IEICE TRANS. FUNDAMENTALS, VOL.E102–A, NO.6 JUNE 2019

LETTER

Propagation-Delay Based Cyclic Interference Alignment with One Extra Time-Slot for Three-User X Channel

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SUMMARY For the three-user X channel, its degree of freedom (DoF) 9/5 has been shown achievable theoretically through asymptotic model with infinite resources, which is impractical. In this article, we explore the propagation delay (PD) feature among different links to maximize the achievable DoF with the minimum cost. Since perfect interference alignment (IA) is impossible for 9 messages within 5 time-slots, at least one extra time-slot should be utilized. By the cyclic polynomial approach, we propose a scheme with the maximum achievable DoF of 5/3 for 10 messages within 6 time-slots. Feasibility conditions in the Euclidean space are also deduced, which demonstrates a quite wide range of node arrangements.

key words: *interference alignment, three-user X channel, propagation delay, time-slot, degree of freedom*

1. Introduction

The X channel has attracted increasing attentions in recent years, since independent and full message transmission between any transmitter and receiver is possible. From the perspective of degree of freedom (DoF) indicating the prelog factor of the capacity, [1] shows that the sum DoF of the $M \times N$ X network with single-antenna nodes is equal to $MN/(M+N-1)$. The key method of the achievability proof is interference alignment (IA), which can be implemented in spatial domain with multiple antenna support such as [2], [3], or in temporal domain exploiting propagation delay (PD) property such as [4] for the two-user X channel (i.e. $M = N = 2$).

It has been shown that when $M \geq 3$ and $N \geq 3$, no perfect IA exists in temporal domain [5] and asymptotic method with infinite symbol extension model is often used to achieve the upper-bound on DoF [1]. In fact, this implies that the DoF upper-bound is not achievable in practice where the available signal space is limited.

As temporal domain is the basic resource which can be always utilized when other kinds of resources are unavailable, we focus on this aspect in the following content. Besides the 2-user X channel, [4] also gives cyclic IA with polynomial model for PD-based K user interference channel. In [6], we propose the implement of prefect IA based on PD with a DoF of $2K/(K+1)$ for $K \times 2$ X channel and give the corresponding feasibility condition in Euclidean space. Further detail of node placements is demonstrated in [7].

The three-user X channel represents the minimal number of M and N with imperfect IA and needs further investigations. In this article, we address this issue by exploiting the PD feature among different links to maximize the achievable DoF with the minimum cost of transmission delay. Since perfect IA is impossible for the required 9 messages within 5 time-slots, at least one extra time-slot should be utilized. By the cyclic polynomial approach, we propose an advanced scheme with the maximum achievable DoF of 5/3 for 10 messages within 6 time-slots. Also, we give its feasibility conditions in Euclidean space along with examples.

Manuscript received November 26, 2018.
Manuscript revised February 10, 2019.

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DOI: 10.1587/transfun.E102.A.854

2. System Model

The three-user X channel model is shown in Fig. 1, where the three transmitters and three receivers equipped with single antenna are denoted by S_1, S_2, S_3, D_1, D_2 and D_3, respectively. W_{ij} denotes the desired message from transmitter S_j to receiver D_i, and τ_{ij} represents the PD between transmitter S_j to receiver D_i. v_j and r_i indicate the polynomials transmitted at S_j and received at D_i, respectively. The channel between each pair of transmitter and receiver is equally divided into time-slots with unit length. Without loss of generality, all messages are normalized into one time-slot. The PD parameters are assumed to be static and non-negative integer multiples of one time-slot. Like the conventional orthogonal multiple-access schemes, the channel access repeats itself after n time-slots and new messages are transmitted cycle by cycle with a period of n. In other words, messages are cyclically right-shifted over this PD-based channel. This procedure can be modeled by circular right-shift polynomial, with a period of n.

Basic rule. For a message W_{ij} transmitted at an offset x^s and delayed by t time-slots, the resulting message can be computed by $x^{(s+t)}W_{ij} \mod (x^n - 1)$.

Encoding procedure. The code-word sent from S_j is encoded into the polynomial $v_j(x)$ by the encoding function

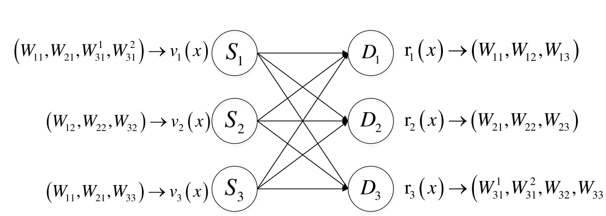

Fig. 1 System model of the three-user X channel with 10 independent messages.

e_j carrying the three messages W_{1j}, W_{2j}, W_{3j}.

$$e_j : (W_{1j}, W_{2j}, W_{3j}) \to v_j(x) \tag{1}$$

Denote the offset (i.e. the index of time-slot) allocated for message W_{ij} by $x^{p_{ij}}$. Then the transmitted polynomial from S_j can be detailed as

$$v_j(x) = \sum_{i=1}^{3} x^{p_{ij}} W_{ij} \mod (x^n - 1) \tag{2}$$

The PD between each pair of transceiver (S_j, D_i) is denoted by τ_{ij}. Before transmission, all nodes are assumed to know the PD polynomial matrix defined by

$$\boldsymbol{D} = \begin{pmatrix} x^{\tau_{11}} & x^{\tau_{12}} & x^{\tau_{13}} \\ x^{\tau_{21}} & x^{\tau_{22}} & x^{\tau_{23}} \\ x^{\tau_{31}} & x^{\tau_{32}} & x^{\tau_{33}} \end{pmatrix} \tag{3}$$

The input polynomial vector is defined by $v = (v_1(x), v_2(x), v_3(x))$. Denote the received polynomial vector as $r = (r_1(x), r_2(x), r_3(x))$. This linear system gives the following input-output relationship

$$\mathbf{r}^T \equiv \boldsymbol{D}\boldsymbol{v}^T \mod (x^n - 1) \tag{4}$$

where $(.)^T$ denotes the transpose of a vector. The received polynomial $r_i(x)$ at the receiver is

$$r_i(x) \equiv \boldsymbol{D}(i, :)\boldsymbol{v}^T \mod (x^n - 1) \tag{5}$$

where $\boldsymbol{D}(i, :)$ indicates the i^{th} row of matrix $\boldsymbol{D}$.

Decoding procedure. The received polynomial is decoded to obtain an estimate of the desired messages

$$f_i : r_i(x) \to (W_{i1}, W_{i2}, W_{i3}) \tag{6}$$

For the above PD-based model, the achieved DoF is defined as the number of total messages K over the period n

$$DoF = K/n \tag{7}$$

3. Proposed Scheme

By adding an extra time-slot to minimize the delay cost (i.e., $n = 6$), we can not only transmit the original 9 messages, but also allow one more message delivered during each transmission period/cycle (i.e., $K = 10$). This is the largest value we can obtain with a period of $n = 6$, since another extra message will cause a higher DoF than the upper bound due to $11/6 > 9/5$ which is impossible.

Generally, exhaustive search among proper space can be used to obtain all potential schemes. However, this method has exponential complexity. Motivated by [6], we find a PD structure which can increase the DoF to 5/3. The extra message is assumed from transmitter S_1 to receiver D_3. To distinguish them, we denote the two independent messages from transmitter S_1 to receiver D_3 by W_{31}^1 and W_{31}^2, respectively.

Firstly, we set the PD polynomial matrix as

$$\boldsymbol{D} = \begin{pmatrix} x^0 & x^1 & x^2 \\ x^0 & x^5 & x^4 \\ x^0 & x^0 & x^0 \end{pmatrix} \tag{8}$$

We should notify that the parameters of $\boldsymbol{D}$ must be known beforehand at all nodes.

Next we show the details of encoding and decoding procedures. To obtain the DoF of 5/3, each receiver should collect its desired messages from different time-slots and align unwanted messages (i.e., interference) into the other available time-slots. Given the above PD matrix, the remaining task is to determine the offset parameters p_{ij} for all the ten messages. Without loss of generality, we can send W_{11} in the first time-slot, i.e., $p_{11} = 0$. Moreover, we can assume the transmitting ordering is as W_{11}, W_{21}, W_{31}^1 and W_{31}^2, i.e.,

$$0 = p_{11} < p_{21} < p_{31}^1 < p_{31}^2 \le 5 \tag{9}$$

Since $\tau_{11} = \tau_{21} = \tau_{31} = 0$, messages W_{i1} from S_1 will occupy the same time-slots at each receiver.

At D_1, the index of time-slot for W_{11} is $p_{11} = 0$, which should not be used by other messages. Since $\tau_{12} = 1$, there is one offset of time-slots for messages from S_2. Therefore, the last time-slot should not be occupied in each cycle at S_2, otherwise the $\mod (x^6 - 1)$ operation will cause an interference in the coming time-slot 0. Since $0 \le p_{ji} \le n - 1 = 5$, we require

$$p_{j2} < 5, \quad \forall j = 1, 2, 3 \tag{10}$$

At the same time $\tau_{13} = 2$, the time-slot offset for messages from S_3 is two, which implies that

$$p_{j3} \neq 4, \quad \forall j = 1, 2, 3 \tag{11}$$

Similar analysis can be done at D_2 and D_3. At D_2, the index of time-slot for W_{21} is p_{21}, which should not be used by other messages. Since $\tau_{22} = 5$, there is 5 offset of time-slots for messages from S_2. To avoid interference at p_{21}, we need

$$p_{j2} + 5 \neq p_{21} \mod 6, \quad \forall j = 1, 2, 3 \tag{12}$$

At the same time $\tau_{13} = 4$, the time-slot offset for messages from S_3 is 4. The above requirement indicates

$$p_{j3} + 4 \neq p_{21} \mod 6, \quad \forall j = 1, 2, 3 \tag{13}$$

At D_3, the indexes of time-slots for W_{31}^1 and W_{31}^2 are p_{31}^1 and p_{31}^2, respectively. Since $\tau_{32} = \tau_{33} = 0$, there is no offset of time-slots for messages from S_2 and S_3. To avoid interference at p_{31}^1 and p_{31}^2, we need

$$p_{j2} \neq p_{31}^1,\ p_{j2} \neq p_{31}^2, \quad \forall j = 1, 2, 3 \tag{14}$$

$$p_{j3} \neq p_{31}^1,\ p_{j3} \neq p_{31}^2, \quad \forall j = 1, 2, 3 \tag{15}$$

After checking the above constraints (9)–(15), we find a feasible solution of the offset of each message

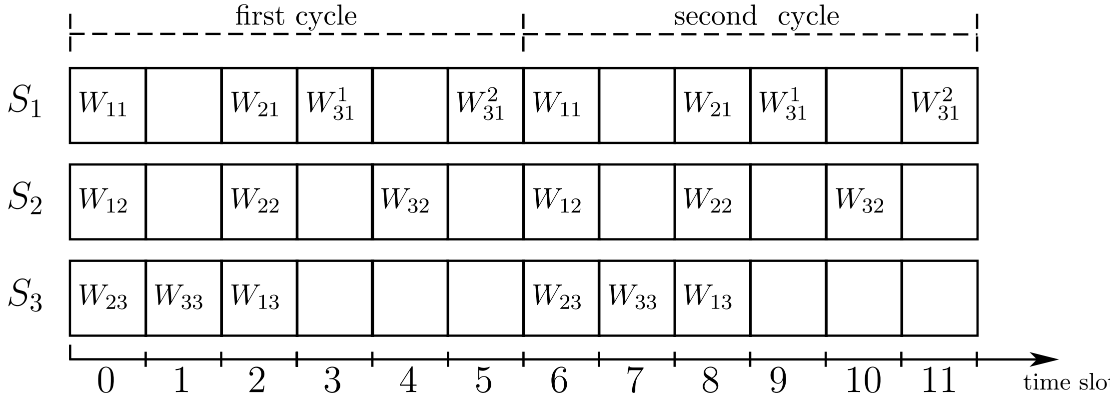

Fig. 2 Demonstration of the transmitting procedure within the first 12 time-slots.

$$\begin{aligned} &p_{11} = p_{12} = p_{23} = 0, && p_{33} = 1, \\ &p_{21} = p_{22} = p_{13} = 2, && p_{13}^1 = 3, \\ &p_{32} = 4, && p_{31}^2 = 5 \end{aligned} \tag{16}$$

Correspondingly, the transmitted polynomials are

$$v_1(x) = W_{11} + x^2 W_{21} + x^3 W_{31}^1 + x^5 W_{31}^2 \mod (x^6 - 1) \tag{17}$$

$$v_2(x) = W_{12} + x^2 W_{22} + x^4 W_{32} \mod (x^6 - 1) \tag{18}$$

$$v_3(x) = W_{23} + x^1 W_{33} + x^2 W_{13} \mod (x^6 - 1) \tag{19}$$

Figure 2 shows the message offsets sent by the three transmitters in the first two cycles.

From (5) combined with $n = 6$, the received polynomials can be computed as below. At D_1, we have

$$\begin{aligned} r_1(x) &\equiv \boldsymbol{D}(1,:)\boldsymbol{v}^T \mod (x^6 - 1) \\ &\equiv x^0 v_1(x) + x^1 v_2(x) + x^2 v_3(x) \mod (x^6 - 1) \\ &\equiv W_{11} + x^2 W_{21} + x^3 W_{31}^1 + x^5 W_{31}^2 \\ &\quad + x^1 (W_{12} + x^2 W_{22} + x^4 W_{32}) + x^2 (W_{23} \\ &\quad + x^1 W_{33} + x^2 W_{13}) \mod (x^6 - 1) \\ &\equiv W_{11} + x^1 W_{12} + x^2 (W_{21} + W_{23}) \\ &\quad + x^3 (W_{31}^1 + W_{22} + W_{33}) + x^4 W_{13} \\ &\quad + x^5 (W_{31}^2 + W_{32}) \mod (x^6 - 1) \end{aligned} \tag{20}$$

Obviously, from (20) we can see that the desired messages W_{11}, W_{12}, and W_{13} are located at the first, second, and fifth time-slots in each cycle without any interference, respectively. Moreover, all other interferences have been aligned in the other time-slots. Similarly, at receiver D_2, we have

$$\begin{aligned} r_2(x) &\equiv \boldsymbol{D}(2,:)\boldsymbol{v}^T \mod (x^6 - 1) \\ &\equiv x^0 v_1(x) + x^5 v_2(x) + x^4 v_3(x) \mod (x^6 - 1) \\ &\equiv W_{11} + x^2 W_{21} + x^3 W_{31}^1 + x^5 W_{31}^2 \\ &\quad + x^5 (W_{12} + x^2 W_{22} + x^4 W_{32}) \\ &\quad + x^4 (W_{23} + x^1 W_{33} + x^2 W_{13}) \mod (x^6 - 1) \\ &\equiv (W_{11} + W_{13}) + x^1 W_{22} + x^2 W_{21} \\ &\quad + x^3 (W_{31}^1 + W_{32}) + x^4 W_{23} \\ &\quad + x^5 (W_{31}^2 + W_{12} + W_{33}) \mod (x^6 - 1) \end{aligned} \tag{21}$$

From (21) the desired messages W_{22}, W_{21}, and W_{23} are located at the second, third, and fifth time-slots in each cycle without any interference, respectively. Finally, the receiver D_3 yields

$$\begin{aligned} r_3(x) &\equiv \boldsymbol{D}(3,:)\boldsymbol{v}^T \mod (x^6 - 1) \\ &\equiv x^0 v_1(x) + x^0 v_2(x) + x^0 v_3(x) \mod (x^6 - 1) \\ &\equiv W_{11} + x^2 W_{21} + x^3 W_{31}^1 + x^5 W_{31}^2 \\ &\quad + W_{12} + x^2 W_{22} + x^4 W_{32} \\ &\quad + W_{23} + x^1 W_{33} + x^2 W_{13} \mod (x^6 - 1) \\ &\equiv (W_{11} + W_{12} + W_{23}) + x^1 W_{33} \\ &\quad + x^2 (W_{21} + W_{22} + W_{13}) \\ &\quad + x^3 W_{31}^1 + x^4 W_{32} + x^5 W_{31}^2 \mod (x^6 - 1) \end{aligned} \tag{22}$$

Equation (22) clearly indicates that the desired four messages W_{33}, W_{31}^1, W_{32} and W_{31}^2 are located at the second, fourth, fifth, and sixth time-slots in each cycle without any interference, respectively.

In summary, the task of sending $K = 10$ messages over $n = 6$ time-slots has been completed by the above scheme, which verifies that the DoF of $5/3$ is achievable.

Remark. We notify that the mod operation makes an effect of bringing the offset back into the integer range of $[0, 5]$, which indicates the circular right-shift of received

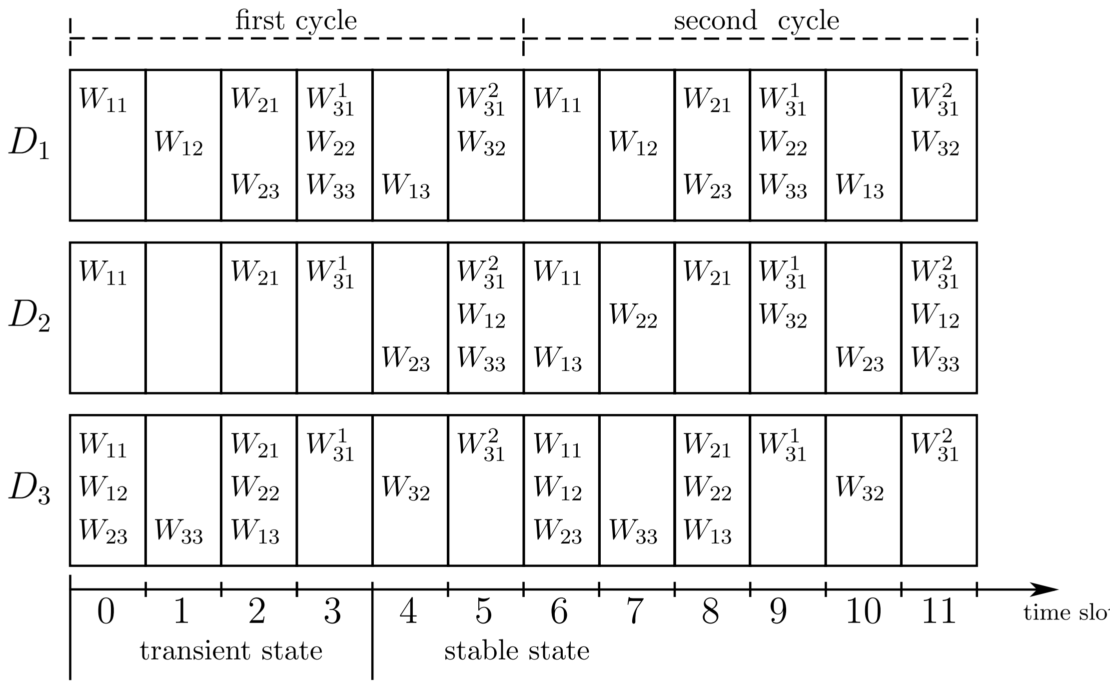

Fig. 3 Demonstration of the receiving procedure within the first 12 time-slots.

messages. To make it clear, we demonstrate it by Fig. 3, which shows the corresponding receiving procedure with respect to (20)–(22) within the first 12 time-slots (i.e., the first two cycles). Remember that the same pattern is repeated at each transmitter as depicted by Fig. 2. However, due to the PD property of the temporal channel, the messages transmitted in the same cycle may arrive at receivers over different cycles, which might cause a transient state before the stable state. Here the term 'stable state' means that the received messages show an exact repeated pattern, while transient state is the stage before that. For the proposed scheme, we can see a length of 4 time-slots (from time-slot 0 to 3) for the transient state. After that the receiving procedure of all receivers keeps in the stable state, which can be easily checked by the same pattern from time-slot 4. In detail, the received messages are exactly composed by the same messages in time-slots k and $k+6$, $\forall k$ when $k \geq 4$. For example, the received messages in time-slots 4 and 10 are W_{13}, W_{23}, and W_{32} for D_1, D_2, and D_3, respectively. This periodic receiving patten exhibits the cyclic property with a stable DoF of 5/3 over every 6 continuous time-slots from time-slot 4.

4. Feasibility in Euclidean Space

In this section we investigate the feasibility in two or three-dimensional Euclidean space for the above scheme. For the sake of simplicity, propagation speed v is assumed to be constant among all links, which means the PD feature also represents the distant relationship. The distance between S_j and D_i can be expressed by $\overline{D_i S_j} = v(\tau_0 + k\tau_{ij})$, where k is the scaling factor and τ_0 is the reference origin of PD. The PD increase in step $\Delta\tau = 1$ in our model indicates the corresponding equal distance increase with a step of $\Delta d = vk\Delta\tau = vk$, while the reference distance is denoted as $d_0 = v\tau_0$. So we have the following conditions between S_j and D_i, $\forall i, \forall j$

$$\overline{D_i S_j} = d_0 + \tau_{ij}\Delta d \tag{23}$$

According to the PD matrix (8), Fig. 4 depicts the equivalent geometric relationship among the transmitters and receivers of the proposed scheme.

Denote the circle/sphere centered at D_i with S_j on it by $O_i(S_j)$. Geometrically, the feasibility conditions can be set up by making sure that the related circles/spheres have the desired intersection points for the node placements of all the three transmitters, which can be expressed as $\forall j = 1, 2, 3$

$$O_1(S_j) \cap O_2(S_j) \cap O_3(S_j) \neq \emptyset \tag{24}$$

In other words, each S_j should locate at the intersection point of the three circles/spheres $O_i(S_j)$, $\forall i = 1, 2, 3$. The above constraint can be decomposed into the triangle relation by any two circles/spheres. Thus, if a node placement scheme is feasible, the side-length variables a, b, and c of the triangle composed by any three nodes should satisfy the triangle inequality

$$\begin{array}{rcccl} |b-c| & \leq & a & \leq & b+c \\ |a-c| & \leq & b & \leq & a+c \\ |a-b| & \leq & c & \leq & a+b \end{array} \tag{25}$$

For convenience, we denote the distance between D_1 and D_2 as $a = \overline{D_1 D_2}$, the distance between D_1 and D_3 as $b = \overline{D_1 D_3}$, and the distance between D_2 and D_3 as $c =$

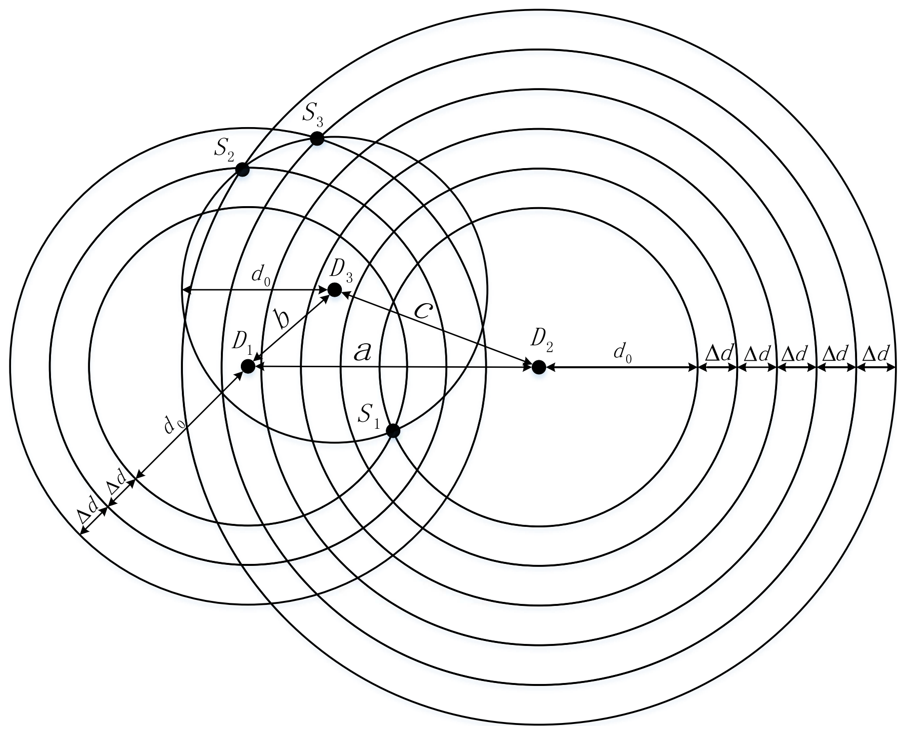

Fig. 4 Feasibility demonstration in Euclidean space.

$\overline{D_2D_3}$.

Firstly, we consider the scenario of each transmitter S_j and any two receivers. For D_1 and D_2, the triangle inequalities are

$$\begin{array}{rcccl} |\overline{D_1S_1}-\overline{D_2S_1}| & \leq & a & \leq & \overline{D_1S_1}+\overline{D_2S_1} \\ |\overline{D_1S_2}-\overline{D_2S_2}| & \leq & a & \leq & \overline{D_1S_2}+\overline{D_2S_2} \\ |\overline{D_1S_3}-\overline{D_2S_3}| & \leq & a & \leq & \overline{D_1S_3}+\overline{D_2S_3} \end{array} \tag{26}$$

for the triangles $\Delta S_1D_1D_2$, $\Delta S_2D_1D_2$, and $\Delta S_3D_1D_2$, respectively. Bringing (23) and (8) into (26), we can obtain

$$\begin{array}{rcccl} 0 & \leq & a & \leq & 2d_0 \\ 4\Delta d & \leq & a & \leq & 2d_0+6\Delta d \\ 2\Delta d & \leq & a & \leq & 2d_0+6\Delta d \end{array} \tag{27}$$

which can be further simplified as

$$4\Delta d \leq a \leq 2d_0 \tag{28}$$

Similarly, the triangle inequalities give the range of b as

$$2\Delta d \leq b \leq 2d_0 \tag{29}$$

and the range of c as

$$5\Delta d \leq c \leq 2d_0 \tag{30}$$

The above conditions require

$$\Delta d \leq 2d_0/5 \tag{31}$$

Secondly, the triangle relation between the three receivers can be further investigated. After some computations, from (25) and (28)–(30) we have

$$\begin{array}{rcccl} \max(4\Delta d, 2d_0-2\Delta d) & \leq & a & \leq & 2d_0 \\ \max(2\Delta d, 2d_0-4\Delta d) & \leq & b & \leq & 2d_0 \\ \max(5\Delta d, 2d_0-2\Delta d) & \leq & c & \leq & 2d_0 \end{array} \tag{32}$$

On the other side, we can consider the scenario of each receiver D_i and any two receivers. For convenience, we denote the distance between S_1 and S_2 as $e = \overline{S_1S_2}$, the distance between S_1 and S_3 as $f = \overline{S_1S_3}$, and the distance between S_2 and S_3 as $g = \overline{S_2S_3}$. The above approach can be applied again. For example, with S_1 and S_2, the triangle inequalities are

$$\begin{array}{rcccl} |\overline{D_1S_1}-\overline{D_1S_2}| & \leq & e & \leq & \overline{D_1S_1}+\overline{D_1S_2} \\ |\overline{D_2S_1}-\overline{D_2S_2}| & \leq & e & \leq & \overline{D_2S_1}+\overline{D_2S_2} \\ |\overline{D_3S_1}-\overline{D_3S_2}| & \leq & e & \leq & \overline{D_3S_1}+\overline{D_3S_2} \end{array} \tag{33}$$

for the triangles $\Delta S_1S_2D_1$, $\Delta S_1S_2D_2$, and $\Delta S_1S_2D_3$, respectively. Bringing (23) and (8) into (33), we can obtain

$$\begin{array}{rcccl} \Delta d & \leq & e & \leq & 2d_0+\Delta d \\ 5\Delta d & \leq & e & \leq & 2d_0+5\Delta d \\ 0 & \leq & e & \leq & 2d_0 \end{array} \tag{34}$$

which can be further simplified as

$$5\Delta d \leq e \leq 2d_0 \tag{35}$$

Similarly, the triangle inequalities give the range of f

as

$$4\Delta d \le f \le 2d_0 \tag{36}$$

and the range of g as

$$\Delta d \le g \le 2d_0 \tag{37}$$

Again, using the triangle relation between the three transmitters, the ranges of e, f, and g can be further determined from (25) and (35)–(37) as

$$\begin{array}{rcccl} \max(5\Delta d, 2d_0 - \Delta d) & \le & e & \le & 2d_0 \\ \max(4\Delta d, 2d_0 - \Delta d) & \le & f & \le & 2d_0 \\ \max(\Delta d, 2d_0 - 4\Delta d) & \le & g & \le & 2d_0 \end{array} \tag{38}$$

We should remark that obviously the arbitrary choice of the reference distance d_0 and the scaling factor k makes the feasibility conditions suitable to wide applications.

Here is an example. For underwater acoustic communication, we often set $v = 1500$m/s. Let the scaling factor k be 0.02, 0.2, and 2, respectively. Then $\Delta d = vk$ is 30 m, 300 m, and 3000 m, respectively. By (31) we have $d_0 \ge 5\Delta d/2$, which gives the lower bound on d_0 as 75 m, 750 m, and 7500 m, respectively. The range of a, b, c, e, f, and g can be obtained by (32) and (38). Given $k = 0.2$ (i.e., $\Delta d = 300$m), the following network instances are provided for demonstration.

(i) When $d_0 = 750$ m, we have

$$\begin{array}{rcccl} 1200 & \le & a & \le & 1500 \\ 300 & \le & b & \le & 1500 \\ 1500 & \le & c & \le & 1500 \\ 1500 & \le & e & \le & 1500 \\ 1200 & \le & f & \le & 1500 \\ 300 & \le & g & \le & 1500 \end{array} \tag{39}$$

where $c = e = 1500$ m is fixed.

(ii) When $d_0 = 3000$ m, we have

$$\begin{array}{rcccl} 5400 & \le & a & \le & 6000 \\ 4800 & \le & b & \le & 6000 \\ 5400 & \le & c & \le & 6000 \\ 5700 & \le & e & \le & 6000 \\ 5700 & \le & f & \le & 6000 \\ 4800 & \le & g & \le & 6000 \end{array} \tag{40}$$

Finally, we notify that as long as the PD relationship is kept, our scheme supports the mobility of transmitters and receivers, which further expands its feasibility.

5. Conclusion

In this letter, we proposed a scheme to show that the maximum DoF of 5/3 can be achieved by proper cyclic IA for the three-user X channel with the minimal delay cost of one extra time-slot. Moreover, feasibility conditions of node arrangements in the two or three- dimensional Euclidean space demonstrate potential wide applications.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (No. 61271283, U1701265, 61472237, 61601283).

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