Local Information, Observable Parameters, and Global View*

Hiroshi SAITO[†a)], *Fellow*

SUMMARY The "Blind Men and an Elephant" is an old Indian story about a group of blind men who encounter an elephant and do not know what it is. This story describes the difficulties of understanding a large concept or global view based on only local information. Modern technologies enable us to easily obtain and retain local information. However, simply collecting local information does not give us a global view, as evident in this old story. This paper gives a concrete model of this story on the plane to theoretically and mathematically discuss it. It analyzes what information we can obtain from collected local information. For a convex target object modeling the elephant and a convex sensing area, it is proven that the size and perimeter length of the target object are the only parameters that can be observed by randomly deployed sensors modeling the blind men. To increase the number of observable parameters, this paper argues that non-convex sensing areas are important and introduces composite sensor nodes as an approach to implement non-convex sensing areas. The paper also derives a model on the discrete space and analyzes it. The analysis results on the discrete space are applicable to some network related issues such as link quality estimation in a part of a network based on end-to-end probing.
key words: local information, global information, sensor, sensing result, shape estimation, integral geometry, network tomography, quality estimation, observability

1. Introduction

The "Blind Men and an Elephant" is an old Indian story about a group of blind men who encounter an elephant and do not know what it is. The first man touches its leg and tells the people in the village that the elephant is like a pillar. The second man touches its tail and tells the villagers that the elephant is like a rope. The third man touches another part of its body, and so on. However, the villagers do not understand what the elephant is like.

This story describes the difficulties in understanding a large concept or global view based on only local information.

Modern technologies enable us to easily obtain and retain local information. Generally speaking, the total amount of local information owned by a distributed crowd tends to become much larger than information owned by a centralized organization. Therefore, collecting such information seems to yield a valuable global view. At the same time, simply collecting local information does not give us a global view, as evident in this old story.

Is this old story true even with current technologies? Can modern technologies help us obtain a global view from collected local information? To obtain a global view, how should we collect and process local information? Under what conditions can we reach a global view based on collected local information? These are basic questions discussed in this paper.

There is a large amount of literature on distributed or decentralized systems. Some compare centralized systems with such systems. However, the main motivation of such studies is methods of implementing these systems. On the other hand, the concepts of wisdom of crowd [1], [2], crowd sourcing [3], social computing [4], and participatory sensing [5] attempt to use local information to achieve a common, centralized, global objective. Although these concepts are in the same direction with the concept discussed in this paper, there have not been any studies discussing the relationship between local information and global view in the real (physical) world. That is, a unique feature of this study is the discussion of observability of global parameters in the real world based on local information obtained through the monitoring of the real world.

In Sect. 2, a concrete model of this old story is defined on the plane to discuss it. By using the mathematical methods of integral geometry and geometric probability provided in Sect. 3, the results of the analysis for this example are given in Sect. 4. With these results, it is shown that the convexity is strongly related to observability. Therefore, in Sect. 5, further results going beyond the convexity are provided. In particular, an additional mechanism called a composite sensor node is explored to enlarge observability. On the other hand, the relationship between local information and global view is discussed on a discrete space in Sect. 6. Because the basic questions asked in this paper are not limited to the cases on the plane, problems appropriately modeled on a discrete space can also be discussed. The results are discussed in Sect. 7. In practice, the modeling on a discrete space and that on the plane are not identical, and their relationships are investigated in Sect. 8. Remaining issues are explored in Sect. 9 and a conclusion is given in Sect. 10.

2. Concrete Example

The questions mentioned above are too abstract. Therefore, let us consider a concrete example similar to the old story.

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Assume that a single target object $\mathcal{T}$ (like the elephant)

(a) modeling on the plane

(b) modeling on discrete space

Fig. 1 Model.

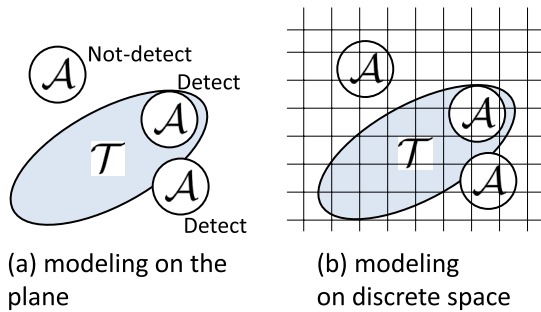**Fig. 2** Parameterization for K.

exists in a convex region $\Omega^{\dagger}$ within a two-dimensional space $\mathbb{R}^2$ (Fig. 1(a)). The shape, size, and location of $\mathcal{T}$ are unknown.

Sensors are the blind men of the story. There are N_s sensors distributed randomly and independently in Ω. Assume that the i-th sensor is located at (x_i, y_i) and the sensing area is rotated by θ_i from the referenced position. Let $\mathcal{A}(x_i, y_i, \theta_i) \subset \mathbb{R}^2$ be its sensing area where $i = 1, 2, \cdots$. Although the sensors in the old story are the blind men and can offer more information than simple binary "detect/not-detect" information, such as local shape, assume that the sensors are binary. The i-th sensor sends a report I_i, where I_i is 1 if it detects the target object and 0 otherwise. That is, $I_i = \mathbf{I}(\mathcal{A}(x_i, y_i, \theta_i) \cap \mathcal{T} \neq \emptyset)$, where $\mathbf{I}(\omega)$ is an indicator function that becomes 1 if a statement ω is true and 0 if otherwise. In addition, the Boolean sensing area model is used. That is, if and only if the target object is in the sensing area, the sensor will detect it. However, no sensor can determine its location or which part of the target object it detects. Therefore, the report does not contain such information.

The global view is the shape, size, and location of the target object. That is, the question is whether we can draw a picture of the target object (elephant) by using the collected information $I_1, I_2, \cdots$.
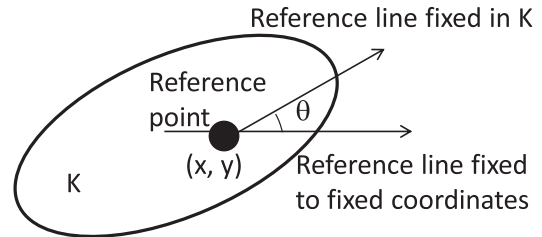
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The key assumption is that the location of each sensor is unknown. Therefore, it is difficult to build a global view from local information. This is an analogy of the story "Blind men and an elephant." If the blind men know their positions, we can draw a picture of the elephant if there are many men.

Because the sensors are binary and their locations are unknown, the only information we can obtain regarding the target object is the number of reports indicating that the target object was or was not detected. Therefore, it seems impossible for us to obtain global information. We discuss the results of analyzing this problem (easily derived from [6]) in Sect. 4.

3. Integral Geometry and Geometric Probability

Before describing the above-mentioned results, we explain the mathematical methods of integral geometry and geometric probability [7] as a preliminary. First, consider a set K

in $\mathbb{R}^2$ whose position is defined by the reference point (x, y) in K and the angle θ formed by a reference line fixed in K with another reference line fixed to the fixed coordinates (Fig. 2). By moving K over a certain domain of the parameter space of (x, y, θ), we obtain the measure $m(K; A)$ of the set of (positions of) K satisfying a condition A. The measure $m(K; A)$ is defined by the area size in which the positions of K satisfies A in the parameter space of (x, y, θ). An example of A is $K \cap K_0 \neq \emptyset$ for a given K_0. This "moving K" is denoted as dK and called the kinematic density. In fact, $dK = dx\, dy\, d\theta$. (In integral geometry, the notation $dK = dx \wedge dy \wedge d\theta$ is often used. Here, $\wedge$ denotes the symbol of the exterior product and is convenient when we transform the coordinates. However, we can ignore this symbol and consider the equation below as a normal Lebesgue integral because we do not transform the coordinates in this paper.) That is, $m(K; A) = \int_A dK = \int_A dx\, dy\, d\theta$.
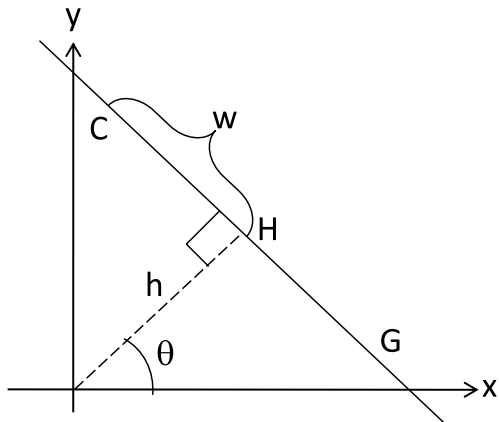
The most important feature of the measure defined by integral geometry is that the measure must be invariant under the group of motions (translation and rotation) in the plane. As a result, convenient parameterization exists to describe a certain class of K. For a bounded K, we can use the natural parameterization (x, y, θ).

Once the measures $m(K; A)$ and $m(K; B)$ ($B \subseteq A$) have been defined, the conditional probability that K satisfying A satisfies B can be defined by the quotient of measures $m(K; B)/m(K; A)$. This is called a geometric probability. Therefore, $m(K; B)$ is like a non-normalized probability.

In addition to a bounded K, the convenient parameterization for a line G and that for a point C on G are now described [7]. Define θ as the angle formed by the direction perpendicular to G with a fixed direction, and define h as the distance of G from the origin O to the intersection point H (Fig. 3). When G is parameterized by θ and h, a measure on

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†The meaning of "Ω is convex" is that, for each pair of points in Ω, the line segment between these two points is in Ω. Therefore, triangles, rectangles, and disks are convex. Polygons with every angle less than π are convex, but those including an angle larger than π are not convex. Neither a region with holes nor a region consisting of non-connected subregions in $\mathbb{R}^2$ is convex. Practically, the assumption of a convex region is equal to the assumption of a single polygon with every angle less than π without holes.

The assumption of convexity simplfies the analysis and results. For example, when $\mathcal{T}$ is a convex polygon, the contour of the center of the disk touching the outside of $\mathcal{T}$ is simple and is made by rectangles and sectors touching the outside of $\mathcal{T}$ (Fig. 4(a)). However, for a non-convex $\mathcal{T}$, it becomes messy because these rectangles overlap (Fig. 4(b)).

Fig. 3 Parameterization for G and C.

(a) convex target object (b) non-convex target object

Fig. 4 (x, y) satisfying $\mathcal{A} \cap \mathcal{T} \neq \emptyset$.

G satisfies motion-invariance. Similarly, C on G is parameterized by (θ, h, w), where w is the distance from C to the foot H of the perpendicular from the origin to G.

4. Results on the Plane

Assume that both the target object $\mathcal{T}$ and sensing area $\mathcal{A}$ are bounded and convex. Let $m(\mathcal{A}; \mathcal{A} \cap \mathcal{T} \neq \emptyset)$ and $m(\mathcal{A}; \mathcal{A} \cap \Omega \neq \emptyset)$ be the measure of the set of $\mathcal{A}$ intersecting $\mathcal{T}$ and that intersecting Ω, respectively.

First, assume that $\mathcal{T}$ is a convex polygon and that $\mathcal{A}$ is a disk with radius r_a. Let us move $\mathcal{A}$ while keeping the condition $\mathcal{A} \cap \mathcal{T} \neq \emptyset$. By setting the reference point of $\mathcal{A}$ at its center and noting that the trajectory of $\mathcal{A}$ is not dependent on θ, we can easily draw a picture of the domain $\mathcal{D}$ in $\mathbb{R}^2$ such that the reference point satisfies the condition $\mathcal{A} \cap \mathcal{T} \neq \emptyset$ (Fig. 4(a)). Therefore, we can easily derive $m(\mathcal{A}; \mathcal{A} \cap \mathcal{T} \neq \emptyset) = \int_0^{2\pi} \int_{(x,y) \subset \mathcal{D}} dx\, dy\, d\theta = 2\pi(r_a|\mathcal{T}| + \pi r_a^2 + \|\mathcal{T}\|)$. For a set K in $\mathbb{R}^2$, $\|K\|$ denotes the size of K, and $|K|$ denotes the perimeter length of K.

Using the results of the theory of integral geometry, this above result under the assumption of a convex polygon $\mathcal{T}$ and a disk $\mathcal{A}$ can be generalized. According to [7], for convex sets K and K_1,

$$m(K; K \cap K_1 \neq \emptyset) = F(K, K_1). \tag{1}$$

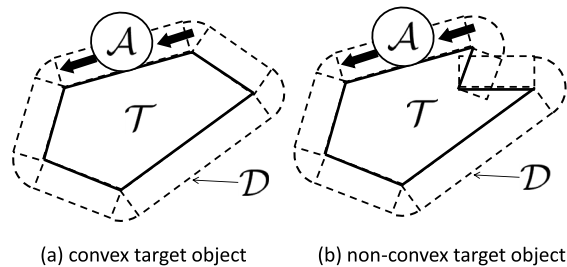Here, $F(K, K_1) \equiv |K| \cdot |K_1| + 2\pi(\|K\| + \|K_1\|)$.

By Applying Eq. (1), $m(\mathcal{A}; \mathcal{A} \cap \mathcal{T} \neq \emptyset) = F(\mathcal{T}, \mathcal{A})$ and $m(\mathcal{A}; \mathcal{A} \cap \Omega \neq \emptyset) = F(\Omega, \mathcal{A})$. Then, we obtain the following result.

Result 1: The probability p_d that a sensor monitoring Ω will detect the target object is

$$p_d = F(\mathcal{T}, \mathcal{A})/F(\Omega, \mathcal{A}) \approx \frac{F(\mathcal{T}, \mathcal{A})}{2\pi\|\Omega\|}, \tag{2}$$

and the expected number of detecting sensors is $N_s p_d$. Its probabilistic distribution is binomial with (N_s, p_d).

We should note that only the parameter p_d determines the probabilistic distribution of the number of sensors detecting $\mathcal{T}$, and that among the shape, size, and location parameters of the target object, only its size and perimeter length appear in p_d. Hence, at least for a convex target object and a convex sensing area, we cannot obtain any information except on the size and perimeter length of the target object from sensor reports. This implies the following.

Result 2: Only the size and perimeter length of the convex target object are observable parameters via randomly deployed convex sensing area sensors.

"Observable parameters" means that we can estimate them by using collecting local information and that their estimates are uniquely converged to exact ones under ideal conditions such as the amount of sensing data being infinitely large. In this sense, increasing the number of observable parameters means approaching an understanding of the global view.

In this example, it is not intuitive that the perimeter length of the target object is observable when the information we have is substantially the number of sensors detecting the target object. This observability comes from the non-zero perimeter length of the sensing area. When the sensing area shape and size and target object size are fixed, the number of sensors detecting the target object increases if the target object perimeter length is longer for the non-zero perimeter length of the sensing area. This is because the sensors detect the target object if only a part of it is detected, however small. Hence, the target object perimeter length can be obtained through the number of sensors detecting the target object when the sensing area shape and size and target object size are given.

In fact, we can estimate the size and perimeter length of the target object through the following procedure: (1) introducing two types of sensors, of which sensing areas are $\mathcal{A}_1$ and $\mathcal{A}_2$, (2) obtaining the number $N(i)$ of type-i sensor reports detecting the target object, (3) describing its expectation as functions of $\|\mathcal{T}\|, |\mathcal{T}|, \|\mathcal{A}_i\|, |\mathcal{A}_i|$ ($i = 1, 2$), that is, $E[N(i)] = N_s(i)\frac{F(\mathcal{T}, \mathcal{A}_i)}{2\pi\|\Omega\|}$ where $N_s(i)$ is the number of type-i sensors, and (4) finding $\|\mathcal{T}\|, |\mathcal{T}|$ by minimizing the sum of the square errors of the observed $N(i)$ and expected $N(i)$ ($i = 1, 2$). The concrete estimation method for size and perimeter length is discussed in [6], and the experimental results are discussed in [8].

However, we cannot estimate any other parameters even if we introduce a large number of and many types of sensors. In addition, this result is robust in the sense that it
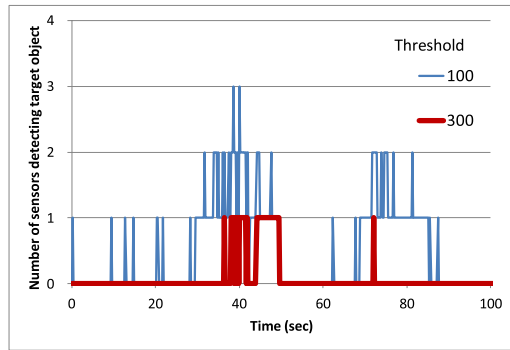
Fig. 5   Sample path of number of sensors detecting target object.

is valid only if the target object $\mathcal{T}$ and sensing area $\mathcal{A}$ are bounded and convex. This result is valid if the deployment of sensors follows a spatially stationary process[†].

Because the results mentioned above are valid at each time t, it is possible for us to use the number of sensors detecting the target object at different sensing times if the target object is time-invariant. This is practically important because, if we can use the sensing result only at a single sensing time, the number of detecting sensors is normally very small and the estimation error becomes very large. Even when the target object is moving, its trajectory and its velocity are not required to estimate the size and perimeter length. This is because Eq. (2) is valid at each time without knowing the positions of the target object. The information we obtain is a time series of the number of sensors detecting the target object. However, the information is used not as a time series but as samples of the number of sensors detecting the target object in the estimation method discussed in [6]. Figure 5 is a sample path of the number of sensors detecting the target object in the experiment in [8] in which there are two detection thresholds. The low threshold corresponds to a large sensing area, and the high threshold corresponds to a small sensing area. As shown in this figure, at each moment, the number of sensors detecting the target object is small. However, as a whole, it becomes sufficient for estimation.

Therefore, the story "Blind men and an elephant" is partially true. Some parts of a global view, the size and perimeter length, can be obtained through local information, that is, sensor reports. However, other parts of a global view cannot be obtained.

5.   Further Results on the Plane

How do we go beyond the results discussed above? One of the key assumptions is convexity. If the target object or sensing area is not convex, the results discussed above may be different. Therefore, some efforts were made to find an equation corresponding to Eq. (2) without the convexity assumption on the target object [9]–[12].

5.1   Extension of Eq. (1)

Studies were conducted to generalize Eq. (1) [9], [10]. It

was found that the following are valid when the target object does not have a dint or a hole that is too small for a sensor to detect. (1) If the sensing area is a line segment of length r, the detectable area size $\int_0^{2\pi} \int_{\mathcal{T} \cap \mathcal{A} \neq \emptyset} dx\, dy\, d\theta$ is given by $2\pi\alpha r^2 + 2r|\mathcal{T}| + 2\pi\|\mathcal{T}\|$, where $\alpha = \frac{1}{4\pi} \sum_i \mathbf{I}(\pi < \phi_i)\{-1 + (-\pi + \phi_i)\cos\phi_i/\sin\phi_i\}$ and ϕ_i is the i-th interior angle of the target object. (2) If the sensing area is a disk of radius r, the detectable area size is given by $2\pi(\alpha r^2 + r|\mathcal{T}| + \|\mathcal{T}\|)$ where $\alpha = \sum_j \{\mathbf{I}(\phi_j < \pi)\frac{\pi - \phi_j}{2} + \mathbf{I}(\phi_j > \pi)\frac{1 + \cos\phi_j}{\sin\phi_j}\}$ (Fig. 4(b)). If and only if a sensor is located in a detectable area of the target object does it detect that object. Thus, the detectable area size is equivalent to $m(\mathcal{A}; \mathcal{A} \cap \mathcal{T} \neq \emptyset)$.

The authors of the above studies conjectured that a similar equation is valid for much broader classes of sensing areas than disk-shaped and line-segment-shaped ones, and proposed the following equation.

$$m(\mathcal{A}; \mathcal{A} \cap \mathcal{T} \neq \emptyset) \approx |\mathcal{A}| \cdot |\mathcal{T}| + 2\pi(\|\mathcal{T}\| + \beta_1|\mathcal{A}|^2 + \beta_2\|\mathcal{A}\|). \quad (3)$$

Here, $\beta_1 = \sum_i \alpha_i$, $\beta_2 = \frac{\sum_j(\pi - \phi_j)}{2\pi}\mathbf{I}(\phi_j < \pi)$, and α_i is a constant determined only by the i-th interior angle larger than π of the target objects. We should note that Eq. (3) is exact when (i) $\mathcal{A}, \mathcal{T}$ are convex, (ii) $\mathcal{A}$ is a line segment, or (iii) $\mathcal{A}$ is a disk. As a result, we can estimate for parameters $|\mathcal{T}|, \|\mathcal{T}\|, \beta_1, \beta_2$ by introducing four types of sensors if Eq. (3) is valid. However, we cannot estimate each ϕ_i. That is, we can obtain additional information β_1 and β_2 but we cannot obtain significant information of the global view.

5.2   Composite Sensor Node

Non-convex sensing areas are theoretically important to obtain additional estimates of parameters rather than the size and perimeter length of the target object. This is because, when the target object is convex, no additional information can be obtained through the convex sensing areas, as stated in Sect. 4. Non-convex sensing areas may enable us to derive additional information even if the target object is convex.

References [11], [12] proposed a composite sensor node, which consists of multiple sensor nodes arranged in a predetermined layout. A sensing area of a composite sensor node is normally not convex no matter what the shape of the sensing area of each sensor is. Therefore, we may be able to derive additional information through the sensing

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[†]The meaning of "spatially stationarity" for the sensor locations $\{(x_1, y_1), \cdots, (x_{N_s}, y_{N_s})\}$ is that $\Pr((x_1, y_1), \cdots, (x_{N_s}, y_{N_s}) \subset \Gamma)$ is invariant under translation for any Γ. That is, the probability distribution of the sensor locations is independent of the location of the origin. Clearly, a homogeneous Poisson process is a spatially stationary process. A double stochastic Poisson process is also spatially stationary. Therefore, if the sensor locations are the realization of such processes, the probability distribution of the sensor locations is invariant under translation. In [6], a double stochastic Poisson process as well as a homogeneous Poisson process is used to place sensors in the numerical example to demonstrate that the estimation method proposed in that study works. However, an inhomogeneous Poisson process is not stationary. The effect of this non-stationarity is numerically discussed in [12].
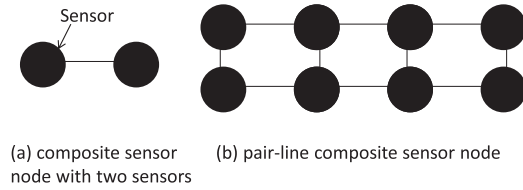
Fig. 6   Examples of composite sensor nodes.

reports. Because the distance between two sensors in a composite sensor node is known (that is, the relative location of each sensor in a composite sensor node is known), we obtain more local information from composite sensor nodes than from single sensor nodes, even though the composite sensor nodes are randomly and independently distributed. (However, the relative location of each sensor in a composite sensor node is not enough to make addtional parameters observable. Non-convexity seems essential. See Appendix.)

A composite sensor node with two sensors [12] and a pair-line composite sensor node consisting of a pair of parallel line segments on which sensors are placed at equal intervals [11] are examples of composite sensor nodes (Fig. 6). Composite sensor nodes are particularly important when there are multiple target objects. Even when each target object is convex, we cannot identify even the size and perimeter length of each target object, even though the total size and total perimeter length can be obtained. As described below, pair-line composite sensor nodes provide us with the number of target objects as well as the angle of each vertex of target objects. The information provided by a composite sensor node with two sensors may enable us to identify the shape of each target object among given basic a priori geometric shapes, such as a disk or rectangle, and to estimate their parameters.

For a composite sensor node with two sensors, the measure $m(i, j)$ of a set of type-j composite sensor nodes such that i sensors in a single composite sensor node detect the target object is derived under the assumption that the shape of a target object is a disk or rectangle [12] where $i = 1, 2$. Parameters of sensing areas, such as the distance between two sensors in a composite sensor node, are different when their types are different. Evaluation of $m(i, j)$ may be possible by moving the positions of the composite sensor nodes where the parameters are (θ, h, w), as described in Sect. 3 (Fig. 7).

Based on $m(i, j)$ and using the parameters of composite sensor nodes, those of target objects, and the number of target objects, we can describe the equations regarding the expected number $E[N(i, j)]$ of type-j composite sensor nodes such that i sensors in a single composite sensor node detect the target object. Therefore, by observing the actual number of such type-j composite sensor nodes, we can estimate the parameters of target objects through the minimum square error method when the number of target objects is given.

Similarly, for pair-line composite sensor nodes, the measure of a set of composite sensor nodes such that specified sensors, such as "second sensors on both lines", in a
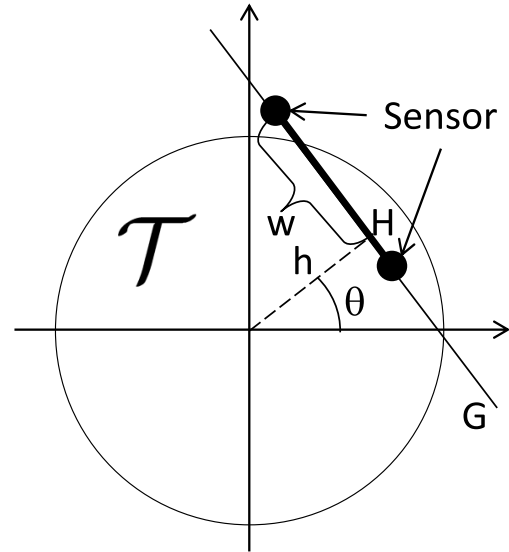

Fig. 7   Evaluation of $m(i, j)$ for disk-shaped target object.

composite sensor node detect the target object, is derived. Based on such a measure and using the parameters of the composite sensor nodes and those of the target objects, we can describe the equations regarding the expected number of composite sensor nodes such that specified sensors in a composite sensor node detect the target object. Because these equations include vertex angles of the target object, we can estimate the number of vertexes and each vertex angle of the target object through the reports from composite sensor nodes [11]. As a result, we can also estimate the number of target objects.

However, there are cases in which we cannot estimate additional parameters related to the target object shape [12]. For example, assume that there are n_T rectangular target objects with side lengths a_i, b_i $(1 \leq i \leq n_T)$ satisfying $l_j - 2r_j \leq \min(a_i, b_i)$, where r_j is the radius of the disk-shaped sensing area and l_j is the distance between two sensors in a type-j composite sensor node with two sensors. The analysis in [12] shows that

$$E[N(2, j)] \approx \lambda_j \left\{ \pi \sum_i a_i b_i + 2(\pi r_j - l_j) \sum_i (a_i + b_i) \right.$$
$$\left. + (4\pi r_j^2 + l_j^2 - 8l_j r_j)n_T \right\}/\pi. \quad (4)$$

Here, $N(2, j)$ is the number of type-j composite sensor nodes in each of which two sensors detect a target object, and λ_j is the mean density of a type-j composite sensor node. Thus, if we can use $E[N(2, j)]$ with various l_j, r_j simultaneously, we can estimate $\sum_i a_i b_i, \sum_i (a_i + b_i)$. However, we cannot estimate each a_i or b_i. (Similarly, $E[N(1, j)]$ does not provide additional information for this case.) That is, each a_i or b_i is not an observable parameter under the assumption mentioned above. Again, we cannot obtain a global view for this case.

On the other hand, if we know that there are n_T rectan-

gular target objects with unknown side lengths and if some a_i, b_i ($1 \le i \le n_T$) do not satisfy $l_j - 2r_j \le \min(a_i, b_i)$, it may be possible for us to estimate each a_i, b_i. In [12], a set of sensor parameters that enables us to estimate each parameter of the target object is called an observing parameter set. In this example, if $a_1^2 + b_1^2 < \cdots < a_{n_T}^2 + b_{n_T}^2$, a set of sensor parameters that satisfies $\max(\sqrt{(a_{i-1} + 2r_{2i})^2 + (b_{i-1} + 2r_{2i})^2}, a_i + 2r_{2i}, b_i + 2r_{2i}) < l_{2i-1} < l_{2i} < \sqrt{a_i^2 + b_i^2} + 2r_{2i}, r_{2i} = r_{2i-1}, l_{2i} = l_{2i-1} + \delta$ is an observing parameter set where δ is a sufficiently small positive scalar [12]. Similarly, when there are n_T disk-shaped target objects and the radius R_i of the i-th target object satisfies $R_1 < \cdots < R_{n_T}$, a set of sensor parameters that satisfies $l_1 - 2r_1 < 2R_1 < l_2 - 2r_2 < 2R_2 < \cdots < l_{n_T} - 2r_{n_T} < 2R_{n_T}$ is an observing parameter set [12].

Result 3: By introducing composite sensor nodes with an observing parameter set, some parameters other than the size and perimeter length of the target object can be identified. Even when composite sensor nodes are used, parameters other than the size and perimeter length of the target object may not be identified with inappropriate parameter values of composite sensor nodes.

The existence of an observing parameter set means that we can approach the understanding of the global view. However, the example mentioned above means that two-sensor composite sensor nodes are not sufficient to obtain a global view because additional information, such as "there are n_T rectangular target objects", is necessary.

Currently, we have no means to obtain a complete global view from local information under the formulation discussed here. However, we developed estimation methods for some parameters. In some applications, a limited number of estimated parameters can be useful, although they are not the global view for the target object [13]. We continuously need to develop methods to obtain a global view.

6. Modeling on Discrete Space

The results described above are based on analysis on the plane, that is, $\mathbb{R}^2$. By making a grid with n_s grid points on the plane, we can convert the problem on $\mathbb{R}^2$ into that on $\{0, 1\}^{n_s}$ (Fig. 1(b)). That is, a target object occupies points on the grid, and a sensor monitors points on the grid and detects whether a point is occupied by the target object. As a result, the problem can be defined on a discrete space.

This modeling on discrete space provides us two important changes. First, the mathematics appropriate to discuss the problem is no longer integral geometry. We can use mathematics familiar to network research engineers such as Boolean algebra. The second change is more important. Discrete space modeling covers many conventional problems [14]. For example,
– Network quality: Each subscriber communicates with another through a network. Some subscribers may submit a report to a network operator concerning quality. The quality of each link is either one of two states, GOOD or BAD.

If and only if all the links included in the route between a source and destination are GOOD, the quality of the source-destination pair is GOOD (this property is called separable [15]). The route occasionally changes due to load balancing or other reasons. Subscriber reports include information about the source, destination, and quality of the source-destination pair, but it does not include the route (assuming that we do not identify the route based on the report). Can a network operator determine the quality of each link based on subscriber reports? This problem is called network tomography [15]–[23], although it sometimes means the inference of routing topology or traffic matrix. Current research can be categorized by the probe types (unicast packet or multicast), variable (binary, probability, real), and with/without probabilistic mixture. In particular, one of the main differences of this study from others is introducing the probabilistic mixture. As discussed later, some current models without probabilistic mixture are covered by a conventional model.
– Quality of experience (QoE): Each subscriber uses applications on the network. A subscriber installs a QoE reporting tool that reports whether QoE is GOOD or BAD with additional information: the application, the terminal, the OS, and the network used. However, the OS version and the route of the network are not reported. Can the QoE monitoring center receiving the report determine which part or parts are BAD?

Of course, the problem in estimating the shape and location of a target object through binary sensor reports monitoring grid points is also covered by this discrete space modeling.

6.1 Notations

In the remainder of this paper, a row vector is denoted in boldface if explicitly indicated otherwise. The following is a list of notations used in this section of the paper.

- $\mathbf{0}^{(j)} \equiv (0, \cdots, 0)$: the j-element zero vector.
- $\mathbf{e}_i \equiv (e_{i,1}, \cdots, e_{i,n_s})$: the i-th elementary n_s-element vector where $e_{i,j} = \mathbf{I}(i = j)$.
- $(\mathbf{x})_i$: the i-th element of any vector $\mathbf{x} = (x_1, \cdots, x_n)$. That is, $(\mathbf{x})_i = x_i$.
- $\mathbf{x} \star \mathbf{y} \equiv \mathbf{I}(\sum_{i=1}^n x_i y_i > 0)$. The operator $\star$ means "sensing", as discussed in a later section.
- $\mathbb{B}(\mathbf{x}) \equiv \sum_{i=1}^n 2^{n-i}(\mathbf{x})_i$ for a binary vector $\mathbf{x}$. That is, $\mathbb{B}(\mathbf{x})$ is an operator that regards $\mathbf{x}$ as a binary number and expresses it as a decimal integer. For a given integer i, we can define an operator of the inverse of $\mathbb{B}(\mathbf{x}) = i$, that is, $\mathbf{x} = \mathbb{B}^{-1}(i)$.
- $|S|$: the number of elements in a discrete set S.

6.2 Model Description

To formally discuss the problem, the discrete space model is described below.

Let $\mathbf{s} = (s_1, \cdots, s_{n_s}) \in S_s$ be a binary state vector, and we would like to know if s_i is 0 or 1, where $S_s \subseteq \{0, 1\}^{n_s}$ is a

given set of possible states. The term $s_i = 1$ (0) means that a target object occupies (does not occupy) position i, and $\mathbf{a}$ denotes a sensing area called a sensing template, where $(\mathbf{a})_i = 1$ means that the sensor can observe s_i. The operator $\star$ means "sensing" because $\mathbf{a} \star \mathbf{s} = \mathbf{I}(\sum_{i=1}^{n_s}(\mathbf{a})_i s_i > 0)$ is the sensing result of $\mathbf{s}$ by $\mathbf{a}$. The term $\mathbf{a} \star \mathbf{s} = 1$ (0) means that the sensing template $\mathbf{a}$ detects (does not detect) a target object in $\mathbf{s}$. That is, when there is a target object occupying at least one of the observed points defined by the sensing template $\mathbf{a}$, $\mathbf{a} \star \mathbf{s} = 1$; otherwise, $\mathbf{a} \star \mathbf{s} = 0$.

Let $A_s \equiv \{\mathbf{a}_1, \cdots, \mathbf{a}_{n_a}\}$ be the set of all available sensing templates, where $n_a \equiv |A_s|$ and $\mathbf{a}_i \neq \mathbf{a}_j$ for any $i \neq j$.

Let $\mathbf{b}(i)$ be an n_a-element column vector of which the j-th element $(\mathbf{b}(i))_j$ is given by $\mathbf{a}_j \star (\mathbb{B}^{-1}(i))$ for $j = 1, \cdots, n_a$ and define the matrices $B(\{0, 1\}^{n_s}) \equiv (\mathbf{b}(0) \cdots \mathbf{b}(2^{n_s} - 1))$ and $B(S_s) \equiv (\mathbf{b}(i_1) \cdots \mathbf{b}(i_j))$, where $i_1 < \cdots < i_j$ and $S_s = \{\mathbb{B}^{-1}(i_1), \cdots, \mathbb{B}^{-1}(i_j)\}$. The term $\mathbf{b}(i)$ is the sensing result by using all the sensing templates available for the state $\mathbb{B}^{-1}(i)$. Therefore, $B(\{0, 1\}^{n_s})$ $(B(S_s))$ is a matrix describing the theoretical sensing results for all the states in $\{0, 1\}^{n_s}$ (S_s).

We now introduce the concept of "classes of sensing templates." If two sensing templates are in the same class, we cannot determine which sensing template corresponds to a sensing result. This is the key assumption related to that in Sect. 2, which is "sensors distributed randomly and independently" and, as a result, "each sensor cannot determine its location". Let n_c be the number of classes, c_i be the set of the i-th class of sensing templates, and $|c_i|$ be the number of sensing templates in class c_i. In c_i, the sensing template $\mathbf{a}_j$ is used with probability $p_{i,j}$, $\sum_{j=1}^{n_a} p_{i,j} = 1$. If the sensing template $\mathbf{a}_j$ is not included in c_i, $p_{i,j} = 0$. Define

$$\mathbf{p}_i \equiv (p_{i,1}, \cdots, p_{i,n_a}) \text{ and } P \equiv \begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_{n_c} \end{pmatrix}. \text{ The term } \mathbf{p}_i \text{ provides a}$$

probability with which each sensing template is used in the i-th class of sensing templates.

Define $u_i(\mathbf{s}) \equiv \sum_{j=1}^{n_a} p_{i,j}(\mathbf{a}_j \star \mathbf{s})$, which is the probability that a sensing result is 1 when a class-i sensing template is used for a given state $\mathbf{s}$, $\mathbf{u}(\mathbf{s}) \equiv (u_1(\mathbf{s}), \cdots, u_{n_c}(\mathbf{s}))$, and $U(S_s) \equiv ({}^t\mathbf{u}(\mathbb{B}^{-1}(i_1)), \cdots, {}^t\mathbf{u}(\mathbb{B}^{-1}(i_j)))$ for $S_s = \{\mathbb{B}^{-1}(i_1), \cdots, \mathbb{B}^{-1}(i_j)\}$.

6.3 Conventional Model

When $|c_i| = 1$, we call this class the reduced class. The conventional model discussed in [15], [24]–[26] is a model of $|c_i| = 1$ for all i and $p_{i,j} = 1$ or 0 for all i, j. That is, it is a model in which all the classes are reduced. Therefore, in the conventional model, we can distinguish each sensing template or the relationship between each sensing template and each sensing result. For the conventional model, $u_i = 1$ or 0.

7. Results on Discrete Space

First, we define "observable" in the discrete space model-

ing: "$\mathbf{s}_1$ is observable by $\{c_1, \cdots, c_{n_c}\}$" means that we can distinguish $\mathbf{s}_1$ from any other state by $\{c_1, \cdots, c_{n_c}\}$. Here, "$\{c_1, \cdots, c_{n_c}\}$ can distinguish $\mathbf{s}_1$ from $\mathbf{s}_2 (\neq \mathbf{s}_1)$" means that $\mathbf{u}(\mathbf{s}_1) \neq \mathbf{u}(\mathbf{s}_2)$ for given $\mathbf{s}_1$ and $\mathbf{s}_2$.

For the conventional model, the definition of "distinguish" given above is identical to that defined in [26].

Because $u_i(\mathbf{s}) = \sum_{j=1}^{n_a}(\mathbf{a}_j \star \mathbf{s})p_{i,j} = \mathbf{p}_i\mathbf{b}(\mathbb{B}(\mathbf{s}))$, $U(S_s) = PB(S_s)$. According to the definition of observability above, we can prove the following.

Result 4: For any S_s, any state is observable if and only if no two columns in $PB(S_s)$ are identical. If $S'_s \subset S_s$, any state observable in S_s is observable in S'_s.

The latter statement is because each column of $B(S'_s)$ is included in the set of columns of $B(S_s)$. This result provides an important suggestion: If we can reasonably limit the state spaces, it is likely that we can identify each state, even when the system is not observable for a large state space.

Because of Result 4, observability is equivalent to that in which no two column vectors in $PB(S_s)$ are equal. If $\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for any $\mathbf{s} \neq \mathbf{s}'$ and if the rank of P is n_a, the two column vectors corresponding to $\mathbf{s}$ and $\mathbf{s}'$ in $PB(S_s)$ cannot be the same. If there exist $\mathbf{s} \neq \mathbf{s}'$ such that $\mathbf{b}(\mathbb{B}(\mathbf{s})) = \mathbf{b}(\mathbb{B}(\mathbf{s}'))$, $P\mathbf{b}(\mathbb{B}(\mathbf{s})) = P\mathbf{b}(\mathbb{B}(\mathbf{s}'))$. We summarize as follows.

Result 5: If $\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for any $\mathbf{s} \neq \mathbf{s}'$ and if the rank of P is n_a, observability is achieved. If there exist $\mathbf{s} \neq \mathbf{s}'$ such that $\mathbf{b}(\mathbb{B}(\mathbf{s})) = \mathbf{b}(\mathbb{B}(\mathbf{s}'))$, observability is not achieved.

Result 5 shows that the rank of P plays an important role. Because the condition "$\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for any $\mathbf{s} \neq \mathbf{s}'$" is the observability for the conventional model, we can say that the elements of P determine the observability of the proposed model. Because "$P\mathbf{b}(\mathbb{B}(\mathbf{s})) = P\mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for $\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$" means that elements of P satisfy certain linear equations, observability is normally achieved almost everywhere in the parameter space in P (anywhere not satisfying these equations) if the system without the probabilistic mixture (that is, the conventional model) is observable. However, it does not mean that no practical problem can almost occur under the observability of the conventional model. As shown in the example below, when the number of sensing data samples is finite, the accuracy of parameter estimation deteriorates around the point at which observability is not achieved in the parameter space. As the number of samples decreases, the range of parameters such that the accuracy of parameter estimation deteriorates becomes larger. Simultaneously, we can say that, if we can design P, we should set $n_c = n_a$ and make P, with rank $n_c (= n_a)$, satisfy observability.

7.1 Simple Example

Consider a ring network with $2n_d + 1$ nodes as a concrete example of "network quality", as mentioned above. The state can be described as $(s_1, \cdots, s_{n_s})$, where $s_i = 0$ (1) means the GOOD (BAD) quality of the link between the i-th node and $(i + 1)$-th node and $n_s = 2n_d + 1$.

Thus, a pair of nodes (a source and a destination) corresponds to a class of sensing templates, and there are two sensing templates corresponding to the shorter and longer routes in each class. Therefore, $n_c = (n_s - 1)n_s/2 = (2n_d + 1)n_d$ and $n_a = 2(2n_d + 1)n_d$. Let $\mathbf{a}_{j+(i-1)n_s}$ be the sensing template corresponding to the i-hop route between node-j and node-$(j+i)$ (mod n_s) through node-$(j+1)$, node-$(j+2)$, ... Therefore, $\mathbf{a}_{j+(i-1)n_s}$ also corresponds to the i-hop route between node-$(j+i)$ to node-j through node-$(j+i-1)$, node-$(j+i-2)$, ... The term $(\mathbf{a}_{j+(i-1)n_s})_k = 1$ (0) means that the link between the k-th node and $(k+1)$-th node is included (not included) in this route.

For example, when s_i denotes the quality of the link between node-i and node-$(i+1)$, the sensing template corresponding to the one-hop route (the short route) between node-j and node-$(j+1)$ is $\mathbf{a}_j = (0, \cdots, 0, 1, 0, \cdots, 0)$, where 1 is at the j-th element, and that corresponding to the (n_s-1)-hop route (the long route) between node-j and node-$(j+1)$ is $\mathbf{a}_{j+(n_s-2)n_s} = (1, \cdots, 1, 0, 1, \cdots, 1)$, where 0 is at the j-th element. Assume that a shorter hop route is chosen with probability p and the other route (a longer hop route) is chosen with probability $1 - p$.

As an example, parameters and variables for $n_d = 1$ (that is, $n_s = 3$) are shown below: Then, $\mathbf{a}_1 = (1, 0, 0)$, $\mathbf{a}_2 = (0, 1, 0)$, $\mathbf{a}_3 = (0, 0, 1)$, $\mathbf{a}_4 = (1, 1, 0)$, $\mathbf{a}_5 = (0, 1, 1)$, $\mathbf{a}_6 = (1, 0, 1)$. When node-i and node-$(i+1)$ is the i-th source-destination pair and corresponds to c_i,

$$P = \begin{pmatrix} p & 0 & 0 & 0 & 1-p & 0 \\ 0 & p & 0 & 0 & 0 & 1-p \\ 0 & 0 & p & 1-p & 0 & 0 \end{pmatrix},$$

$$B(\{0, 1\}^{n_s}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, U(B(\{0, 1\}^{n_s}))$$

$$= \begin{pmatrix} 0 & 1-p & 1-p & 1-p & p & 1 & 1 & 1 \\ 0 & 1-p & p & 1 & 1-p & 1-p & 1 & 1 \\ 0 & p & 1-p & 1 & 1-p & 1 & 1-p & 1 \end{pmatrix}.$$

7.2 Observability for Simple Example

This subsection discusses observability for the simple example mentioned above. The possible state set S_s we assume is the set of states in which at most n_B links are BAD.

Note that $\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for any $\mathbf{s} \neq \mathbf{s}'$. (This is because of the following reason: if $\mathbf{s} \neq \mathbf{s}'$, there is an integer i such that $(\mathbf{s})_i \neq (\mathbf{s}')_i$. Then, for the sensing template $\mathbf{a} = \mathbf{e}_i$, $\mathbf{a} \star \mathbf{s} \neq \mathbf{a} \star \mathbf{s}'$. This means $\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq \mathbf{b}(\mathbb{B}(\mathbf{s}'))$.) We should also note that $(P\mathbf{b}(\mathbb{B}(\mathbf{s})))_i = p\mathbf{a}_s\star\mathbf{s}+(1-p)\mathbf{a}_l\star\mathbf{s}$, where $\mathbf{a}_s$ is a sensing template for the shorter hop route between a certain pair of nodes and $\mathbf{a}_l$ is that for the longer hop route between these nodes. For two states $\mathbf{s}, \mathbf{s}'$ ($\mathbf{s} \neq \mathbf{s}'$), there exists $\mathbf{a}_s$ such that $\mathbf{a}_s \star \mathbf{s} \neq \mathbf{a}_s \star \mathbf{s}'$. This is because, if $(\mathbf{s})_i \neq (\mathbf{s}')_i$, $\mathbf{a}_s = \mathbf{e}_i$ makes $\mathbf{a}_s \star \mathbf{s} \neq \mathbf{a}_s \star \mathbf{s}'$. Thus, we can assume that $\mathbf{a}_s \star \mathbf{s} = 1$, $\mathbf{a}_s \star \mathbf{s}' = 0$ without loss of generality. Then, if $\mathbf{a}_l \star \mathbf{s} \geq \mathbf{a}_l \star \mathbf{s}'$, $P\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq P\mathbf{b}(\mathbb{B}(\mathbf{s}'))$ for $p > 0$. If $\mathbf{a}_l \star \mathbf{s} = 0$, $\mathbf{a}_l \star \mathbf{s}' = 1$,

$(P\mathbf{b}(\mathbb{B}(\mathbf{s})))_i = p \neq 1 - p = (P\mathbf{b}(\mathbb{B}(\mathbf{s}')))_i$ for $p \neq 1/2$. As a result, if $p \neq 0, 1/2$, $P\mathbf{b}(\mathbb{B}(\mathbf{s})) \neq P\mathbf{b}(\mathbb{B}(\mathbf{s}'))$. According to Results 4 and 5, the system is observable for any n_B for $p \neq 0, 1/2$. In addition, if $p = 1/2$, $P\mathbf{b}(\mathbb{B}(\mathbf{e}_i)) = P\mathbf{b}(\mathbb{B}(\mathbf{e}_j))$, that is, the system is not observable.

7.3 Numerical Results for Simple Example

The relationship between the observability and state estimation errors is now investigated for the simple example. The theoretical results mentioned in this paper are based on the sensing results $\mathbf{u}(\mathbf{s})$. However, we cannot actually obtain $\mathbf{u}(\mathbf{s})$ because of the finite number of sensing samples. Therefore, even when the system is observable, it may be difficult to estimate the system state with a finite number of sensing samples.

We assume that n_R reports are available for each pair of nodes i, j but do not include the route information and that $100r_B(i, j)\%$ of these n_R reports state BAD quality. (The assumption of no route information is validated when the ring network is a physical network and is a logically mesh network where traffic into an incoming layer-3 node is transferred to an outgoing layer-3 node through layer 1 or 2 functions at the nodes between them.) Define $\mathbf{r}_B \equiv (r_B(1, 2), \cdots, r_B(n_s - 1, n_s))$. Based on $r_B(i, j)$, we estimate the state of the network by finding $\mathbf{s}$ among the possible states to minimize $\min_\mathbf{s} |E[\mathbf{r}_B] - \mathbf{r}_B| = \min_\mathbf{s} |\mathbf{u}(\mathbf{s}) - \mathbf{r}_B|$. (Although this estimation method is brute force, the following result is almost insensitive to the estimation method used.)

The results using a computer simulation are shown in Fig. 8, where the true state $s_{TRUE} \neq \mathbf{0}^{(n_s)}$ was randomly chosen and 100 cases of s_{TRUE} were used for each point in this figure. Figure 8 suggests the following. (1) At $p = 0.5$, the possibility of correct estimation seriously deteriorates. This is because some states among possible states are not distinguishable at $p = 0.5$. Around $p = 0.5$, the estimation error becomes large. Therefore, judgment concerning observability is critical. (2) Observability is not achieved only at $p = 0.5$. However, estimation deterioration occurs around $p = 0.5$. When the number n_R of reports is small, the range of p at which estimation errors can occur becomes larger. Practically, caution is needed for estimation around the point where observability is not achieved, particularly
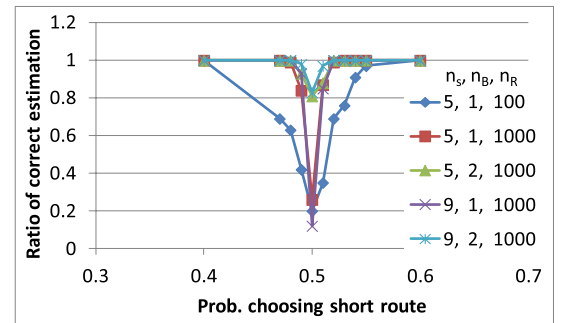

Fig. 8 Results for ring network [14].

when the number of reports is small. (3) The possibility of correct estimation for $n_B = 2$ is higher than that for $n_B = 1$. In particular, the ratio of correct estimation at $p = 0.5$ is sensitive to n_B. This is because any state in which one link is BAD cannot be distinguished among other such states. Thus, when $n_B = 1$ and $p = 0.5$, the estimation method is equivalent to randomly choosing among the possible states ($\neq \mathbf{0}^{(n_s)}$) and the ratio of correct estimation is about $1/n_s$. On the other hand, when $n_B = 2$ even if $p = 0.5$, a state $\mathbf{s}_2$ in which two links are BAD can be distinguished from any state $\mathbf{s}_1$ in which one link is BAD. This is because $u_i(\mathbf{s}_2) = 1$ can occur for a certain i, but $u_i(\mathbf{s}_1) = 1$ never occurs for any i if $p \neq 0, 1$. In addition, we can show that any two states $\mathbf{s}_2, \mathbf{s}_2'$ in which two links are BAD can be distinguished even if $p = 0.5$. Consequently, for $n_B = 2$, if the true states are any states in which two links are BAD, the true state can be identified even if $p = 0.5$. Therefore, the correct estimation ratio is better for $n_B = 2$ than for $n_B = 1$ at $p = 0.5$, although any state in which one link is BAD cannot be observable for $n_B = 2$. (4) The deterioration of the correct estimation ratio at $p = 0.5$ becomes sharper as the number of nodes increases. This may be due to the fact that the total number of samples used increases as the number of nodes increases.

8. Relationship between Results for Discrete Space and Those for the Plane

The major difference in the model for the discrete space from that for the plane is the class of sensing templates. The objective of introducing the class of sensing templates is to model the fact that we cannot distinguish sensing results. This fact is not identical to the fact that we cannot distinguish the sensing results wherever sensing is executed. The concept of the class of sensing templates can model broader cases. For example, the class of sensing templates can model cases in which we cannot distinguish sensing results from sensors monitoring many points and from those monitoring a small number of points. In this sense, the class of sensing templates is a part of an extension of the model for the plane. Therefore, by introducing the class of sensing templates in the model for the plane, we may be able to discuss other new topics.

When each sensing template corresponds to the location of sensors and $p_{i,j}$ is independent of i, j, this is the model in which sensors are uniformly distributed. Under this model, a similar result to Result 2 is obtained. However, it becomes too simple because concepts, such as convexity and spatial stationarity, are lost.

One of the conclusions for the discrete space is that observability is also normally achieved almost everywhere in the parameter space in P. The observability defined for the discrete space means that, when the number of samples is sufficiently large, we can distinguish the sensing result of each $\mathbf{s}$. As a result, for each point s_i, we can determine whether the point is occupied. Therefore, the conclusion suggests that we can determine the position and shape of

the target object in many cases. In the discussion for the plane in Sects. 4 and 5, we have no idea about how to determine the location of the target object. However, we may be able to estimate the location of the target object under the discrete space. This is because we cover the non-stationary distribution of sensors in the discrete space model. We now refer to a method for estimating the location of the target object when P is location dependent. With this method, sensors are intentionally deployed with some bias dependent on the location [14], and the biased sensing results are used to estimate the locations of the target object as well as other parameters of the target object. (As shown in Sect. 7.2, however, if the sensing templates in a class are chosen with equal probability, it is likely that many parameters including the location of the target object are not observable.) However, the method using non-stationary sensor distribution requires us to know the non-stationary distribution of sensors. This means that this method loosens the original assumption in that we have no idea about the locations of sensors. In some applications, such as participatory sensing [13], this method may be appropriate because, although we cannot identify the location of each sensor, it is valid to assume that the sensor distribution is similar to the population distribution.

9. Remaining Research Issues

Many issues remain for future research, but they depend on what the "blind men" are and what the "elephant" is. They also depend on modeling: how and where we model them, in a physical space or in an abstract space including a discrete state space.

- Judgement: Is it possible to determine whether a parameter can be estimated for a generically shaped, time-variant, and moving target object through the use of binary sensors, composite sensors, or advanced function sensors? If yes, how do we determine it?
- Additional mechanisms: By introducing additional mechanisms, the range of observable parameters may increase. What kinds of mechanisms are useful to increase the range of observable parameters?
- Estimation method: Many parameters of a global view exist, for example shape and location. We have learned how to estimate only a few such parameters. When additional mechanisms are introduced, how do we modify the estimation methods? In the near future, we will propose a method for estimating the parameters of shape and location of a time-variant target object when binary sensors are used [13].
- Implementation: Even when we can derive an estimation method, it requires a network to collect the necessary local information and may require the processing of a huge amount of data. Existing networks, such as mobile networks, may be available but they may not be suitable for sensor networking or machine-to-machine communication. We have already proposed a network dedicated to such an objective [27], [28], but it may not

be the final solution. In addition to the transport network, name resolution and ID remain important issues [29].

Implementation of composite sensor nodes, advanced function nodes, and additional mechanisms if necessary are also for future work. Finally, developing efficient data processing algorithms is another issue to address.

10. Conclusion

This paper discussed the observability of parameters of a target object in the plane when local information on whether each location-unknown sensor detects the target object is given. Observability is achieved only for the size and perimeter length of the target object when the target object and sensing area are convex. A composite sensor node as an example of a mechanism for breaking this observation limit was proposed. By using this mechanism, we can increase the observable parameters. However, we have not achieved observability of the shape and location of a target object or captured the global view of the target object.

This paper also discussed observability in the discrete space. In the discrete space model, some network applications can be considered as problems of observability.

If collecting a large amount of information enables us to obtain a global view, the need and importance of a network become higher. Our network community should make an effort to satisfy such needs by offering methods and mechanisms for obtaining a global view.

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Appendix: Observability with Convex Combined Sensing Area

Divide a convex region $\mathcal{A}$ into two exclusive convex regions $\mathcal{A}_1$ and $\mathcal{A}_2$ (Fig. A·1). Assume a composite sensor node with two sensors of which sensing areas are $\mathcal{A}_1$ and $\mathcal{A}_2$.

Let $m_{i,j}$ be the measure of a set of composite sensor nodes such that the status of the first sensor is $i \in \{0, 1\}$ and that of the second sensor is $j \in \{0, 1\}$ in a single composite sensor, where the status being equal to 0 (1) means not-detecting (detecting) the target object. For example, $m_{1,1}$
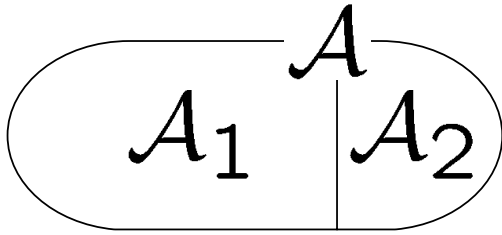
Fig. A·1 Composite sensor node with convex combined sensing area.

means that both the first and second sensors detect the target object. According to Eq. (1), for a convex $\mathcal{T}$,

$$m_{1,0} + m_{1,1} = F(\mathcal{T}, \mathcal{A}_1),$$
$$m_{0,1} + m_{1,1} = F(\mathcal{T}, \mathcal{A}_2),$$
$$m_{0,1} + m_{1,0} + m_{1,1} = F(\mathcal{T}, \mathcal{A}).$$

Let $\Pr(i, j)$ be the probability that a pair of the statuses of the first and second sensors is (i, j). Because $\|\mathcal{A}\| = \|\mathcal{A}_1\| + \|\mathcal{A}_2\|$ and $\Pr(i, j) = m_{i,j}/F(\Omega, \mathcal{A})$,

$$\Pr(1, 1) = \{2\pi\|\mathcal{T}\| + |\mathcal{T}|(|\mathcal{A}_1| + |\mathcal{A}_1| - |\mathcal{A}|)\}/F(\Omega, \mathcal{A}),$$
$$\Pr(1, 0) = \{2\pi\|\mathcal{A}_1\| + |\mathcal{T}|(|\mathcal{A}| - |\mathcal{A}_2|)\}/F(\Omega, \mathcal{A}),$$
$$\Pr(0, 1) = \{2\pi\|\mathcal{A}_2\| + |\mathcal{T}|(|\mathcal{A}| - |\mathcal{A}_1|)\}/F(\Omega, \mathcal{A}).$$

These equations do not include the target object parameters other than $\|\mathcal{T}\|$ and $|\mathcal{T}|$. Therefore, other parameters cannot be observable if $\mathcal{A}_1$ and $\mathcal{A}_2$ are exclusive and convex and $\mathcal{A}_1 \cup \mathcal{A}_2$ is convex.

Hiroshi Saito graduated from the University of Tokyo with a B.E. degree in Mathematical Engineering in 1981, an M.E. degree in Control Engineering in 1983 and received his Dr.Eng. in Teletraffic Engineering in 1992. He joined NTT in 1983. He is currently an Executive Research Engineer at NTT Network Technology Labs. He received the Young Engineer Award of the Institute of Electronics, Information and Communication Engineers (IEICE) in 1990, the Telecommunication Advancement Institute Award in 1995 and 2010, and the excellent papers award of the Operations Research Society of Japan (ORSJ) in 1998. He served as an editor and a guest editor of technical journals such as Performance Evaluation, IEEE Journal of Selected Areas in Communications, and IEICE Trans. Communications, the organizing committee chairman of a few international conferences, a TPC member of more than 30 international conferences, and the director of Journal and Transactions of IEICE. He is currently an editorial board member of Computer Networks. Dr. Saito is a fellow of IEEE, IEICE, and ORSJ, and a member of IFIP WG 7.3. His research interests include traffic technologies of communications systems, network architecture, and ubiquitous systems. More information can be found at http://www9.plala.or.jp/hslab.