LETTER

L_0-Norm Based Adaptive Equalization with PMSER Criterion for Underwater Acoustic Communications

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SUMMARY Underwater acoustic channels (UWA) are usually sparse, which can be exploited for adaptive equalization to improve the system performance. For the shallow UWA channels, based on the proportional minimum symbol error rate (PMSER) criterion, the adaptive equalization framework requires the sparsity selection. Since the sparsity of the L_0 norm is stronger than that of the L_1, we choose it to achieve better convergence. However, because the L_0 norm leads to NP-hard problems, it is difficult to find an efficient solution. In order to solve this problem, we choose the Gaussian function to approximate the L_0 norm. Simulation results show that the proposed scheme obtains better performance than the L_1 based counterpart.

key words: *underwater acoustic channels, sparsity selection, PMSER, L_0 norm approximation, adaptive equalization*

1. Introduction

The environment of the underwater acoustic (UWA) channel is complex. Multi-path fading leads to serious inter-symbol interference (ISI) [1], [2]. Due to the low carrier frequency, Doppler effect has a heavier effect on the UWA channels than the terrestrial radio. Measurements showed that the shallow UWA channels with slow-time-varying coherent multipath characteristics are sparse and can be considered as time-invariant [3], where most energy only focuses on several time-delay and Doppler spreads. Improving sparsity can achieve better system performance.

How to improve sparsity has become a popular topic in recent years. For this goal, the basis pursuit (BP) algorithm was used in [4] and [5], which is based on the L_1 norm to reconstruct the sparse signal. However, the BP algorithm is often too complex to reconstruct the sparse signal efficiently. In addition, there are many sparse reconstruction algorithms, such as the orthogonal matching pursuit (OMP) [6], and the iteratively reweighted least squares (IRLS) [7]. However, the above algorithms are usually not efficient enough.

Compression sensing (CS) method recovers all information of the original signal [8], through a measure far less than the collected signal data amount. The most sparse one is found under the constraints of satisfying $\boldsymbol{Ax} = \boldsymbol{b}$. The L_p norm of a vector $\boldsymbol{x}$ is defined by

$$||\boldsymbol{x}||_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} \tag{1}$$

In fact, $\boldsymbol{Ax} = \boldsymbol{b}$ represents a straight line in the Euclidean space, and $||\boldsymbol{x}||_p$ can be represented as a L_p norm sphere. We can represent them on the two-dimensional axis, observing the existing relationship between them.

- When $p = 1$, the L_p ball at this time can be described as a diamond. The L_p ball and the space line $\boldsymbol{Ax} = \boldsymbol{b}$ have an intersection, which must be on the coordinate axis. The number of intersections on the axis is the sparsity of the vector. The sparsity of the L_1 norm is 1.
- When $p = 0$, the radius of the L_p sphere is zero, the L_p sphere is two lines on the coordinate axis. The L_p ball and the space line $\boldsymbol{Ax} = \boldsymbol{b}$ should have two intersections on two axes. So the sparsity of the L_0 norm is 2. Thus the L_0 norm has better sparsity than the L_1. The L_0 norm is used to measure the number of non-zero elements in a vector [9], which is also known as the vector's sparsity.

Considering the inherent sparsity of the UWA channels, minimum symbol error rate (MSER) criterion was applied to the adaptive equalizer design in [10]. Then the proportional MSER criterion (PMSER) based sparse equalization algorithm was proposed to reduce the SER [11], where its fast convergence is achieved by adding sparse matrix with the L_1 norm.

Motivated by the better sparsity of the L_0 norm, the L_1 norm can be replaced by the L_0 norm in the adaptive equalization design. Unfortunately, since the L_0 will lead to a NP-hard problem [12], it's difficult to find a solution in closed form. In this letter, we use the Gaussian function to approximate the L_0 norm for a better solution to adaptive equalization with PMSER criterion for UWA communications. Simulation results show the proposed the scheme has a faster convergence and a lower symbol error rate (SER).

2. System Model

The system model is demonstrated in Fig. 1, where a point-to-point UWA communication with adaptive decision-feedback equalizer (DFE) is considered as that in [13]. The input signal sequence is denoted by $r(k)$, and the time-invariant channel is represented by $h(k)\,(k = 0, 1, \cdots, L-1)$ where L is the channel length. Then the output sequence of the channel can be expressed as

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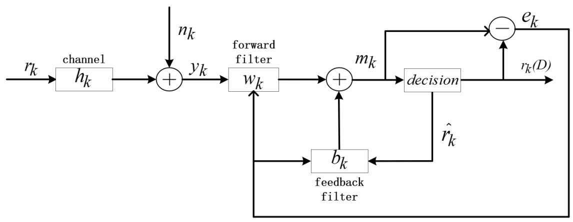

Fig. 1 System model.

$$y(k) = \sum_{l=0}^{L-1} h(l)\, r(k-l) + n(k) \tag{2}$$

where $y(k)$ is the received signal, and $n(k)$ is the additive white Gaussian noise (AWGN) of the channel with zero mean and σ_0^2 variance. The transmit power of $\{r_k\}$ is normalized as $P = 1$ to obtain the same average power under different transmitter processing, and to provide fair comparison under the same condition. The above sequence model can be transformed into matrix form as

$$\boldsymbol{y}_k = \boldsymbol{H}\boldsymbol{r}_k + \boldsymbol{n}_k \tag{3}$$

where $\boldsymbol{y}_k = \left[y(k), \cdots, y\left(k - N_f - 1\right)\right]^T$ is the received column vector with N_f as the number of forward filter taps, $\boldsymbol{H}$ is the channel matrix with Toeplitz structure, $\boldsymbol{r}_k = \left[r(k), \cdots, r\left(k - L - N_f - 1\right)\right]^T$ is the transmitted signal vector, $\boldsymbol{n}_k = \left[n(k), \cdots, n\left(k - N_f - 1\right)\right]^T$ is the noise vector, and $(\cdot)^T$ is the transpose operation of a vector/matrix.

The output of the decision feedback equalizer can be expressed as

$$m(k) = \boldsymbol{w}_k^T \boldsymbol{y}_k + \boldsymbol{b}_k^T \hat{\boldsymbol{r}}_k \tag{4}$$

where $m(k)$ is the output of the decision feedback equalizer, $\boldsymbol{w}_k = \left[w_k(0), w_k(1), \cdots, w_k\left(N_f - 1\right)\right]^T$, $\boldsymbol{b}_k = [b_k(0), b_k(1), \cdots, b_k(N_b - 1)]^T$, $\hat{\boldsymbol{r}}_k = [\hat{r}_k(D-1), \cdots, \hat{r}_k(D - N_b)]^T$, $w_k(d)$ represents forward filter coefficient at time slot k, $b_k(d)$ represents backward filtering coefficients at time slot k, $\hat{r}_k$ is the past estimated symbols, N_b represents the number of feedback filter taps, and D is the delay of the equalizer.

The error of the signal can be defined as

$$e_k = m_k - r_k(D) \tag{5}$$

where $r_k(D)$ is obtained by the decision of m_k.

Our goal is to get the minimum value of e_k, so as to minimize the SER performance.

3. Proposed Scheme

In order to realize a faster convergence of the algorithm, we replaces the L_1 norm by the L_0 norm to increase the sparsity of the PMSER algorithm. The iterative formula of $\boldsymbol{w}_k$ can be expressed as

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k} \boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k} \boldsymbol{y}_k} - ||\hat{\boldsymbol{h}}||_0 \tag{6}$$

where $\boldsymbol{G}_{f,k}$ and $\boldsymbol{G}_{b,k}$ are diagonal sparse matrices, μ indicates the impact of all scalars, and $||\hat{\boldsymbol{h}}||_0$ represents the sparsity selection of the L_0 norm.

The L_0 norm leads to the NP hard problem, which is difficult to obtain a closed form. In order to solve this problem, we use the following Gaussian method to obtain an approximation of $||\hat{\boldsymbol{h}}||_0$.

Firstly, the Gaussian function with zero mean and variance σ^2 is used

$$\varphi_\sigma(x_i) = exp\left(-x_i^2/2\sigma^2\right) \tag{7}$$

From [14], we have

$$\lim_{\sigma\to 0} \varphi_\sigma(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases} \tag{8}$$

which is equivalent to

$$\varphi_\sigma(x) = \begin{cases} 1, & |x| \leq \sigma \\ 0, & |x| > \sigma \end{cases} \tag{9}$$

For simplification, we define the following function

$$u(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases} \tag{10}$$

Combining the above Eqs. (8)–(10) we can get

$$||\boldsymbol{x}||_0 = \sum_{i=0}^{L-1} u(x_i) \tag{11}$$

Thus, the estimation of the L_0 norm of $\boldsymbol{x}$ can be transformed into the expression with function u. Through the above analyses, we can get

$$\lim_{\sigma\to 0} \varphi_\sigma(x) = 1 - u(x) \tag{12}$$

and

$$\lim_{\sigma\to 0} \sum_{i=0}^{L-1} \varphi_\sigma(x_i) = \sum_{i=0}^{L-1} [1 - u(x_i)] \tag{13}$$

By defining

$$\phi_\sigma(\boldsymbol{x}) = \sum_{i=0}^{L-1} \varphi_\sigma(x_i) \tag{14}$$

we rewrite (13) as

$$\phi_\sigma(\boldsymbol{x}) = L - ||\boldsymbol{x}||_0 \tag{15}$$

So we can get the expression of the L_0 norm as

$$||\boldsymbol{x}||_0 = L - \phi_\sigma(\boldsymbol{x}) \tag{16}$$

With the above analysis and current research [15], a general expression of the L_0 norm is obtained

$$||\boldsymbol{x}||_0 = \sum_{i=0}^{L-1} \left[1 - e^{-x_i^2/2\sigma^2}\right] \tag{17}$$

Substituting the above expression of the L_0 norm into the model, we can get

$$||\boldsymbol{w}_{k-1}||_0 = \sum_{l=0}^{L-1}\left[1 - e^{-\beta|\boldsymbol{w}_{k-1}(l)|}\right] \tag{18}$$

where β is the coefficient parameter of the Gaussian function. As in [16], in order to take advantage of the sparsity of the L_0 norm, gradient descend algorithm is used as

$$||\hat{\boldsymbol{h}}||_0 = \frac{||\boldsymbol{w}_{k-1}||_0}{\partial \boldsymbol{w}_{k-1}} = \beta sgn\left(|\boldsymbol{w}_{k-1}|\right) e^{-\beta|\boldsymbol{w}_{k-1}|} \tag{19}$$

Bringing (19) into (6), we can get

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k}\boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k}\boldsymbol{y}_k} - \beta sgn\left(|\boldsymbol{w}_{k-1}|\right) e^{-\beta|\boldsymbol{w}_{k-1}|} \tag{20}$$

where $sgn(\cdot)$ is defined as

$$sgn(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & elsewhere \end{cases} \tag{21}$$

To reduce the computational complexity of (14), especially that caused by the last term, the first order Taylor series expansions of exponential functions is taken into consideration

$$e^{-\beta|\boldsymbol{w}_{k-1}|} \approx 1 - \beta|\boldsymbol{w}_{k-1}| \tag{22}$$

Combining (20)–(22), we have

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k}\boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k}\boldsymbol{y}_k} - \beta\frac{\boldsymbol{w}_{k-1}}{|\boldsymbol{w}_{k-1}|}\left(1 - \beta|\boldsymbol{w}_{k-1}|\right) \tag{23}$$

Since the exponential function is larger than zero, the approximation of (22) is bounded to be positive. Thus we discuss the following two cases.

Case I: when $-\frac{1}{\beta} < \boldsymbol{w}_{k-1} < 0$

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k}\boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k}\boldsymbol{y}_k} - \beta\frac{\boldsymbol{w}_{k-1}}{-\boldsymbol{w}_{k-1}}\left(1 + \beta\boldsymbol{w}_{k-1}\right) \tag{24}$$

Case II: when $0 < \boldsymbol{w}_{k-1} < \frac{1}{\beta}$

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k}\boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k}\boldsymbol{y}_k} - \beta\frac{\boldsymbol{w}_{k-1}}{\boldsymbol{w}_{k-1}}\left(1 - \beta\boldsymbol{w}_{k-1}\right) \tag{25}$$

The above expressions (24) and (25) can be unified as

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \frac{\mu \boldsymbol{G}_{f,k}\boldsymbol{y}_k e_k}{\boldsymbol{y}_k^T \boldsymbol{G}_{f,k}\boldsymbol{y}_k} + f_\beta\left(\boldsymbol{w}_{k-1}\right) \tag{26}$$

where

$$f_\beta(x) = \begin{cases} \beta^2 x + \beta, & -\frac{1}{\beta} < x < 0 \\ \beta^2 x - \beta, & 0 < x < \frac{1}{\beta} \\ 0, & elsewhere \end{cases} \tag{27}$$

is operated on each element of the vector.

Usually the selection of matrices $\boldsymbol{G}_{f,k}$ and $\boldsymbol{G}_{b,k}$ should take into account the sparseness of the equalizer. We use the measure function of sparsity as that in [17]

$$S\left(\boldsymbol{w}_k\right) = \frac{L}{L - \sqrt{L}}\left(1 - \frac{||\boldsymbol{w}_k||_1}{\sqrt{L}||\boldsymbol{w}_k||_2}\right) \tag{28}$$

where $||\boldsymbol{w}_k||_1$ and $||\boldsymbol{w}_k||_2$ are the L_1 norm and L_2 norm of $|\boldsymbol{w}_k|$, respectively. Then the tap step length of the forward filter can be updated by

$$\zeta_{f,k} = \lambda_f \zeta_{f,k-1} + \left(1 - \zeta_{f,k-1}\right) S\left(\boldsymbol{w}_{k-1}\right) \tag{29}$$

where λ is the forgetting factor.

To avoid the problem of overlong steps, we should assign smaller steps to filter taps, which can also effectively improve the convergence speed. So, we use $||\boldsymbol{w}_{k-1}||_0$ instead of $||\boldsymbol{w}_{k-1}||_1$ to calculate the step length in (29).

By adopting the Lagrangian relaxation method, we obtain the element of the forward sparse matrix $\boldsymbol{G}_{f,k}$ as

$$g_{f,k}(l) = \frac{(1-\alpha)\zeta_{f,k}}{2N_f} + (1+\alpha)\frac{1 - e^{-\beta|\boldsymbol{w}_{k-1}|}}{2\sum_{l=0}^{L-1}\left[1 - e^{-\beta|\boldsymbol{w}_{k-1}|}\right]} \tag{30}$$

and the element of the feedback sparse matrix $\boldsymbol{G}_{b,k}$ as

$$g_{b,k}(l) = \frac{(1-\alpha)\zeta_{b,k}}{2N_b} + (1+\alpha)\frac{1 - e^{-\beta|\boldsymbol{b}_{k-1}|}}{2\sum_{l=0}^{L-1}\left[1 - e^{-\beta|\boldsymbol{b}_{k-1}|}\right]} \tag{31}$$

where $\alpha \in [-1, 1]$. And the sparsity of sparse matrix elements is determined by parameter α.

In similar way, we can obtain the iterative expression of the feedback filter as

$$\boldsymbol{b}_k = \boldsymbol{b}_{k-1} + \frac{\mu \boldsymbol{G}_{b,k}\hat{\boldsymbol{r}}_k e_k}{\hat{\boldsymbol{r}}_k^T \boldsymbol{G}_{b,k}\hat{\boldsymbol{r}}_k} + f_\beta\left(\boldsymbol{b}_{k-1}\right) \tag{32}$$

3.1 Complexity Analysis

Since the index of w_k is from 0 to $L-1$, by analyzing the iterative processing of the related methods, the complexity of the L_0 algorithm is found to be increased by $2L$-fold compared with that of the L_1 algorithm.

4. Simulation Results

In the simulation, information bits are modulated by the BPSK constellation. Parameter configuration is given as [10]: the sampling rate is 48 kHz, the date transmission rate is 2 kHz, the maximum delay spread of the channel is about 100 symbol periods, $\alpha = -0.5$, and $\beta = 0.5$. The UWA channel instance used for simulation is plotted in Fig. 2.

For comparison, we use the MSER algorithm and the PMSER algorithm based on the L_1 norm (which is referred

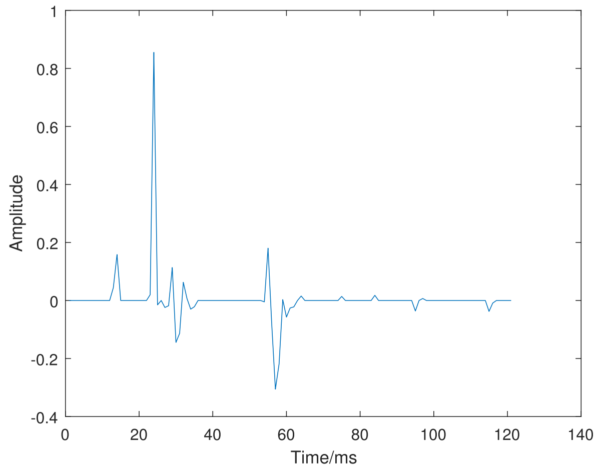

Fig. 2 The UWA channel instance used for simulation.

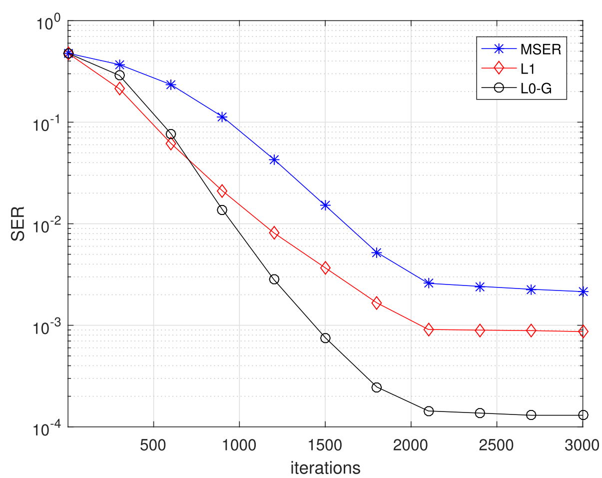

Fig. 3 Convergence performance at SNR=17 dB.

to as L1). The proposed sparse PMSER algorithm based on the approximation with Gaussian function is abbreviated as L0-G.

To show the convergence behavior of the related schemes, Fig. 3 is given with SNR=$10\log\frac{P}{\sigma_0^2}$=17 dB. It is clear that as the iteration goes on, the SER performance of all these schemes converges, while the proposed scheme shows the fastest convergence and lowest SER among them. In detail, all curves converge after 2000 iterations. The proposed L0-G scheme curve has a significant improvement than the MSER scheme and the L1 scheme. Obviously, both PMSER algorithms have a faster decreasing within iterations of 2000 than the MSER. The MSER takes about 1600 iterations to achieve a SER of 10^{-2} and 3000 iterations to a SER of near 2×10^{-3}. The L1 takes about 1100 and 1700 iterations to achieve the SER of 10^{-2} and 2×10^{-3}, respectively. In contrast, the iterations needed by the L0-G are about 950 and 1250, respectively. With 3000 iterations, the converged SER is 2.1×10^{-3}, 8.9×10^{-4}, and 1.4×10^{-4}, for the three schemes, respectively.

Then we show the performance under different SNR

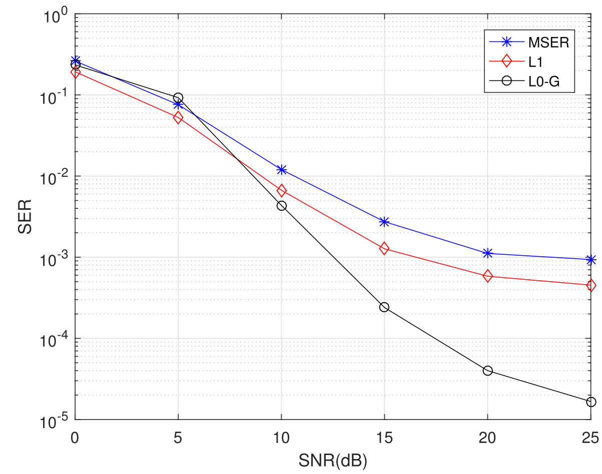

Fig. 4 SER performance versus SNR with 3000 iterations.

in Fig. 4, where the iteration numbers are set to be 3000. By observing the curves in Fig. 4, we can find that with the increase of SNR, the SER of the three algorithms are gradually decreasing as expected and the proposed scheme shows the best SER performance. In detail, the proposed L0-G scheme significantly outperforms the L1 counterpart and the both curves show better performance of the SER in high SNR region than the MSER scheme. On the other hand, at the SNR of 25 dB, the SER is 9.2×10^{-4}, 4.5×10^{-4}, and 17×10^{-5} for the MSER, L1, and L0-G, respectively. The SER performance in high SNR region indicates that the L0-G has the highest diversity gain, while the MSER has the lowest diversity gain. When the SNR is between 0 dB and 8 dB, the L0-G shows inefficient SER performance. However, the performance gap between them becomes more and more big after the SNR of 10 dB. For example, the SNR gains of the L0-G over the L1 are about 0.5 dB and 3.5 dB at the SER of 10^{-2} and 10^{-3}, respectively. Particularly, the MSER scheme achieves a SER of 10^{-3} at 24 dB of SNR. In contrast, the L1 method obtains the SER of 10^{-3} about 16 dB, while the L0-G achieve it at a SNR of about 12.5 dB.

5. Conclusion

In this paper, we proposed a new PMSER adaptive equalization scheme, which is based on the L_0 norm. The Gaussian function is used to approximate the L_0 norm to achieve the sparse selection. The simulation results showed that Gaussian approximation of L_0 norm has faster convergence and lower SER than the L_1 method, while the both PMSER schemes significantly outperform the MSER method.

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