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New Binary Sequences with Low Odd Correlation via Interleaving Technique

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SUMMARY Even correlation and odd correlation of sequences are two kinds of measures for their similarities. Both kinds of correlation have important applications in communication and radar. Compared with vast knowledge on sequences with good even correlation, relatively little is known on sequences with preferable odd correlation. In this paper, a generic construction of sequences with low odd correlation is proposed via interleaving technique. Notably, it can generate new sets of binary sequences with optimal odd correlation asymptotically meeting the Sarwate bound.

key words: *binary sequence, periodic correlation, odd correlation, interleaving technique*

1. Introduction

Sequences with low correlation have been widely used in digital communication systems, radar, sonar, and cryptography [4], [8], [16]. When applied in direct sequence code division multiple access (DS-CDMA) systems, the odd correlation function of sequences affects the output of the correlator when the information symbols change over one integration interval, while their even correlation function affects the output of the correlator when the information symbols do not change. As shown in [17] and [20], the odd correlation and even correlation of sequences are equally important in CDMA systems. Therefore, both the even correlation and odd correlation should be taken into consideration.

In last decades, binary sequences with low even correlation have attracted a lot of attention. Please refer to [8] and [2] for good monograph and survey on this topic. However, the research on sequences with odd correlation is scarce and only a few work have been reported. The pioneering work on sequences with odd correlation began from 1955. In [13], Lüke and Schotten firstly presented a class of almost perfect binary sequences with period $q + 1$ and one zero element, where q is an odd prime power. By replacing the zero element with ± 1, a class of binary sequences with odd autocorrelation magnitude 2 were obtained. Later, binary sequences of even period with low odd autocorrelation were summarized by Lüke, Schotten and Hadinejad-Mahram [14]. Recently, some binary sequences with optimal odd autocorrelation obtained from the interleaving technique were constructed in [22], [23].

Table 1 Binary sequences with optimal odd autocorrelation.

Period N	Restriction	References
$q+1$	q is an odd prime power	[13]
$2p$	p is an odd prime	[14]
$2p$	p is an odd prime	[22], [23]
$2p(p+2)$	p, $p+2$ are twin primes	[22], [23]
$2(2^m-1)$	$m \geq 1$ is even	[22], [23]

Except for sequences derived from basic sequences and transformations (i.e., product transformation, BTP transformation [14]), few constructive results on binary sequences with low odd autocorrelation have been presented. We list parameters of relevant sequences in Table 1.

In practical communication systems, binary sequence sets [7], [9], [25] are of importance due to their easy implementation of modulators. In 2018, Yang and Tang proposed a generic construction of binary sequences of period $2N$ with optimal odd correlation magnitude based on quaternary sequences of odd period N [22]. Li and Yang provided a complete answer to three conjectures of Parker on low autocorrelation of sixteen cyclotomic binary sequences and constructed new binary sequences with low odd autocorrelation [12]. Generally, it is interesting to construct families of sequences with low odd correlation and large size. This is our motivation to present binary sequence sets based on the interleaving technique.

This paper is organized as follows. Section 2 introduces some basic results of sequences including interleaving technique and Parker's transformation. In Sect. 3, we propose a generic construction of sequence sets by means of the interleaving technique, and give several classes of binary sequence sets with low odd correlation. In particular, a class of binary sequence sets has optimal odd correlation magnitude with respect to Sarwate bound. Finally, some concluding remarks are given in Sect. 4.

2. Preliminaries

2.1 Correlation Functions

Correlation is a measure of the similarity of two sequences. Let $\mathbf{a} = (a(i))_{i=0}^{N-1}$ and $\mathbf{b} = (b(i))_{i=0}^{N-1}$ be two binary sequences of the same period N, where $a(i), b(i) \in \{0, 1\}$, the even correlation function and the odd correlation function of $\mathbf{a}$ and $\mathbf{b}$ at a shift τ are defined as

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$$\theta_{\mathbf{a},\mathbf{b}}(\tau)=\sum_{i=0}^{N-1}(-1)^{a(i)+b(i+\tau)}$$

and

$$\hat{\theta}_{\mathbf{a},\mathbf{b}}(\tau)=\sum_{i=0}^{N-1}(-1)^{a(i)+b(i+\tau)+\lfloor\frac{i+\tau}{N}\rfloor}$$

respectively, where $\lfloor x\rfloor$ is the largest integer smaller than or equal to x. If $\mathbf{a}=\mathbf{b}$, then $\theta_{\mathbf{a},\mathbf{b}}$ and $\hat{\theta}_{\mathbf{a},\mathbf{b}}$ are called the even autocorrelation function and odd autocorrelation function, denoted by $\theta_{\mathbf{a}}$ and $\hat{\theta}_{\mathbf{a}}$ for short, respectively.

For any set $\mathbf{X}$ consisting of K binary sequences of period N, define the maximum even autocorrelation magnitude and the maximum even crosscorrelation magnitude as follows:

$$\begin{aligned}\psi_a(\mathbf{X}) &= \max\{|\theta_{\mathbf{u}}(\tau))| : \mathbf{u}\in\mathbf{X}, 1\le\tau<N\},\\ \psi_c(\mathbf{X}) &= \max\{|\theta_{\mathbf{u},\mathbf{v}}(\tau))| : \mathbf{u},\mathbf{v}\in\mathbf{X}, \mathbf{u}\neq\mathbf{v}, 0\le\tau<N\}.\end{aligned}$$

Then the maximum odd correlation magnitude of the sequence set $\mathbf{X}$ is given by

$$\psi_{\max}(\mathbf{X})=\max\{\psi_a(\mathbf{X}),\psi_c(\mathbf{X})\}.$$

For simplicity, $\mathbf{X}$ is said to be an $(N,K,\psi_{\max})$ sequence set with even correlation.

Similarly, for any set $\mathbf{X}$ consisting of K binary sequences of period N, define the maximum odd autocorrelation magnitude and the maximum odd crosscorrelation magnitude as follows:

$$\begin{aligned}\hat{\psi}_a(\mathbf{X}) &= \max\{|\hat{\theta}_{\mathbf{u}}(\tau))| : \mathbf{u}\in\mathbf{X}, 1\le\tau<N\},\\ \hat{\psi}_c(\mathbf{X}) &= \max\{|\hat{\theta}_{\mathbf{u},\mathbf{v}}(\tau))| : \mathbf{u},\mathbf{v}\in\mathbf{X}, \mathbf{u}\neq\mathbf{v}, 0\le\tau<N\}.\end{aligned}$$

Then the maximum odd correlation magnitude of the sequence set $\mathbf{X}$ is given by

$$\hat{\psi}_{\max}(\mathbf{X})=\max\{\hat{\psi}_a(\mathbf{X}),\hat{\psi}_c(\mathbf{X})\}.$$

Accordingly, $\mathbf{X}$ is said to be an $(N,K,\hat{\psi}_{\max}(\mathbf{X}))$ sequence set with odd correlation.

2.2 Interleaving Technique

Gong proposed the interleaving technique of sequences in [5], [6]. Let $\{\mathbf{a}_0,\mathbf{a}_1,\ldots,\mathbf{a}_{K-1}\}$ be a sequence set consisting of K sequences of period N, where $\mathbf{a}_i=(a_i(0),a_i(0),\ldots,a_i(N-1))$ and $0\le i<K$. A matrix is formed by placing the sequence $\mathbf{a}_i$ in the i-th column $(0\le i<K)$, i.e.,

$$\mathbf{M}=[\mathbf{a}_0,\mathbf{a}_1,\ldots,\mathbf{a}_{K-1}].$$

By concatenating the each consecutive rows of matrix $\mathbf{M}$, one can construct an interleaving sequence $\mathbf{u}$ of period NK. The interleaving sequence $\mathbf{u}$ can be written as

$$\mathbf{u}=I(\mathbf{a}_0,\mathbf{a}_1,\ldots,\mathbf{a}_{K-1}),\tag{1}$$

where I denotes the interleaving operator.

Let $\mathbf{v}=I(\mathbf{b}_0,\mathbf{b}_1,\ldots,\mathbf{b}_{K-1})$ be another interleaving sequence obtained from K sequences $\mathbf{b}_0,\mathbf{b}_1,\ldots,\mathbf{b}_{K-1}$, where $\mathbf{b}_i=(b_i(0),b_i(1),\ldots,b_i(N-1))$ and $0\le i<K$. Define a left cyclic shift operator $L^\tau(\mathbf{v})$, where $\tau=K\tau_1+\tau_2$, $0\le\tau_1<N$, and $0\le\tau_2<K$. Then $L^\tau(v)$ is another interleaving sequence. We have

$$L^\tau(\mathbf{v})=I(L^{\tau_1}(\mathbf{b}_{\tau_2}),\ldots,L^{\tau_1}(\mathbf{b}_{K-1}),L^{\tau_1+1}(\mathbf{b}_0),\ldots,L^{\tau_1+1}(\mathbf{b}_{\tau_2-1})).\tag{2}$$

The even correlation between the interleaving sequence $\mathbf{u}$ and $\mathbf{v}$ at the shift τ can be expressed as

$$\theta_{\mathbf{u},\mathbf{v}}(\tau)=\sum_{i=0}^{K-\tau_2-1}\theta_{\mathbf{a}_i,\mathbf{b}_{i+\tau_2}}(\tau_1)+\sum_{K-\tau_2}^{K-1}\theta_{\mathbf{a}_i,\mathbf{b}_{i+\tau_2-K}}(\tau_1+1),\tag{3}$$

which is the summation of the inner products between the pairwise column sequences in (1) and (2).

2.3 Parker's Transformation

In [18], Parker gave a very effective transformation from even periodic autocorrelation to odd autocorrelation. Specifically, let $\mathbf{s}=(s(i))_{i=0}^{N-1}$ and $\mathbf{t}=(t(i))_{i=0}^{N-1}$ be two binary sequences of same length N. For a sequence $\mathbf{s}$, we define a new binary sequence

$$\mathbf{u}=(s(0),s(1),\cdots,s(N-1),s(0)+1,s(1)+1,\cdots,s(N-1)+1),$$

which is a concatenation of the sequence $\mathbf{s}$ and its complement $\mathbf{s}+1$. For simplicity, write $\mathbf{u}$ and $\mathbf{v}$ as $\mathbf{u}=\mathbf{s}||(\mathbf{s}+1)$ and $\mathbf{v}=\mathbf{t}||(\mathbf{t}+1)$ respectively, where $||$ denotes the concatenation operation. The following lemma establishes a generic bridge from even correlation to odd correlation for binary sequences. and will be useful to prove the main result of this paper.

Lemma 1 ([18]): With notations as above, we have

$$\hat{\theta}_{\mathbf{s},\mathbf{t}}(\tau)=\frac{\theta_{\mathbf{u},\mathbf{v}}(\tau)}{2}\text{ for each }0<\tau<N.\tag{4}$$

2.4 Bounds on the Odd Correlation Magnitude

Similar to the Welch bound, Sarwate established the following lower bound on the autocorrelation magnitude and the odd crosscorrelation magnitude [19].

Lemma 2 (Theorem 4 of [19]): For any set $\mathbf{X}$ consisting of K sequences of period N satisfying $\hat{\theta}_{\mathbf{u}}(0)=N$ for all $\mathbf{u}\in X$, then one has

$$\frac{\hat{\psi}_a^2}{N}+\frac{(N-1)}{N(K-1)}\frac{\hat{\psi}_c^2}{N}\ge 1,\tag{5}$$

which results in

$$\hat{\psi}_{\max}(\mathbf{X})\ge N\sqrt{\frac{K-1}{NK-1}}.\tag{6}$$

Table 2 Binary sequences with low odd correlation.

Seed sequence	Set size	Length	$\hat{\psi}_{\max}(\mathbf{S})$	Constraint condition	Reference
Kasami set [9]	$2^{\frac{m}{2}}$	$2^{m+1}-2$	$2+2^{\frac{m+2}{2}}$	$m \equiv 0 (\mathrm{mod}\ 2)$	
Bent set [15]	$2^{\frac{m}{2}}$	$2^{m+1}-2$	$2+2^{\frac{m+2}{2}}$	$m \equiv 0 (\mathrm{mod}\ 2)$	
Gold set [7]	2^m+1	$2^{m+2}-2$	$2+2^{\frac{m+3}{2}}$	$m \equiv 1 (\mathrm{mod}\ 2)$	
Gold set [7]	2^m+1	$2^{m+1}-2$	$2+2^{\frac{m+4}{2}}$	$m \equiv 2 (\mathrm{mod}\ 4)$	
Gold-like set [1]	2^m	$2^{m+1}-2$	$2+2^{\frac{m+4}{2}}$	$m \equiv 0 (\mathrm{mod}\ 4)$	
Large Kasami set [10]	$2^{\frac{m}{2}}(2^m+1)$	$2^{m+1}-2$	$2+2^{\frac{m+4}{2}}$	$m \equiv 2 (\mathrm{mod}\ 4)$	
Large Kasami set [10]	$2^{\frac{m}{2}}(2^m+1)-1$	$2^{m+1}-2$	$2+2^{\frac{m+4}{2}}$	$m \equiv 0 (\mathrm{mod}\ 4)$	This paper
Yu-Gong set [21]	$2^{n\rho}$	$2^{n+1}-2$	$2+2^{\frac{n+2\rho+1}{2}}$	n is odd, ρ is an integer	
Yu-Gong set [21]	$2^{n\rho}$	$2^{n+1}-2$	$2+2^{\frac{n}{2}+\rho+1}$	n is even, ρ is an integer	
Generalized Kasami sequence [25]	$2^{3n/2}+2^{n/2}$	$2^{n+1}-2$	$2+2^{n/2+2}$	$n/2$ is odd	
Generalized Kasami sequence [25]	$2^{3n/2}+2^{n/2}-1$	$2^{n+1}-2$	$2+2^{n/2+2}$	$n/2$ is even	
Gold-like set [1]	2^r+1	$2^{r+1}-2$	$2+2^{(r+3)/2}$	$r=2s+1$	
Generalized modified Gold sequences [26]	$2^{ln}+2^{(l-1)n}+\ldots+1$	$2^{n+1}-2$	$2+2^{m+2}$	$n=2m+1, \gcd(n,k)=1$	
Family $\mathcal{A}/\mathcal{S}$ [27], [28]	2^n+1	$2^{n+1}-2$	$2+2^{(n+1)/2}$	n is odd	[22]

3. Construction of Sequence Sets with Low Odd Correlation via Interleaving Technique

In this section, we will propose a generic construction for sequence sets with low correlation by means of the interleaving technique. The interleaving representation helps us find out the structure of the sequences, and investigate their correlation property in a more lucid way based on Parker's transformation.

Construction 1: Interleaved construction of sequence set with low odd correlation.

Step 1: Choose a binary sequence set

$$\mathbf{A} = \{\mathbf{a}_i = (a_i(0), a_i(1), \cdots, a_i(N-1)) : i = 1, 2, \cdots, K\}$$

with low even correlation consisting of K sequences, where $N = 2^n - 1$.

Step 2: Construct a binary sequence set

$$\mathbf{U} = \{\mathbf{u}_i = (u_i(0), u_i(1), \cdots, u_i(4N-1)) : i = 1, 2, \cdots, K\}$$

from the sequences in $\mathbf{A}$ via interleaving technique, where

$$\mathbf{u}_i = I(\mathbf{a}_i, L^{1/4}(\mathbf{a}_i), L^{1/2}(\mathbf{a}_i)+1, L^{3/4}(\mathbf{a}_i)+1). \quad (7)$$

Herein and hereafter, $1/k$ denotes the inverse of k in the ring $\mathbb{Z}_N$ of integer modulo N for any k with $\gcd(N, k) = 1$. Such as, $3/4$ means the three times of the inverse of 4 in the ring $\mathbb{Z}_N$.

Step 3: Obtain the desired binary sequence set

$$\mathbf{S} = \{\mathbf{s}_i = (s_i(0), s_i(1), \cdots, s_i(2N-1)) : 1 \le i \le K\}$$

by deleting the last half part of $\mathbf{u}_i$ in Step 2, that is, $s_i(t) = u_i(t)$ for each $0 \le t \le 2N-1$.

We have the following result.

Theorem 1: The set $\mathbf{S}$ generated by Construction 1 is a sequence set consisting of K sequences of length $2N$ with maximum odd correlation

$$\hat{\psi}_{\max}(\mathbf{S}) = 2\psi_{\max}(\mathbf{A}). \quad (8)$$

Proof 1: For any two sequences $\mathbf{u}_i$ and $\mathbf{u}_j$ ($1 \le i, j \le K$) in $\mathbf{U}$, it follows from (3) that

1) $\tau = 4\tau_1$,

$$L^{\tau}(\mathbf{u}_j) = (L^{1+\tau_1}(\mathbf{a}_j), L^{1/4+\tau_1}(\mathbf{a}_j), L^{1/2+\tau_1}(\mathbf{a}_j)+1, L^{3/4+\tau_1}(\mathbf{a}_j)+1),$$

then,

$$\theta_{\mathbf{u}_i,\mathbf{u}_j}(\tau) = \theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1).$$

2) $\tau = 4\tau_1 + 1$,

$$L^{\tau}(\mathbf{u}_j) = (L^{1/4+\tau_1}(\mathbf{a}_j), L^{1/2+\tau_1}(\mathbf{a}_j)+1, L^{3/4+\tau_1}(\mathbf{a}_j)+1, L^{1+\tau_1}(\mathbf{a}_j)),$$

then,

$$\theta_{\mathbf{u}_i,\mathbf{u}_j}(\tau) = \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/4+\tau_1) - \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/4+\tau_1) - \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/4+\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/4+\tau_1).$$

3) $\tau = 4\tau_1 + 2$,

$$L^{\tau}(\mathbf{u}_j) = (L^{1/2+\tau_1}(\mathbf{a}_j)+1, L^{3/4+\tau_1}(\mathbf{a}_j)+1, L^{1+\tau_1}(\mathbf{a}_j), L^{1/4+\tau_1+1}(\mathbf{a}_j)),$$

then,

$$\theta_{\mathbf{u}_i,\mathbf{u}_j}(\tau) = -\theta_{\mathbf{a}_i,\mathbf{a}_j}(1/2+\tau_1) - \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/2+\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/2+\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(1/2+\tau_1).$$

4) $\tau = 4\tau_1 + 3$,

$$L^{\tau}(\mathbf{u}_j) = (L^{3/4+\tau_1}(\mathbf{a}_j)+1, L^{1+\tau_1}(\mathbf{a}_j), L^{1/4+\tau_1+1}(\mathbf{a}_j), L^{1/2+\tau_1+1}(\mathbf{a}_j)+1),$$

then,

$$\theta_{\mathbf{u}_i,\mathbf{u}_j}(\tau) = -\theta_{\mathbf{a}_i,\mathbf{a}_j}(3/4+\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(3/4+\tau_1) + \theta_{\mathbf{a}_i,\mathbf{a}_j}(3/4+\tau_1) - \theta_{\mathbf{a}_i,\mathbf{a}_j}(3/4+\tau_1).$$

Above all,

$$\theta_{\mathbf{u}_i,\mathbf{u}_j}(\tau) = \begin{cases} 4\theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1), & \tau = 4\tau_1, \\ 0, & \tau = 4\tau_1 + 1, \\ -4\theta_{\mathbf{a}_i,\mathbf{a}_j}(\tau_1 + 1/2), & \tau = 4\tau_1 + 2, \\ 0, & \tau = 4\tau_1 + 3, \end{cases} \quad (9)$$

where $\tau = 4\tau_1 + \tau_2$ with $0 \le \tau_1 < N$ and $0 \le \tau_2 < 4$. We then show that $\mathbf{u}_i = \mathbf{s}_i \| (\mathbf{s}_i + 1)$ for each $1 \le i \le K$. To this end, we only need to show that $u_i(t+2N) = 1 + u_i(t)$ for each $0 \le t < 2N$. Note that $2N = 4N_1 + 2$ with $N_1 = \frac{N-1}{2}$. By (7) and (2), we have

$$\begin{aligned} L^{2N}(\mathbf{u}_i) &= I(L^{1/2+N_1}(\mathbf{a}_i) + 1, L^{3/4+N_1}(\mathbf{a}_i) + 1, \\ &\qquad L^{N_1+1}(\mathbf{a}_i), L^{1/4+N_1+1}(\mathbf{a}_i)) \quad (10) \\ &= I(\mathbf{a}_i + 1, L^{1/4}(\mathbf{a}_i) + 1, L^{1/2}(\mathbf{a}_i), L^{3/4}(\mathbf{a}_i)). \end{aligned}$$

Compared (7) with (10), we immediately get $u_i(t + 2N) = u_i(t) + 1$ for each for each $0 \le t < 2N$. This together with Lemma 1 and (9) implies $\hat{\psi}_{\max}(\mathbf{S}) = 2\psi_{\max}(\mathbf{A})$. The proof of this theorem is finished.

Remark 1: Construction 1 is simple but generic in the sense that it works for any set of binary sequences with length $2^n - 1$. In the sequel, we shall employ this generic construction to obtain binary sequence sets with low odd correlation from known ones with low even correlation.

Thanks to known abundant results on binary sequences with low even correlation, we can obtain the corresponding binary sequences with low odd correlation, whose parameters are summarized in Table 2.

Remark 2: For the binary sequences listed in Table 2, we have the following comments.

- As far as the odd correlation property, the sequence sets from the Kasami set [9] and the bent sequence set [15] have the lowest odd correlation. Specifically, the resultant set $\mathbf{S}$, consisting of $2^{\frac{m}{2}}$ binary sequences of length $2^{m+1} - 2$, has the maximum odd correlation magnitude $\hat{\psi}_{\max}(\mathbf{S}) = 2 + 2^{\frac{m+2}{2}}$. Let $x = 2^{\frac{m}{2}}$. Note that

$$\begin{aligned} \lim_{m\to\infty} \frac{\hat{\psi}_{\max}(\mathbf{S})}{(2N)\cdot\sqrt{\frac{K-1}{(2N)\cdot K-1}}} &= \lim_{m\to\infty} \frac{2 + 2^{\frac{m+2}{2}}}{(2^{m+1} - 2)\sqrt{\frac{2^{\frac{m}{2}}-1}{2^{\frac{m}{2}}(2^{m+1}-2)-1}}} \\ &= \lim_{x\to\infty} \frac{2 + 2x}{(2x^2 - 2)\sqrt{\frac{x-1}{x(2x^2-2)-1}}} \\ &= \sqrt{2}. \end{aligned} \quad (11)$$

  This means that the resultant sequence set $\mathbf{S}$ is nearly optimal with respect with the Sarwate bound of (6).
- Compared with known low odd correlation binary sequence set, the seed sequence set in this paper is more abundant than that in [22], which leads to the results of Construction 1 is more flexible as shown in Table 2.
- Recently, it turns out in [11] that binary sequence with large family size and low correlation can be used to support massive machine type communication (mMTC), which is one of the core components of 6G communication systems to fulfil the demand of massive connectivity of billions of Internet-of-Things (IoT) devices. Note that the sequence sets listed in Table 2 have flexible tradeoff between family size and correlation, and thus provide flexible choices for such applications in mMTC of IoT devices.

4. Conclusion

According to the relationship between the binary set with even correlation and the binary set with odd correlation, we can obtain many kinds of binary sets with low odd correlation from known binary set. Based on the interleaving technique, we proposed a generic construction method for binary sequence sets of even length with low odd correlation from sets with low even correlation. The proposed construction can generate new binary sequence sets with low odd correlation which can not be produced by earlier methods.

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