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LETTER

Novel Auto-Calibration Method for 7-Elements Hexagonal Array with Mutual Coupling

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SUMMARY Mutual coupling between antenna array elements will significantly degrade the performance of the array signal processing methods. Due to the Toeplitz structure of mutual coupling matrix (MCM), there exist some mutual coupling calibration algorithms for the uniform linear array (ULA) or uniform circular array (UCA). But few methods for other arrays. In this letter, we derive a new transformation formula for the MCM of the 7-elements hexagonal array (HA-7). Further, we extend two mutual coupling auto-calibration methods from UCA to HA by the transformation formula. Simulation results demonstrate the validity of the proposed two methods.

key words: *array signal processing, mutual coupling, hexagonal array, auto-calibration*

1. Introduction

Beamforming and direction-of-arrival (DOA) estimation have been two classical problems in array signal processing in the last decades. Many excellent methods have been proposed [1], [2], which have attracted interest in many fields, such as radar, sonar, and mobile communication. Most of these methods work with the assumption that the array is ideal. However, in practice, the ideal array model is often broken by some characteristics of the array antenna, like the mutual coupling between the array elements. It will degrade the performance of these methods [3], [4].

Many methods have been proposed to calibrate the array mutual coupling. Generally, these methods can be divided into two categories, offline calibration algorithms [5]–[8] and auto-calibration algorithms [9]–[15]. The offline calibration algorithms need a set of auxiliary sources. The number and DOA of sources are usually known in advance. Literature [7] uses a set of time-disjoint sources to calibrate the phase-amplitude mutual coupling errors for ULA or UCA by solving the constructed optimization model. Literature [8] handles array errors in the DFT beam space and only uses a single auxiliary source to accomplish the calibration of phase-amplitude and mutual coupling errors for UCA. Although the offline method's computational complexity is low, the effect of this kind of method will degrade with time and environmental changes. Therefore, the auto-calibration method, which can estimate the direction of arrival angle and the mutual coupling coefficient simultaneously, has aroused much interest these years. Literature [12] uses the GEESE method to complete the estimation of DOA and mutual coupling coefficients for ULA; Literature [13] proposed an iterative method to estimate DOA and mutual coupling coefficients for UCA. However, most offline or auto-calibration methods for mutual coupling are limited to UCA or ULA because the MCMs of these two arrays have a Toeplitz structure, which other arrays do not have.

Hexagonal arrays have great potential in many applications such as satellite and radar because they provide compact triangular-grid element arrangements with the ability to steer high-gain directive beams in the full azimuth. Moreover, hexagonal arrays show lower sidelobe amplitudes than rectangular arrays and UCA [16], [17]. However, the MCM of HA does not have the Toeplitz structure, and HA has attracted little attention in the mutual coupling calibration.

In this letter, a kind of mutual coupling calibration method for an HA-7 is presented. First, A transformation formula for the MCM of the hexagonal array is derived. This formula allows most of the mutual coupling calibration algorithm for UCA or ULA to be extended to HA-7. Then, combining with the methods of literature [10] and [13], two mutual coupling auto-calibration algorithms for HA-7 are proposed. Simulation results show that the performance of the two proposed methods on HA-7 is close to the methods of literature [10] and [13] on UCA.

2. Signal Model and Problem Formulation

Consider an HA-7 consisting of 7 antenna elements, as shown in Fig. 1, where the distance between any two adjacent sensors is d, and the position vectors (x, y, z) of elements are $[\boldsymbol{p_1}, \boldsymbol{p_2}, \cdots, \boldsymbol{p_7}] = [(d, 0, 0), (d/2, \sqrt{3}d/2, 0), (-d/2, \sqrt{3}d/2, 0), (-d, 0, 0), (-d/2, -\sqrt{3}d/2, 0), (d/2, -\sqrt{3}d/2, 0), (0, 0, 0)]$. Assume the antenna array receives K uncorrelated narrow-band signals, $s_1(t), s_2(t), \cdots, s_K(t)$, with the wavelength λ, and the DOA of the k-th signal is $\varphi_k, (k = 1, 2, \cdots, K)$. Define $\boldsymbol{v_k} = [\cos\varphi_k, \sin\varphi_k, 0], (k = 1, 2, \cdots, K)$. The received signal of

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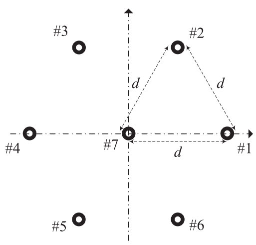

Fig. 1 Planar 7-elements hexagonal array geometry.

i-th antenna element in the ideal array can be written as:

$$r_i(t) = \sum_{k=1}^{K} s_k(t) e^{\frac{-j2\pi \boldsymbol{v_k} \boldsymbol{p_i}^T}{\lambda}} + n_i(t), \tag{1}$$

where $n_i(t)$ is the white Gaussian noise with a mean of 0 and variance of σ^2. Further, the vector form of the received signal of HA-7 can be written as:

$$\begin{aligned} \boldsymbol{r}(t) &= [r_1(t), \cdots, r_7(t)]^T = \sum_{k=1}^{K} \boldsymbol{a}(\varphi_k) s_k(t) + \boldsymbol{n}(t) \\ &= \boldsymbol{AS}(t) + \boldsymbol{n}(t), \end{aligned} \tag{2}$$

where $\boldsymbol{a}(\varphi_k) = [e^{-j2\pi \boldsymbol{v_k} \boldsymbol{p}_1^T/\lambda}, \cdots, e^{-j2\pi \boldsymbol{v_k} \boldsymbol{p}_7^T/\lambda}]^T$ is the steering vector, $\boldsymbol{A} = [\boldsymbol{a}(\varphi_1), \cdots, \boldsymbol{a}(\varphi_K)]$ is the array manifold matrix, $\boldsymbol{S}(t) = [s_1(t), \cdots, s_K(t)]^T$ is the signal vector, and $\boldsymbol{n}(t) = [n_1(t), \cdots, n_7(t)]^T$ is the noise vector.

When the mutual coupling between each array element exists, Eq. (2) should be modified as:

$$\boldsymbol{r}(t) = \boldsymbol{CAS}(t) + \boldsymbol{n}(t), \tag{3}$$

where $\boldsymbol{C}$ is the MCM of the HA-7. Reference [14] points out the conclusion that the mutual coupling coefficients are inversely proportional to the distance between the elements, and the coupling between any two equally spaced sensors is the same. Reference [18] present the absolute value of the elements of the full electromagnetic $\boldsymbol{C}$ and verifies the conclusion proposed by Reference [14]. Therefore the MCM of the HA-7 can be denoted as:

$$\boldsymbol{C} = \left[\begin{array}{cccccc:c} c_1 & c_2 & c_3 & c_4 & c_3 & c_2 & c_2 \\ c_2 & c_1 & c_2 & c_3 & c_4 & c_3 & c_2 \\ c_3 & c_2 & c_1 & c_2 & c_3 & c_4 & c_2 \\ c_4 & c_3 & c_2 & c_1 & c_2 & c_3 & c_2 \\ c_3 & c_4 & c_3 & c_2 & c_1 & c_2 & c_2 \\ c_2 & c_3 & c_4 & c_3 & c_2 & c_1 & c_2 \\ \hdashline c_2 & c_2 & c_2 & c_2 & c_2 & c_2 & c_1 \end{array} \right]. \tag{4}$$

Actually, $\boldsymbol{C}_{1:6,1:6}$, the first six rows and six columns of $\boldsymbol{C}$, is the MCM of the 6-elements UCA (UCA-6), and it has the Toeplitz structure but $\boldsymbol{C}$ does not. Further, the covariance matrix of the array received signal can be written as:

$$\boldsymbol{R} = \mathrm{E}[\boldsymbol{r}(t)\boldsymbol{r}^H(t)] = \boldsymbol{CAR_s A}^H \boldsymbol{C}^H + \sigma^2 \boldsymbol{I}, \tag{5}$$

where $\boldsymbol{R_s} = \mathrm{E}[\boldsymbol{S}(t)\boldsymbol{S}^H(t)]$ is the desired signal covariance matrix, and $\boldsymbol{I}$ is an identity matrix.

Further, the eigendecomposition of $\boldsymbol{R}$ is written as:

$$\boldsymbol{R} = \sum_{i=1}^{7} \lambda_i \boldsymbol{e_i} \boldsymbol{e_i}^H = \boldsymbol{E_S \Sigma_S E_S^H} + \boldsymbol{E_N \Sigma_N E_N^H}, \tag{6}$$

where λ_i is the i-th eigenvalue, and $\lambda_i > \lambda_{i+1}$. $\boldsymbol{e_i}$ is the eigenvector of λ_i. $\boldsymbol{E_S} = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_K]$, the signal subspace, contains K eigenvectors corresponding to the K maximum eigenvalues. $\boldsymbol{E_N} = [\boldsymbol{e}_{K+1}, \cdots, \boldsymbol{e}_7]$, the noise subspace, contains the rest of eigenvectors. The subspace principle can be denoted as follows:

$$\mathrm{span}\{\boldsymbol{CA}\} = \mathrm{span}\{\boldsymbol{E_S}\} \perp \mathrm{span}\{\boldsymbol{E_N}\}. \tag{7}$$

Therefore, it can be derived as follow:

$$\|\boldsymbol{E_N^H C a}(\varphi)\|^2 = 0. \tag{8}$$

Without considering the mutual coupling, $\boldsymbol{C}$ is an identity matrix, Eq. (8) will hold when the φ is the actual DOA of the signal, and the DOA estimation can be obtained by searching in 0–360 degrees. However, the above method will be invalid when mutual coupling exists.

3. Review of Calibration Methods for UCA

In this section, two UCA auto-calibration methods proposed in [10] and [13] will be reviewed. For the UCA, the MCM $\boldsymbol{C_{uca}}$ has Toeplitz structure, so there exists transformation $\mathrm{T}[\cdot]$ as follow:

$$\boldsymbol{C_{uca} a}(\varphi) = \mathrm{T}[\boldsymbol{a}(\varphi)] \boldsymbol{c_{uca}}, \tag{9}$$

where $\boldsymbol{c_{uca}} = [c_{1,1}, c_{1,2}, \cdots, c_{1,q}]^T$, $c_{1,i}$ is the element in row 1, column i of $\boldsymbol{C_{uca}}$, $q = \lfloor M/2 + 1 \rfloor$, and M is the number of UCA antenna elements. Define the matrix $\mathrm{T}[\boldsymbol{a}(\varphi)]$ as:

$$\mathrm{T}[\boldsymbol{a}(\varphi)] = \boldsymbol{T_1} + \boldsymbol{T_2} + \boldsymbol{T_3} + \boldsymbol{T_4}, \tag{10}$$

$$[\boldsymbol{T_1}]_{ij} = \begin{cases} a(\varphi)_{i+j-1} & i+j \leq M+1 \\ 0 & otherwise, \end{cases} \tag{11a}$$

$$[\boldsymbol{T_2}]_{ij} = \begin{cases} a(\varphi)_{i-j+1} & i \geq j \geq 2 \\ 0 & otherwise, \end{cases} \tag{11b}$$

$$[\boldsymbol{T_3}]_{ij} = \begin{cases} a(\varphi)_{M+1+i-j} & i < j \leq 2 \\ 0 & otherwise, \end{cases} \tag{11c}$$

$$[\boldsymbol{T_4}]_{ij} = \begin{cases} a(\varphi)_{i+j-M-1} & 2 \leq j \leq p, i+j \geq M+2 \\ 0 & otherwise. \end{cases} \tag{11d}$$

Where $p = \lfloor (M+1)/2 \rfloor$, and $a(\varphi)_i$ is the i-th element of $\boldsymbol{a}(\varphi)$. According to Eq. (9), the $\mathrm{T}[\cdot]$ transforms the problem of estimating the MCM $\boldsymbol{C_{uca}}$ into estimating the vector $\boldsymbol{c_{uca}}$, which reduces the computational complexity significantly. Moreover, the transformation provides a new idea for DOA estimation with mutual coupling.

Combined with Eqs. (9) and (10), Eq. (8) can be written as:

$$\boldsymbol{c_{uca}^H Q}(\varphi) \boldsymbol{c_{uca}} = 0, \tag{12}$$

where $\boldsymbol{Q}(\varphi) = \mathrm{T}[\boldsymbol{a}(\varphi)]^H \boldsymbol{E_N E_N^H} \mathrm{T}[\boldsymbol{a}(\varphi)]$. In general, $\boldsymbol{Q}(\varphi)$ is full rank, and the rank reduction will take place only when φ coincides with one of the signal directions $\{\varphi_k\}_{k=1}^{K}$ [10]. Therefore, reference [10] (C.Qi method) proposed a novel DOA estimator:

$$f(\varphi) = \frac{1}{\det\{\boldsymbol{Q}(\varphi)\}}. \tag{13}$$

It can be seen from Eq. (12) that $\boldsymbol{c_{uca}}$ is the eigenvector corresponding to the eigenvalue 0 of $\boldsymbol{Q}(\varphi)$. The estimation of $\boldsymbol{c_{uca}}$ can be written as:

$$\tilde{\boldsymbol{c}}_{uca} = \mathrm{v}_{\mathbf{min}}\left\{\sum\nolimits_{k=1}^{K} \boldsymbol{Q}(\tilde{\varphi}_k)\right\}, \tag{14}$$

where $\mathrm{v}_{\mathbf{min}}\{\cdot\}$ means obtaining the eigenvector of the minimum eigenvalue, $\tilde{\varphi}_k$ is the estimated k-th DOA.

Reference [13] proposed an iterative method (M.Wang method) to estimate DOA and mutual coupling coefficients. It constructs a quadratic minimization problem as follows:

$$(\{\varphi_k\}_{k=1}^{K}, \boldsymbol{c_{uca}}) = \underset{\{\varphi_k\}_{k=1}^{K}, \boldsymbol{c_{uca}}}{\arg\min}\ \boldsymbol{c}_{\boldsymbol{uca}}^{H}\boldsymbol{Q}(\varphi)\boldsymbol{c_{uca}}. \tag{15}$$

Then the spatial spectrum can be expressed as:

$$p(\varphi) = J^{-1} = \left(\boldsymbol{c}_{\boldsymbol{uca}}^{H}\boldsymbol{Q}(\varphi)\boldsymbol{c_{uca}}\right)^{-1}. \tag{16}$$

With Eqs. (14) and (16), the M.Wang method can obtain the accurate estimations of DOA and $\boldsymbol{c_{uca}}$ by times iterations.

4. Proposed Calibration Method for HA-7

Obviously, the transformation $\mathrm{T}[\cdot]$ is the basis of the above two methods can work normally on UCA. However, the MCM of HA-7 is not a Toeplitz matrix, and the transformation $\mathrm{T}[\cdot]$ cannot be applied to the HA-7. In this section, a new transformation for the HA-7 is derived. Further, using the new transformation, we extend the above two methods to the HA-7. For the convenience of analysis, Eq. (4) can be rewritten as:

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C_{TOP}} & c_5 \times \mathbf{1} \\ c_5 \times \mathbf{1}^T & c_6 \end{bmatrix}, \tag{17}$$

where $\boldsymbol{C_{TOP}} = \boldsymbol{C}_{\mathbf{1:6,1:6}}$, and $\mathbf{1} \in \mathbb{C}^{6,1}$ is a column vector with six 1-elements. In Eq. (17), c_2 and c_1 in the last row and column of $\boldsymbol{C}$ are substituted as c_5 and c_6, and it is just a symbols substitution.

Further, the equation can be obtained as follow:

$$\boldsymbol{C}\boldsymbol{a}(\varphi) = \begin{bmatrix} \boldsymbol{C_{TOP}} \times \boldsymbol{a}(\varphi)_{1:6} + c_5 \times a(\varphi)_7 \times \mathbf{1} \\ c_5 \times \boldsymbol{a}(\varphi)_{1:6} \times \mathbf{1}^T + c_6 \times a(\varphi)_7 \end{bmatrix}. \tag{18}$$

Define the mutual coupling vector of the HA-7 as:

$$\boldsymbol{c} = [c_1, c_2, c_3, c_4, c_5, c_6]^T. \tag{19}$$

Combined with Eq. (9), we rewrite Eq. (18) as:

$$\begin{aligned} \boldsymbol{C}\boldsymbol{a}(\varphi) &= \begin{bmatrix} \mathrm{T}[\boldsymbol{a}(\varphi)_{1:6}]\boldsymbol{c}_{\mathbf{1:4}} + c_5 a(\varphi)_7 \mathbf{1} \\ c_5 \sum_{i=1}^{6} a(\varphi)_i + c_6 a(\varphi)_7 \end{bmatrix} = \mathrm{T_H}[\boldsymbol{a}(\varphi)]\boldsymbol{c} \\ &= \begin{bmatrix} \mathrm{T}[\boldsymbol{a}(\varphi)_{1:6}] & a(\varphi)_7 \mathbf{1} & \mathbf{0} \\ \mathbf{0}^T & \sum_{i=1}^{6} a(\varphi)_i & a(\varphi)_7 \end{bmatrix}\boldsymbol{c}, \end{aligned} \tag{20}$$

where $\mathbf{0} \in \mathbb{C}^{6,1}$ is a column vector with six 0-elements. It can be seen that the derived transformation $\mathrm{T_H}[\cdot]$ has the same form as $\mathrm{T}[\cdot]$. Thus, the C.Qi method and M.Wang method can be extended to HA-7 easily. For the HA-7, Eqs. (12), (13), (14), and (16) can be modified as follow:

$$\boldsymbol{c}^H \mathrm{T_H}[\boldsymbol{a}(\varphi)]^H \boldsymbol{E_N}\boldsymbol{E_N}^H \mathrm{T_H}[\boldsymbol{a}(\varphi)]\boldsymbol{c} = \boldsymbol{c}^H \boldsymbol{Q_H}(\varphi)\boldsymbol{c} = 0, \tag{21}$$

$$f(\varphi) = \frac{1}{\det\{\boldsymbol{Q_H}(\varphi)\}}, \tag{22}$$

$$\tilde{\boldsymbol{c}} = \mathrm{v}_{\mathbf{min}}\left\{\sum\nolimits_{k=1}^{K} \boldsymbol{Q_H}(\tilde{\varphi}_k)\right\}, \tag{23}$$

$$p(\varphi) = J^{-1} = \left(\boldsymbol{c}^H \boldsymbol{Q_H}(\varphi)\boldsymbol{c}\right)^{-1}. \tag{24}$$

The two extended calibration methods, called the extended C.Qi method for HA-7 (ECH) and the extended M.Wang method for HA-7 (EMH), respectively, are shown as follow:

Algorithm 1 Proposed ECH Algorithm

1: Calculate the covariance matrix $\boldsymbol{R}$ with equation (5), and get the noise subspace $\boldsymbol{E_N}$ with equation (6).
2: Construct spatial spectrum with equation (21) and (22), and search for the K highest peaks of the spatial spectrum as the estimated DOA $\{\tilde{\varphi}_k\}_{k=1}^{K}$.
3: Obtain the estimated mutual coupling vector $\tilde{\boldsymbol{c}}$ with equation (23).

Algorithm 2 Proposed EMH Algorithm

1: **Initial** $\tilde{\boldsymbol{c}}^{(0)} = [1, 0, \cdots, 0]^T$, and set $l = 0$, and the threshold δ. calculate the covariance matrix $\boldsymbol{R}$ with equation (5), and get the noise subspace $\boldsymbol{E_N}$ with equation (6).
2: Construct spatial spectrum with $\tilde{\boldsymbol{c}}^{(0)}$ and equation (24), search for the K highest peaks of the spatial spectrum as the estimated DOA $\{\tilde{\varphi}_k^{(0)}\}_{k=1}^{K}$, and calculate $J^{(0)}$.
3: Obtain the estimated mutual coupling vector $\tilde{\boldsymbol{c}}^{(l+1)}$ with equation (23).
4: Construct spatial spectrum with $\tilde{\boldsymbol{c}}^{(l+1)}$ and equation (24), search for the K highest peaks of the spatial spectrum as the estimated DOA $\{\tilde{\varphi}_k^{(l+1)}\}_{k=1}^{K}$, and calculate $J^{(l+1)}$.
5: **Update** iteration coefficient. $l = l + 1$.
6: **Judge** whether the condition $\left|J^{(l)} - J^{(l-1)}\right| \leq \delta$ is true. If true, process ends, and consider $\{\tilde{\varphi}_k^{(l+1)}\}_{k=1}^{K}$ and $\tilde{\boldsymbol{c}}^{(l+1)}$ as final estimated DOAs and mutual coupling vector, respectively. If not, **jump** to step 3.

It is worth mentioning that, in addition to the two methods proposed above, transformation $\mathrm{T_H}[\cdot]$ can extend any UCA calibration method based on $\mathrm{T}[\cdot]$ to HA-7.

5. Simulation Results

In this section, simulations on HA-7 will be presented to illustrate the validity of the proposed methods. For comparison, the C.Qi method and M.Wang method will be simulated on UCA-6 which can be seen as an HA-7 without the center

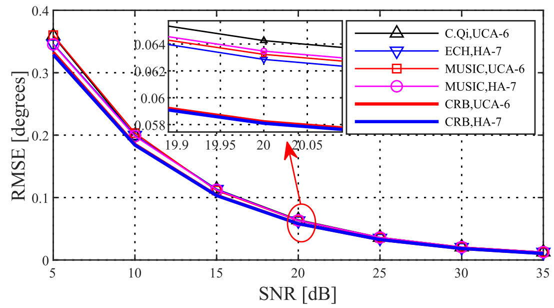

Fig. 2 RMSE of the DOAs estimation and the corresponding CRBs versus SNR in Simulation 1, K=512.

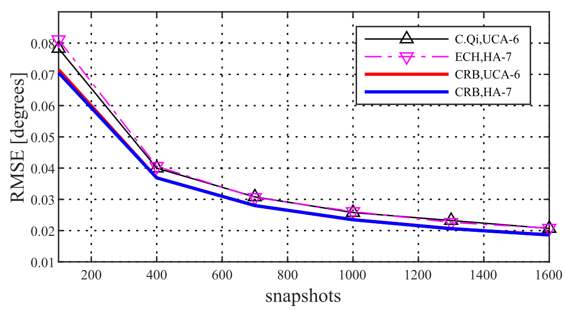

Fig. 3 RMSE of the DOAs estimation and the CRBs versus snapshot number in Simulation 1, SNR=25 dB.

element. The distance between any two adjacent elements of HA-7 is $d = 0.7\lambda$, the mutual coupling vector is $\boldsymbol{c} = [1, 0.7 + 0.44j, 0.39 + 0.24j, 0.31 + 0.2, 0.39 + 0.24j, 0.7 + 0.44j, 0.7 + 0.44j]^T$. In order to ensure the same simulation conditions, the value of d and $\boldsymbol{c}$ are chosen from reference [13]. For UCA, the distance d is same as HA-7's, and $\boldsymbol{c_{uca}} = \boldsymbol{c_{1:4}}$. Define the root mean square errors (RMSE) of DOA estimation and mutual coupling coefficient as follow:

$$RMSE_{\varphi_i} = \sqrt{\left(\sum_{n=1}^{N} \left(\tilde{\varphi}_i{}^{(n)} - \varphi_i\right)\right)^2 / N}, \tag{25}$$

$$RMSE_{\boldsymbol{c}} = \sqrt{\left(\sum_{n=1}^{N} \left(\|\tilde{\boldsymbol{c}}^{(n)} - \boldsymbol{c}\|\right) / \|\boldsymbol{c}\|\right)^2 / N}. \tag{26}$$

where $\tilde{\varphi}_i{}^{(n)}$ and $\tilde{\boldsymbol{c}}^{(n)}$ are the n-th estimated results of DOA φ_i and mutual coupling coefficient $\boldsymbol{c}$, respectively. In this section, the RMSE results are averaged over 2000 trials.

Simulation 1 is performed to verify the validity of EMH. Meanwhile, the C.Qi method is simulated. Assume there is a source from $\varphi = 60°$ impinges on the arrays. In this simulation, there are two experiments, called experiments 1 and 2. The experiments are to know the relationship between the performance of methods and SNR, and snapshot number, respectively. Results are shown in Fig. 2, Fig. 3, and Table 1.

In experiment 1, the snapshot number K is 512, SNR range from 5 dB to 35 dB. Figure 2 plots the RMSE of the DOA estimation versus SNR. Besides the C.Qi method and the EMH, the DOA estimation performances of MUSIC with known mutual coupling coefficients, and the corresponding CRBs are also plotted in Fig. 2. Meanwhile, the RMSE of the mutual coupling coefficients is given in the second and third rows of Table 1. In experiment 2, the SNR is 25 dB, snapshot number K ranges from 100 to 1600, and Fig. 3 plots the RMSE of the DOA estimation versus snapshot number.

From Fig. 2 and Fig. 3, it can be seen that (1) the ECH method can achieve the auto mutual coupling calibration for the HA-7; (2) The DOA estimation performance of the ECH is close to that of MUSIC and CRB; (3) The DOA estimation performance of the ECH for HA-7 is close to that of the C.Qi method for UCA-6.

The validity of the EMH method is verified by simulation 2, and the M.Wang method is also simulated. Assume there are two uncorrelated sources from $\varphi_1 = 60°$, $\varphi_2 = 120°$ impinge on arrays. In this simulation, three experiments are presented, called experiments 3–5. The simulation purposes and conditions of experiments 3, 4 are the same as those of experiments 1, 2. Figure 4 plots the RMSE of the DOA estimation versus SNR, and the RMSE of the mutual coupling coefficients is given in the 4-th and 5-th rows of Table 1. Figure 5 plots the RMSE of the DOA estimation versus snapshots number. Experiment 5 is to compare the spatial spectrum of the EMH and uncalibrated MUSIC on HA-7, the SNR is 35 dB, snapshot number K is 512, and the result is depicted in Fig. 6.

Table 1 Mutual coupling coefficients RMSE of the methods.

SNR(dB)	C.Qi	ECH	M.Wang	EMH
5	1.72E-02	2.21E-02	1.42E-02	1.83E-02
10	9.46E-03	1.25E-02	7.88E-03	1.02E-02
15	5.37E-03	7.00E-03	4.35E-03	5.84E-03
20	3.02E-03	3.89E-03	2.45E-03	3.21E-03
25	1.71E-03	2.22E-03	1.36E-03	1.80E-03
30	9.51E-04	1.24E-03	7.75E-04	1.01E-03
35	5.35E-04	6.96E-04	4.46E-04	5.86E-04

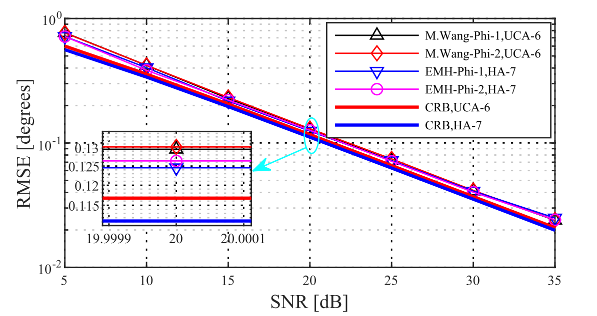

Fig. 4 RMSE of the DOAs estimation and the corresponding CRBs versus SNR in Simulation 2, K=512.

From Fig. 4, Fig. 5 and Fig. 6, it can be seen that (1) the EMH method can handle the mutual coupling auto-calibration for the HA-7, and the performance of the MEH is close to that of the M.Wang method for UCA; (2) Under the mutual coupling, the MUSIC cannot form the peak at DOAs. However, the EMH method can estimate the DOA of desired signals accurately.

From Table 1, it can be seen that both the ECH and EMH can achieve an estimation of the mutual coupling for HA-7. However, the mutual coupling estimation performances of

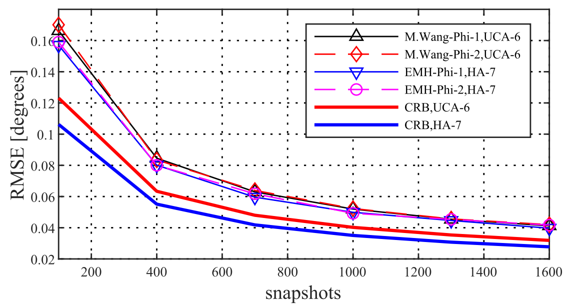

Fig. 5 RMSE of the DOAs estimation and the corresponding CRBs versus snapshot number in Simulation 2, SNR=25 dB.

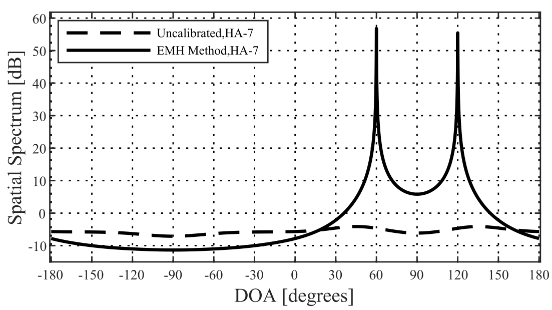

Fig. 6 Comparison of spatial spectrums for HA-7, SNR=35 dB, K=512.

the ECH and EMH are lower than that of C.Qi and M.Wang, respectively. The main reason for this phenomenon is that the substitution in equation (17) makes the estimations of c_5, c_6 independent of c_1, c_2. Further, it will introduce additional mutual coupling estimation errors.

6. Conclusion

In this letter, we derive a new transformation formula for the MCM of the HA-7. Based on the derived transformation, we extend two mutual coupling auto-calibration methods from UCA to HA-7 and get new methods, ECH and EMH. Computer simulations show that the ECH and EMH can effectively handle the mutual coupling problem for HA-7, and the performances of the two methods are close to that of the C.Qi and M.Wang methods on UCA.

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