters at the relay, the application does not help to increase the number of the eigenvalues. This means that the performance improvement by the selection of the relays is limited. While the noise-based selection is regarded as the optimum selection in terms of the SNR, the eigenvalue-based technique is a near optimum technique [32]. This is the reason why the eigenvalue based technique achieves a small performance gain.

Figure 2 shows the BER performance of the proposed technique in various overloaded MIMO channels. Because the noise based selection achieves the best performance in all the techniques, the noise based selection is used in the figure. The number of the antennas on the relay is fixed to 2, while the number of the antennas on the terminal is changed from 2 to 6. Because the number of the transmit antennas is the same to that of the spatial multiplexing signal streams, in a word, overloading ratio is set from 1 to 3. While the dotted line shows the performance in a non-overloaded
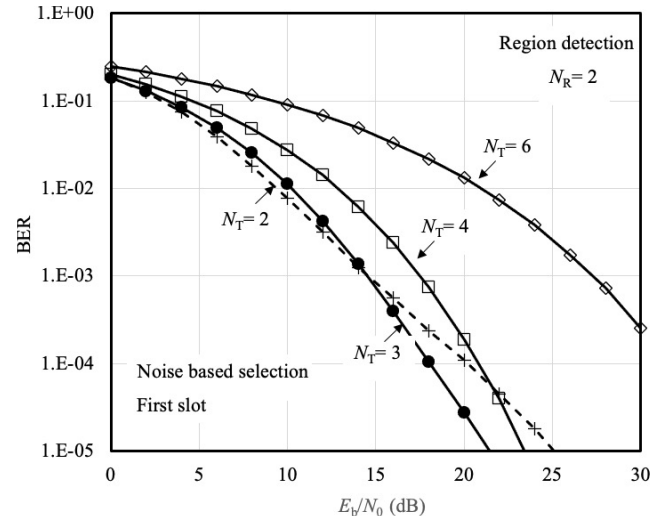
MIMO channel, the solid lines indicate the performance in overloaded MIMO channels. As is shown in the figure, although the transmission performance gets worse as the number of the spatial multiplexed signal streams increases in the overloaded channels, the higher speed signal transmission is made possible by the proposed communication. On the contrary, the proposed technique achieves better BER performance in the channel with overloading ratio of 1.5 than in that of 1.0 at the BER of less than 10^{-3}. The proposed overloaded precoder in conjunction with the precoder at the relay enables the system to attain the diversity gain. As the number of the signal streams is increased from 3 to 4, the BER curve gets steeper in the range of the BER less than 10^{-3}. However, the diversity gain is difficult to obtain when the number signal streams is 6[†].

5.2 Performance in Second Slot

Figure 3 compares the proposed selection technique in terms of the BER between the relay and terminal in the second slot. Those performances shown in the figure are the BER in the channel in only the second slot when one of the proposed relay filter selection techniques is used. The number of the antennas on the relay is 2, whereas that on the terminal is 4 in the channel. Because 4 spatial multiplexed signal steams are received at the both terminals with 4 antennas, the channel in the second slot is not overloaded, which means that any MIMO detectors can be applied at the terminals. However, we apply the maximal likelihood detector (MLD) to the terminals for the signal reception in order to show the performance upper bound. As is shown in the figure, the BER performance does not depend on the selection of the relay filter **G**. In addition, the performance is a little bit better than that in the first slot.

[†]This is only the observation. The performance will be proven theoretically as one of our future works.
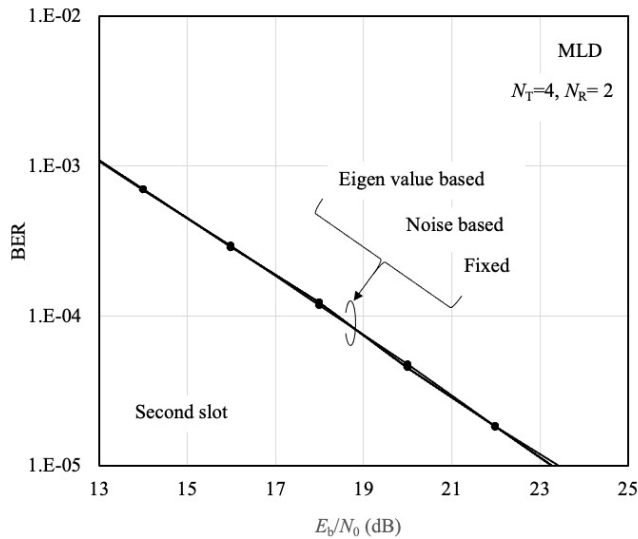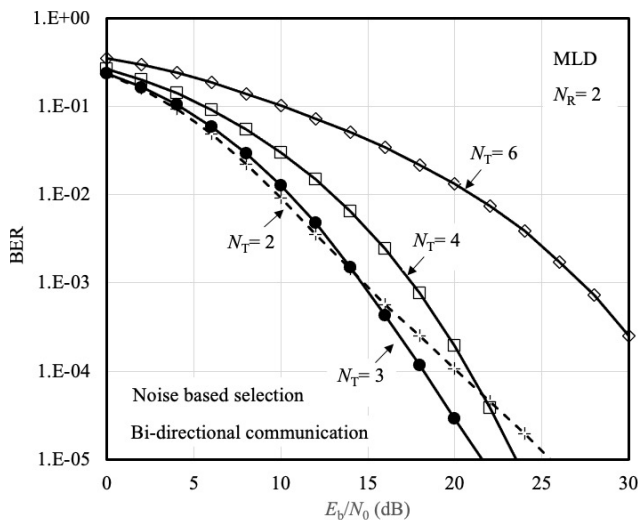
Fig. 3 BER performance in second slot.

Fig. 4 BER performance of bidirectional communication.

5.3 Performance of Bi-Directional Communications

Figure 4 shows the BER performance of the proposed overloaded MIMO bi-directional communications with the PLNC where N_T spatial multiplexed signal streams are exchanged between the terminals in the 2-hop relay systems, in spite of the number of the relay antennas. In the performance evaluation, the noise based selection is applied and the MLD is used for the signal detection at the terminals. Since the BER performance in the second slot is better than that in the first slot as described above, the BER performance of the bi-directional communication with the PLNC is dominated by the performance in the first slot. In other words, the BER performances of the bi-directional communication with the PLNC is almost the same to those in the first slot.

Summarizing, although the transmission performance gets worse as the number of the spatial multiplexed sig-

nal streams increases in the overloaded channels, the higher speed signal transmission is made possible by the proposed communication. On the contrary, the proposed overloaded MIMO bi-directional communication achieves better BER performance than the non-overloaded bi-directional communication in the higher BER region, as far as the overloading ratio is less than 3, even though the transmission speed is increased.

6. Conclusion

This paper has proposed overload MIMO bi-directional communication with physical layer network coding that performs high speed signal transmission with high frequency utilization efficiency in wireless heterogeneous multihop networks. A relay filter is applied to raise the number of the spatial multiplexed signal streams equal to that of the antennas on the terminal, even when the number of the antennas on the relay is less than that on the terminal. We propose a relay filter configuration made from the SVD of the channels, based on which some relay filter candidates can be produced. Some techniques are proposed to select the best relay filter in the candidates.

The performance of the proposed overloaded MIMO bi-directional communication with PLNC is evaluated in wireless 2-hop networks by computer simulation. Even when the number of the antennas on the relay is 2, the proposed overloaded MIMO bi-directional communication with PLNC is shown to enable more than 2 spatial multiplexed signal transmission. The proposed relay filter based on the proposed noise based selection achieves about 1.5 dB better BER performance than that without selection at the BER of 10^{-5}. The proposed MIMO bi-directional communication achieves higher diversity gain in overloaded MIMO channels than that in a non-overloaded channel.

Acknowledgments

References

[1] Z. Md. Fadlullah, H. Nishiyama, N. Kato, H. Ujikawa, K. Suzuki, and N. Yoshimoto, "Smart WiFi networks: Challenges and solutions for QoS and green communications," IEEE Intell. Syst., vol.28, no.2, pp.86–91, 2013.

[2] F. Kojima and H. Harada, "A study on IEEE 802.15.4e compliant low-power multi-hop SUN with frame aggregation," Proc. IEEE International Conf. Commun. (ICC), 2012.

[3] J. Wang, X. Liu, K. Chi, and X. Zhao, "Complex field network-coded cooperation based on multi-user detection in wireless networks," J. Syst. Eng. Electron., vol.24, no.2, pp.215–221, April 2013.

[4] P. Popovski and H. Yomo, "Physical network coding in two way wireless relay channels," IEEE ICC, 2007.

[5] T.K. Akino, P. Popovski, and V. Tarokh, "Optimized constellations for two-way wireless relaying with physical network coding," IEEE J. Sel. Areas Commun., vol.27, no.5, pp.773–787, 2009.

[6] R.H.Y. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: Performance analysis and comparison," IEEE Trans. Wireless Commun., vol.9, no.2, pp.764–777, Feb. 2010.

[7] M. Ju and I.M. Kim, "Error performance analysis of BPSK modulation in physical-layer network coded bi-directional relay networks," IEEE Trans. Commun., vol.58, no.10, pp.2770–2775, Sept. 2010.

[8] I.E. Telatar, "Capacity of multi-antenna Gaussian channels," European Trans. Telecommun., vol.10, no.6, pp.585–595, 1999.

[9] S. Yang and L. Hanzo, "Fifty years of MIMO detection: The road to large-scale MIMOs," IEEE Commun. Surveys Tuts., vol.17, no.4, pp.1941–1988, Fourthquarter 2015.

[10] E.G. Larsson, O. Edfors, F. Tufvesson, and T.L. Marzetta, "Massive MIMO for next generation wireless systems," IEEE Commun. Mag., vol.52, no.2, pp.186–195, Feb. 2014.

[11] L. Lu, G.Y. Li, A.L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," IEEE J. Sel. Topics Signal Process., vol.8, no.5, pp.742–758, Oct. 2014.

[12] M. Sakai, K. Kamohara, H. Iura, H. Nishimoto, K. Ishioka, Y. Murata, M. Yamamoto, A. Okazaki, N. Nonaka, S. Suyama, J. Mashino, A. Okamura, and Y. Okumura, "Experimental field trials on MU-MIMO transmissions for high SHF wide-band massive MIMO in 5G," IEEE Trans. Wireless Commun., vol.19, no.4, pp.2196–2207, April 2020.

[13] K.K. Wong, A. Paulraj, and R.D. Murch, "Efficient high-performance decoding for overloaded MIMO antenna systems," IEEE Trans. Wireless Commun., vol.6, no.5, pp.1833–1843, May 2007.

[14] N. Surajudeen-Bakinde, X. Zhu, J. Gao, and A.K. Nandi, "Improved signal detection approach using genetic algorithm for overloaded MIMO systems," Proc. 4th International Conference on Wireless Commun., Networking and Mobile Computing, pp.1–4, 2008.

[15] X. Lian and D. Li, "On the application of sphere decoding algorithm in overload MIMO systems," IEEE International Conference on Information Theory and Information Security, pp.989–993, Dec. 2010.

[16] I. Shubhi and Y. Sanada, "Joint turbo decoding for overloaded MIMO-OFDM systems," IEEE Trans. Veh. Technol., vol.66, no.1, pp.433–442, Jan. 2017.

[17] Y. Sanada, "Performance of joint maximum-likelihood decoding for block coded signal streams in overloaded MIMO-OFDM system," International Symposium on Intelligent Signal Processing and Communication Systems, pp.775–780, Nov. 2013,

[18] S. Yoshikawa, S. Denno, and M. Morikura, "Complexity reduced lattice-reduction-aided MIMO receiver with virtual channel detection," IEICE Trans. Commun., vol.E96-B, no.1, pp.263–270, Jan. 2013.

[19] R. Hayakawa, K. Hayashi, and M. Kaneko, "Lattice reduction-aided detection for overloaded MIMO using slab decoding," IEICE Trans. Commun., vol.E99-B, no.8, pp.1697–1705, Aug. 2016.

[20] R. Hayakawa and K. Hayashi, "Convex optimization-based signal detection for massive overloaded MIMO systems," IEEE Trans. Wireless Commun., vol.16, no.11, pp.7080–7091, Nov. 2017.

[21] S. Denno, Y. Kawaguchi, T. Inoue, and Y. Hou, "A novel low complexity lattice reduction-aided iterative receiver for overloaded MIMO," IEICE Trans. Commun., vol.E102-B, no.5, pp.1045–1054, May 2019.

[22] M. Eslamifar, W.H. Chin, C. Yuen, and Y.L. Guan, "Performance analysis of two-step bi-directional relaying with multiple antennas," IEEE Trans. Wireless Commun., vol.11, no.12, pp.4237–4242, Nov. 2012.

[23] L. Shi, T. Yang, K. Cai, P. Chen, and T. Guo, "On MIMO linear physical layer network coding: Full-rate full-diversity design and optimization," IEEE Trans. Wireless Commun., vol.17, no.5, pp.3498–3511, March 2018.

[24] Y.T. Kim, K. Lee, M. Park, K.J. Lee, and I. Lee, "Precoding designs based on minimum distance for two-way relaying MIMO systems with physical network coding," IEEE Trans. Commun., vol.6, no.10,

pp.4151–4160, Oct. 2013.

[25] S. Denno and D. Umehara, "Simplified maximum likelihood detection with unitary precoding for XOR physical layer network coding," IEICE Trans. Commun., vol.E100-B, no.1, pp.167–176, Jan. 2017.

[26] L. Cao and S. Denno, "Nonlinear precoding for XOR physical layer network coding in bi-directional MIMO relay systems," IEICE Trans. Commun., vol.E100-B, no.3, pp.440–448, March 2017.

[27] S. Denno, Y. Nagai, and Y. Hou, "XOR physical layer network coding with non-linear precoding for quadrature amplitude modulations in bi-directional MIMO relay systems," IEICE Trans. Commun., vol.E102-B, no.10, pp.2073–2081, Oct. 2019.

[28] K. Kusume, M. Joham, W. Utschick, and G. Bauch, "Cholesky factorization with symmetric permutation applied to detecting and precoding spatially multiplexed data streams," IEEE Trans. Wireless Commun., vol.55, no.6, pp.3089–3103, June 2007.

[29] A.K. Lenstra, H.W. Lenstra, Jr., and L. Lovász, "Factoring polynomials with rational coefficients," Math. Ann., vol.261, no.4, pp.515–534, 1982.

[30] D. Wübben, D. Seethaler, J. Jalden, and G. Mats, "Lattice reduction," IEEE Signal Process. Mag., vol.28, no.3, pp.70–91, 2011.

[31] C. Windpassinger, R.F.H. Fischer, and J.B. Huber, "Lattice-reduction-aided broadcast precoding," IEEE Trans. Commun., vol.52, no.12, pp.2057–2060, 2004.

[32] S. Denno, K. Yamamoto, and Y. Hou, "Eigenvalue based relay selection for XOR-physical layer network coding in bi-directional wireless relaying networks," IEICE Trans. Commun., E104-B, no.10, pp.1336–1344, Oct. 2021.

[33] W.C. Jakes, Microwave Mobile Communications, IEEE Press, 1994.

Appendix: Spatial Filter

The channel in the second slot is regarded as a non-overloaded channels where the number of the antennas on the transmitter N_R is less than that on the receiver N_T. If a precoder based on the MMSE is applied to the channel, however, the channel is transformed to one of overloaded channels. Hence, the number of the spatial multiplexed signal streams becomes same to that of the number of the antennas on the terminal N_T, in spite of the number of the antennas on the relay. A precoded signal vector sent from the relay is expressed as,

$$\mathbf{X}_{ms,R} = g_{ms,R}\mathbf{H}_\Omega \left(\mathbf{H}_\Omega^H \mathbf{H}_\Omega + \gamma \mathbf{I}_{N_T}\right)^{-1} \dot{\mathbf{D}}_R. \quad (A\cdot 1)$$

$g_{ms,R} \in \mathbb{R}$, $\mathbf{X}_{ms,R} \in \mathbb{C}^{N_R}$ and $\dot{\mathbf{D}}_R \in \mathbb{C}^{N_T}$ in $(A\cdot 1)$ denote a normalization factor, a signal vector precoded with the MMSE filter, a modulation signal vector added with Gaussian integer multiples, which are transmitted from the relay. When the precoded signal vector is transmitted from the relay, the received signal vector at the terminal Ω, $\mathbf{Y}_{ms,\Omega} \in \mathbb{C}^{N_R}$, can be expressed as,

$$\mathbf{Y}_{ms,\Omega} = \mathbf{H}_\Omega^H \mathbf{X}_{ms,R} + \mathbf{N}_\Omega$$
$$= g_{ms,R}\mathbf{H}_\Omega^H \mathbf{H}_\Omega \left(\mathbf{H}_\Omega^H \mathbf{H}_\Omega + \gamma \mathbf{I}_{N_T}\right)^{-1} \dot{\mathbf{D}}_R + \mathbf{N}_\Omega. \quad (A\cdot 2)$$

As is written above, while the number of the antennas on the relay is N_R, the N_T signals contained in the modulation signal vector $\dot{\mathbf{D}}_R$ are broadcast for the terminals. Though the number of the antennas on the relay is only N_R, in a word, the N_R spatial multiplexed streams can be transmitted with

only precoder based on the MMSE in the second slot. We apply such signal transmission to that in the first slot, which is made possible with the proposed relay filter defined in (20). The relay filter transforms the channel matrix $\mathbf{H}_\Omega$ as,

$$\mathbf{G}^{(\Omega)}\mathbf{H}_\Omega = \mathbf{U}_\Omega \mathbf{\Lambda}_\Omega \mathbf{U}_\Omega^H. \qquad (A\cdot 3)$$

Even though two vectors are received at the relay in the first slot, we only focus on the signal transmission from one of the terminal, which is modeled as,

$$\tilde{\mathbf{Y}}_R = \mathbf{H}_\Omega \mathbf{X}_\Omega^{(G)} + \mathbf{N}, \qquad (A\cdot 4)$$

where $\tilde{\mathbf{Y}}_R \in \mathbb{C}^{N_R}$ denotes a signal vector received at the relay when only one of the two terminals sends the signals. If the orthogonality of the matrices such as $\mathbf{S}_\Omega$ and $\mathbf{U}_\Omega$ is taken into account, by substituting the term in the left hand side of (A·3) for (4) and (5), the filtered received signal vector can be rewritten as follows.

$$
\begin{aligned}
\mathbf{G}\tilde{\mathbf{Y}}_R &= g_\Omega \mathbf{U}_\Omega \mathbf{\Lambda}_\Omega^2 \mathbf{U}_\Omega^H \left(\mathbf{U}_\Omega \mathbf{\Lambda}_\Omega^2 \mathbf{U}_\Omega^H + \gamma \mathbf{I}_{N_T} \right)^{-1} \dot{\mathbf{D}}_\Omega + \mathbf{GN} \\
&= g_\Omega \mathbf{H}_\Omega^H \mathbf{H}_\Omega \left(\mathbf{H}_\Omega^H \mathbf{H}_\Omega + \gamma \mathbf{I}_{N_T} \right)^{-1} \dot{\mathbf{D}}_\Omega + \mathbf{GN} \\
&= \kappa_\Omega \mathbf{H}_\Omega^H \mathbf{X}_{\mathrm{ms,R}} + \mathbf{N}_G, \qquad (A\cdot 5)
\end{aligned}
$$

where $\mathbf{N}_G \in \mathbb{C}^{N_T}$ and $\kappa_\Omega \in \mathbb{R}$ represent a filtered noise vector defined as $\mathbf{N}_G = \mathbf{GN}$ and a ratio of the normalization factors defined as $\kappa_\Omega = \frac{g_\Omega}{g_{\mathrm{ms,R}}}$.

As is shown in (A·5), the system model in the first slot is converted to that in the second slot by using the proposed relay filter, although the normalization factors could be different from each other. Since the filter defined in (5) is the same to that in (A·1) except for the normalization factors, however, the normalization factors can be set to the same value, if we apply the same unimodular matrix to those filters. In a word, those filter can be made same, i.e., $\kappa_\Omega = 1$. Though the filtered noise vector $\mathbf{N}_G$ is a colored noise, i.e., $\mathrm{E}\left[\mathbf{N}_G \mathbf{N}_G^H\right] \neq 2\sigma^2 \mathbf{I}_{N_T}$, the power of the filtered noise is kept same to that of the original noise vector $\mathbf{N}$. The characteristics of the noise could have little influence on the transmission performance. We think that the use of the proposed relay filter converts the channel model in the first slot to that in the second slot. Even though such channel transformation is performed only in the channel between the relay and the terminal A or B, the use of the proposed relay filter is expected to balance the performances in the first time slot and the second time slot.

Satoshi Denno received the M.E. and Ph.D. degrees from Kyoto University, Kyoto, Japan in 1988 and 2000, respectively. He joined NTT radio communications systems labs, Yokosuka, Japan, in 1988. He was seconded to ATR adaptive communications research laboratories, Kyoto, Japan in 1997. From 2000 to 2002, he worked for NTT DOCOMO, Yokosuka, Japan. In 2002, he moved to DOCOMO communications laboratories Europe GmbH, Germany. From 2004 to 2011, he worked as an associate professor at Kyoto University. Since 2011, he is a full professor at graduate school of natural science and technology, Okayama University. From the beginning of his research career, he has been engaged in the research and development of digital mobile radio communications. In particular, he has considerable interests in channel equalization, array signal processing, Space time codes, spatial multiplexing, and multimode reception. He won the Best paper award of the 19th international symposium on wireless personal multimedia communications (WPMC2016), the outstanding paper award of the 23rd international conference on advanced communications technology (ICAT2021), and the contribution to academic research award of the 27th international symposium on wireless personal multimedia communications (WPMC2022). He received the excellent paper award and the best paper award from the IEICE in 1995 and from the IEICE communication society in 2020, respectively.

Tomoya Tanikawa received B.S. and M.S. degrees from Okayama University, Japan, in 2020 and 2022, respectively. He joined with NTT Data, inc. in 2022. His research interests include wireless communication systems, network coding, and overloaded MIMO signal processing.

Yafei Hou received his Ph.D. degrees from Fudan University, China and Kochi University of Technology (KUT), Japan in 2007. He was a post-doctoral research fellow at Ryukoku University, Japan from August 2007 to September 2010. He was a research scientist at Wave Engineering Laboratories, ATR Institute International, Japan from October 2010 to March 2014. He was an Assistant Professor at the Graduate School of Information Science, Nara Institute of Science and Technology, Japan from April 2014 to March 2017. He became an assistant professor at the Graduate School of Natural Science and Technology, Okayama University, Japan from April 2017. He is a guest research scientist at Wave Engineering Laboratories, ATR Institute International, Japan from October 2016. His research interest are communication systems, wireless networks, and signal processing. He received IEICE (the Institute of Electronics, Information and Communication Engineers) Communications Society Best Paper Award in 2016, 2020, and Best Tutorial Paper Award in 2017. Dr. Hou is a senior member of IEEE and member of IEICE.