LETTER

Geometric Dilution of Precision for Received Signal Strength in the Wireless Sensor Networks

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SUMMARY Geometric dilution of precision (GDOP) is a measure showing the positioning accuracy at different spatial locations in location systems. Although expressions of GDOP for the time of arrival (TOA), time difference of arrival (TDOA), and angle of arrival (AOA) systems have been developed, no closed form expression of GDOP are available for the received signal strength (RSS) system. This letter derives an explicit GDOP expression utilizing the RSS measurement in the wireless sensor networks.
key words: *geometric dilution of precision, received signal strength systems*

1. Introduction

GDOP is an indicator that provides the information regarding the degree of location accuracy affected by the geometric relation between the source(s) and the sensors [1]. The received signal strength (RSS) measurements are important and commonly used in indoor location solutions based on Wi-Fi, cellular net-works or Bluetooth [2]–[4], [8]. Therefore, it is significant to do quantitative analysis of GDOP in the RSS positioning systems. When RSS are used to estimate the source locations, the positioning accuracy is related to the accuracy of RSS measurement as well as the geometric relation between the source and the sensors [8]. This letter investigates the effect of geometric relation to the positioning accuracy based on RSS positioning systems in 3 different scenarios. In particular, the root mean square (RMS) position error is used as GDOP in the RSS system.

RSS is commonly expressed in terms of the unknown emitting source power, the priori path loss parameter (PLP) and the distance between the sensors and source. The first two parameters are independent of the geometric relation between the source and the sensors. GDOP in GPS system was calculated by considering all the errors generated by different parameters of TOA measurements [5]. In actual project the parameters are difficult to obtain normally and the GDOP with unknown parameters is different from the GDOP with known parameters. Therefore, three expressions of GDOP are derived in this letter. One is for the condition that PLP and the source power are known. The second GDOP is under the known PLP, and the third one is for unknown PLP and unknown source power.

The innovation of this letter is as follows: this letter utilizes the calculating formula of CRB to calculate GDOP, which follows the thought in paper [7]. This letter first deduces expressions of GDOP based on RSS systems in 3 different scenarios and closed form expressions for the first and second scenarios.

2. Derivation of GDOP

The signal strength received by the sensor k can be defined as [6]

$$P_k = \eta_0 - 10\alpha \log_{10} d_k + n_k^{\boldsymbol{\beta}} \tag{1}$$

for $k = 1, 2 \cdots, K$. P_0 is the unknown emitting source power and $\eta_0 = P_0 + 10\alpha \log_{10} d_0$ is the equivalent unknown emitting source power. $d_k = |\boldsymbol{u} - \boldsymbol{x}_k|$ is the distance between the kth sensor and the source. α is PLP and $n_k^{\boldsymbol{\beta}}$ is independently identically distributed (i.i.d.) Gaussian noise with zero mean and variance $\sigma_{\boldsymbol{\beta}}^2$.

GDOP can be obtained by connecting the contour lines of CRB on the area of interest according to the relationship between CRB and GDOP [7]. The CRB for RSS-based localization provided in [8] is an approximate expression, not in the closed form. Thus, the provided CRBs are not exact values.

The signal strength of Eq. (1) can be rewritten as

$$n_k^{\boldsymbol{\beta}} = c\left(\ln P_k - \alpha \ln d_k - \ln \eta_0\right) \tag{2}$$

The set of all the measurements is denoted as $\boldsymbol{\beta} = c\begin{bmatrix}\ln P_1 & \ldots & \ln P_K\end{bmatrix}^T$ and $c = 10/\ln 10$. The conditional probability of the measurement error can be expressed as [8]

$$p\left(\zeta|\boldsymbol{u}, \eta_0, \alpha\right) = const \cdot \exp\left\{\frac{1}{2\sigma_{\boldsymbol{\beta}}^2}\left(\boldsymbol{\beta} - \boldsymbol{h}_{\boldsymbol{\beta}}\left(\boldsymbol{u}, \eta_0, \alpha\right)\right)^T \left(\boldsymbol{\beta} - \boldsymbol{h}_{\boldsymbol{\beta}}\left(\boldsymbol{u}, \eta_0, \alpha\right)\right)\right\} \tag{3}$$

where *const* is a constant independent of the localization parameters $(\boldsymbol{u}, \eta_0, \alpha)$. In Eq. (3), $\boldsymbol{h}_{\boldsymbol{\beta}}$ can be expressed as

$$\boldsymbol{h}_{\boldsymbol{\beta}}\left(\boldsymbol{u}, \eta_0, \alpha\right) = c\begin{bmatrix}\alpha \ln d_1 + \ln \eta_0 \\ \vdots \\ \alpha \ln d_k + \ln \eta_0\end{bmatrix} \tag{4}$$

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The Fisher matrix can be constructed by [7], [8]

$$FIM = \boldsymbol{H}\boldsymbol{Q}^{-1}\boldsymbol{H} \tag{5}$$

where

$$\boldsymbol{H} = \begin{bmatrix}\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}} & \boldsymbol{H}_{\eta_0}^{\boldsymbol{\beta}} & \boldsymbol{H}_{\alpha}^{\boldsymbol{\beta}}\end{bmatrix}^T \tag{6}$$

$$\boldsymbol{Q} = \sigma_\beta^2 \boldsymbol{I} \tag{7}$$

$$\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}} = \frac{\partial \boldsymbol{h}_{\boldsymbol{\beta}}^T}{\partial \boldsymbol{u}} = c\alpha \begin{bmatrix} d_1^{-1}\cos\theta_1 & \cdots & d_1^{-K}\cos\theta_K \\ d_1^{-1}\sin\theta_1 & \cdots & d_1^{-K}\sin\theta_K \end{bmatrix} \tag{8}$$

$$\boldsymbol{H}_{\boldsymbol{\eta}_0}^{\boldsymbol{\beta}} = \frac{\partial \boldsymbol{h}_{\boldsymbol{\beta}}^T}{\partial \eta_0} = c\eta_0^{-1}\begin{bmatrix}1 & \cdots & 1\end{bmatrix} = c\eta_0^{-1}\cdot \boldsymbol{1}^T \tag{9}$$

$$\boldsymbol{H}_{\alpha}^{\boldsymbol{\beta}} = \frac{\partial \boldsymbol{h}_{\boldsymbol{\beta}}^T}{\partial \alpha} = c\begin{bmatrix}\ln d_1 & \cdots & \ln d_K\end{bmatrix}. \tag{10}$$

The CRB of location parameters with respect to power is shown as [8]

$$CRB = (FIM)^{-1}. \tag{11}$$

The CRB_1 of the localization with known η_0 and α is obtained from Eq. (11) and can be expressed as

$$CRB_1 = \left(\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}}\boldsymbol{Q}^{-1}\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}T}\right)^{-1}. \tag{12}$$

The 1st GDOP is defined as [7]

$$\mathrm{GDOP}_1 = \sqrt{\mathrm{tr}\,(CRB_1)}, \tag{13}$$

where $\mathrm{tr}(\boldsymbol{X})$ is the trace of matrix $\boldsymbol{X}$. After arrangement, we have

$$\mathrm{GDOP}_1 = \frac{\sigma_\beta}{c\alpha}\left\{\left(\sum_{k=1}^{K} d_k^{-2}\right)/\mathrm{A}\right\}^{-1/2} \tag{14}$$

where

$$A = \left[\sum_{k=1}^{K}\sum_{m\neq k}^{K}\sin^2(\theta_k-\theta_m)\,d_k^{-2}d_m^{-2}\right]^{-1} \tag{15}$$

The CRB of the localization with known α can be derived from Eq. (11)

$$CRB_2 = \sigma_\beta^2\left[\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}}\left(\boldsymbol{I}-(1/K)\boldsymbol{1}\boldsymbol{1}^T\right)\left(\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}}\right)^T\right]^{-1} \tag{16}$$

The 2nd GDOP is defined as [7]

$$\mathrm{GDOP}_2 = \sqrt{\mathrm{tr}\,(CRB_2)} \tag{17}$$

After some manipulations, we have

$$\mathrm{GDOP}_2 = \left\{A\sum_{k=1}^{K} d_k^{-2} + A^2\cdot\left[(B\cdot F - C\cdot E)^2 + (C\cdot F - B\cdot D)^2\right]/[K - A\cdot \mathrm{G}]\right\}^{1/2}\cdot\sigma_\beta/(c\alpha) \tag{18}$$

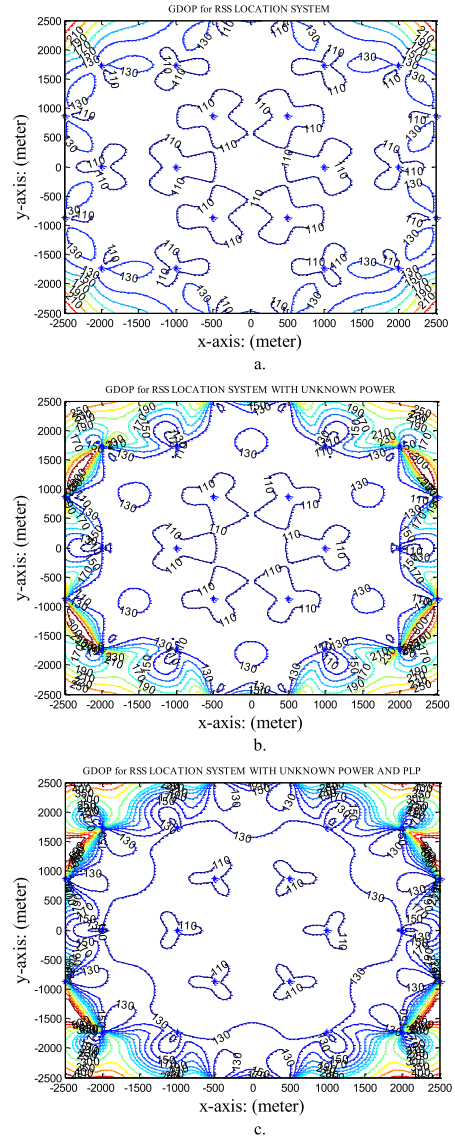

Fig. 1 GDOP for RSS with 24 sensors and positive hexagon distribution. * denotes the sensor. a: GDOP_1, b: GDOP_2, c: GDOP_3

$$B = \sum_{k=1}^{K}\cos\theta_k d_k^{-1} \tag{19}$$

$$C = \sum_{k=1}^{K}\sin\theta_k d_k^{-1} \tag{20}$$

$$D = \sum_{k=1}^{K}\sin^2\theta_k d_k^{-2} \tag{21}$$

$$E = \sum_{k=1}^{K}\cos^2\theta_k d_k^{-2} \tag{22}$$

$$F = \sum_{k=1}^{K}\cos\theta_k\sin\theta_k d_k^{-2} \tag{23}$$

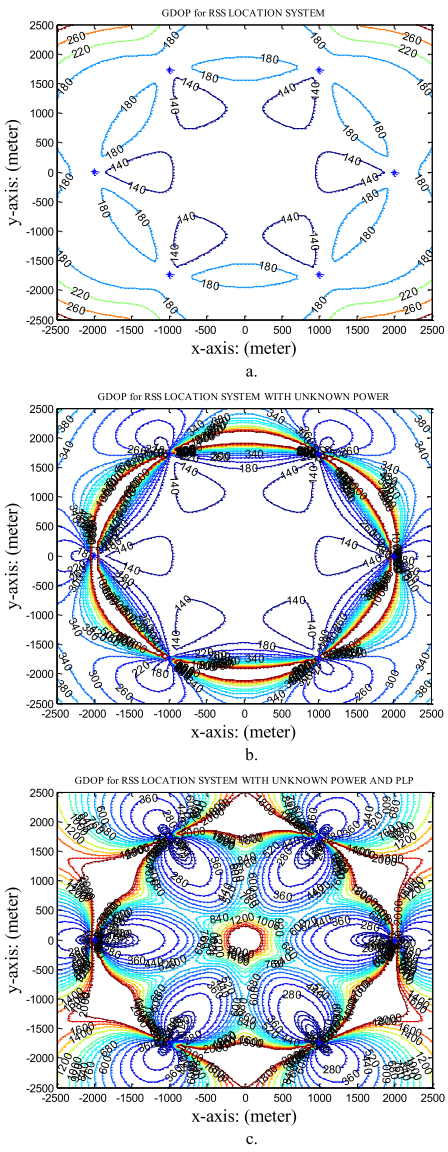

Fig. 2 GDOP for RSS with 6 sensors and positive hexagon distribution. a: GDOP_1, b: GDOP_2, c: GDOP_3

$$G = \sum_{m=1}^{K} [B \sin\theta_m - C \cos\theta_m]^2 \, d_m^{-2}. \tag{24}$$

Similarly, the CRB of the localization with unknown η_0 and α can be derived as

$$CRB_3 = \sigma_\beta^2 \left[\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}} \left(\boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}}\right)^T - \boldsymbol{H}_{\boldsymbol{u}}^{\boldsymbol{\beta}} \left(\boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}}\right)^T \left(\boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}} \left(\boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}}\right)^T\right)^{-1} \boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}} \left(\boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}}\right)^T \right] \tag{25}$$

where

$$\boldsymbol{H}_{\boldsymbol{a}}^{\boldsymbol{\beta}} = \left[\boldsymbol{H}_{\eta_0}^{\boldsymbol{\beta}} \quad \boldsymbol{H}_{\alpha}^{\boldsymbol{\beta}} \right]^T \tag{26}$$

The 3rd GDOP is defined as

$$\mathrm{GDOP}_3 = \sqrt{\mathrm{tr}\,(CRB_3)} \tag{27}$$

3. Simulation Study

Two scenarios of GDOP for received signal strength are examined. In scenario 1, 24 sensors are set as the cellular layout (positive hexagon) with side length of 1 km and marked with * symbols in the figure. The RSS measurement error $n_k^{\boldsymbol{\beta}}$ is the white Gaussian noise with zero mean and 2 dB variance. The propagation factor α is set as 2. Scenario 2 has the same layout as scenario 1, but with only 6 sensors. The variance of RSS measurement error is 1 dB and α is 3.

Figure 1 shows the contour maps of the RSS localization for scenario 1. Figure 1a is for GDOP_1, Fig. 1b for GDOP_2, and Fig. 1c for GDOP_3. It can be seen from Fig. 1 that the positioning accuracy is between 110 and 170 meters in most regions whether or not the equivalent source power η_0 and the PLP are unknown. Close examination of Fig. 1a, 1b, and 1c shows that the positioning error along the anchor-anchor line gradually increases along with the increase of unknown conditions.

Figure 2 illustrates the contour maps of the RSS localization for scenario 2. It can be seen from Fig. 2a that the positioning accuracy is satisfactory when PLP and the source power are known. It is observed from Fig. 2b and Fig. 2c that the RSS localization results deteriorate rapidly along and off the anchor-anchor line, and become irregular when both PLP and source power are not known.

4. Conclusion

Three GDOP expressions for RSS-based positioning systems are presented. Simulation results for different number of sensors are given to illustrate the effects of the GDOP.

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