LETTER

Removing Boundary Effect of a Patch-Based Super-Resolution Algorithm

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SUMMARY In a patch-based super-resolution algorithm, a low-resolution patch is influenced by surrounding patches due to blurring. We propose to remove this boundary effect by subtracting the blur from the surrounding high-resolution patches, which enables more accurate sparse representation. We demonstrate improved performance through experimentation. The proposed algorithm can be applied to most of patch-based super-resolution algorithms to achieve additional improvement.
key words: *super-resolution, dictionary, sparse representation, patch*

1. Introduction

High-resolution images contain high frequency detail and demand for them has increased. However, spatial resolution is limited by an image sensor or an image acquisition system [1]. Super-resolution (SR) algorithms can overcome the physical limitation of the imaging system at low-cost.

In a SR algorithm based on a sparse representation, an image patch is represented by a sparse linear combination from an overcomplete dictionary. Assuming that a low-resolution (LR) patch and the corresponding high-resolution (HR) patch have the same sparse representation vector, we estimate a HR patch for each LR patch [2]. Yang *et al.* trained LR and HR dictionaries jointly such that corresponding HR and LR patches have the same sparse representation. After obtaining a sparse representation of an LR patch, corresponding HR dictionaries are employed to recover a HR patch. Thus the LR dictionary and HR dictionary pairs are crucial for obtaining the SR image. Zeyde *et al.* [3] trained LR dictionary with a high pass filtered LR image, while HR dictionary is constructed from the difference between the HR and LR image. This feature extraction facilitates more accurate SR recovery. We adopt the Zeyde algorithm for boundary effect elimination.

An LR image is a blurred and downsampled version of the HR image, therefore; an LR patch is a superposition of blurs from the corresponding HR patch and its neighbors. This blur will result in error in LR sparse representation which in turn degrades HR recovery. To remove the blur from the surrounding patches, we propose to subtract the blur of surrounding patches from the observed LR image. We use an estimated HR image to calculate the blur, and the blur removal and improved SR estimation loop is iterated. A block diagram of the blur removal is shown in Fig. 1.

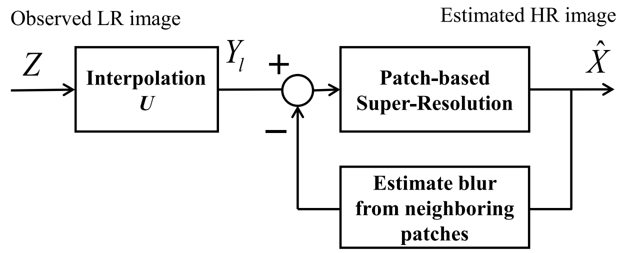

Fig. 1 Diagram of the proposed super-resolution method.

The patch-based SR algorithm using sparse representation is described in Sect. 2. Then, the proposed boundary effect reduction method is presented in Sect. 3. Experimental results including PSNR and PSF comparison are shown in Sect. 4 and finally conclusions are summarized in Sect. 5.

2. Patch-Based Super-Resolution Algorithm

A SR algorithm is a technique to restore a HR image from an observed LR image, which restores high frequency details and removes the degradation during image acquisition.

Let Z be an LR image, which is a blurred and downsampled from a HR image X. Then, an image observation model is as follows:

$$Z = SHX, \tag{1}$$

where H is a blur operator and S represents a downsampling operator. Additive noise is disregarded in this work for simplicity. A patch $x \in \mathbb{R}^n$ denotes a HR patch and an observed patch $z \in \mathbb{R}^l$ represents the LR patch of x. The relationship of an LR and HR patches is not the same as Eq. (1) due to blurring from surrounding patches, i.e.,

$$z = SHx + w, \tag{2}$$

where w is the blur from the surrounding patches. In many algorithms the spurious blur w is ignored. To take this influence into consideration, Yu *et al.* [4] proposed to increase the size of the patch considering the size of the blur support region. However, it also increases the size of the sparse representation vector, which complicates the calculation of sparse representation.

In this paper, we aim to remove the influence of surrounding patches during SR recovery as well as dictionary training stage. During the dictionary training stage, a HR

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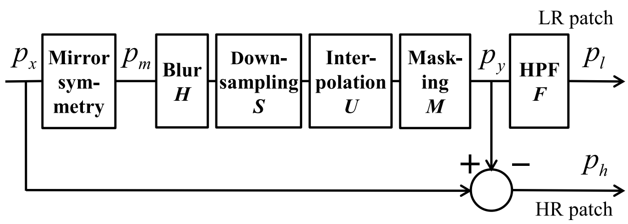

Fig. 2 Proposed patch-pair construction for low- and high-resolution dictionary training.

patch is extended by mirror symmetry as shown in Fig. 2, so that the blurred patch is not affected by neighboring patches at all. Therefore we can obtain an LR and HR dictionary pair without spurious blur effect. We also need to remove blur smeared from neighbors during SR recovery. We calculate the smeared blur using an estimated SR image and subtract it from an observed LR patch, and then we add a blur from mirror symmetry of the estimated HR center patch to generate an LR data in the same way as dictionary training. More details will be given in the following section.

In this paper, we adopt the Zeyde algorithm [3] for patch-based SR algorithm and then remove the boundary effect; however, the proposed boundary effect reduction can be applied to other patch-based SR algorithms to achieve further improvements. We briefly describe the Zeyde algorithm in this section, which will be used in our algorithm in the following section.

The Zeyde algorithm consists of dictionary learning and SR restoration. First, the dictionary learning algorithm constructs patch-pairs using the HR image in the training set. The LR image Z is obtained by blurring and down-sampling a HR image X in the training set using Eq. (1). To make the size of Z and X equal, an LR image Y is obtained after interpolation as in Eq. (3):

$$Y = UZ = USHX, \tag{3}$$

where U is an interpolation operator. We adopt bicubic interpolation in our experiments.

LR and HR dictionaries are constructed from high frequency features to recover the high frequency component faithfully. The LR patch p_l is calculated from the high pass filtered LR image Y, and then K-SVD [5] dictionary training is applied to obtain the LR dictionary D_l, whose atoms are incoherent each other [6]. The HR dictionary D_h is obtained from HR patches and the sparse representation of LR patches using pseudo-inverse.

The sparse representation vector q is obtained by minimizing Eq. (4) for an LR patch p_l and the LR dictionary D_l:

$$q = \min \|p_l - D_l q\|_2^2 \quad \text{s.t.} \ \ \|q\|_0 \leq L. \tag{4}$$

Finally the sparse representation vector q is multiplied by the HR dictionary D_h, thus the HR patch p_h is estimated as in Eq. (5):

$$p_h = D_h q. \tag{5}$$

The above SR algorithm is applied to each LR patch, and then an HR image is obtained by stitching the HR patches.

3. Proposed Method

To estimate a HR patch p_h from an LR patch p_l, p_h and p_l are assumed to have the same sparse representation vector q for each LR dictionary D_l and HR dictionary D_h pair. Therefore the dictionary pairs play a significant role in restoring HR features precisely from an LR patch. Recently, Peleg and Elad [7] proposed a statistical prediction model on sparse representation for further improvement. During the dictionary training stage, LR patches are obtained after blurring, down-sampling, interpolation, and feature extraction of HR patches. Existing methods [2], [3] applied the above processes for the entire image, thus the LR patch is influenced by the spurious blur from surrounding patches and the relation of patch-pair is not accurate. Unwanted superposition of the blurring from the neighboring patches, denoted as w in Eq. (2), causes error in sparse representation of an LR patch, which also causes degradation of HR patch restoration. To eliminate this boundary problem, Yu *et al.* [4] increased the size of the patch considering the size of the blur kernel and extract the center portion from the restored patch. However, the proposed method excludes the boundary effect by subtracting the blur from the neighboring HR patches which have been estimated previously. In our experiments, the estimation process is iterated five times, which achieves saturated improvement of around 0.3 dB as shown in Fig. 4.

Let us briefly review the patch-pair composition method in the dictionary learning algorithm [3], and then present an improved method which removes the blur from the surrounding patches.

3.1 Patch-Pair Construction for Dictionary Learning

Let p_x be a partitioned HR patch in the training set whose size is $\sqrt{n} \times \sqrt{n}$. To generate an LR patch, we need to apply blurring, down-sampling, and interpolation. Before blurring we extend the patch p_x using mirror symmetry as shown in Fig. 2; zero padding is not acceptable due to abrupt change at the boundary, which would cause severe artifacts after high pass filtering. The mirror symmetric extended version is denoted as p_m. The size of p_m should be increased in the same way as Yu *et al.* [4], however, we extract only $\sqrt{n} \times \sqrt{n}$ center portion after blurring, which is represented as a masking operation M. Let p_y be the blur, down-sampling, interpolation and masking of p_m:

$$p_y = MUSHp_m. \tag{6}$$

The LR patch p_l and the HR patch p_h are obtained after extracting features as in Zeyde *et al.* [3]. The LR patch p_l is obtained after high pass filtering p_y

$$p_l = Fp_y, \tag{7}$$

where F is the high pass filter. The HR patch p_h is obtained by subtracting p_l from p_x:

$$p_h = p_x - p_y. \tag{8}$$

The dictionary pair is obtained from the LR and HR patches using the Zeyde algorithm.

3.2 Removing Boundary Effect from LR Patches

During the dictionary learning, we have the training set of HR images, therefore we can obtain an LR patch from a mirror symmetric HR patch from the HR image. However, in the SR recovery step, we only have an LR image, thus we need an iteration to employ HR patches which have been estimated from the previous iteration. A detailed diagram of the blur effect estimation is shown in Fig. 3.

An initial HR patch $\hat{p}_{x,0}$ is estimated from the Zeyde algorithm. To update the HR patch $\hat{p}_{x,k}$ at iteration k, the previous HR image $\hat{X}_k$ and the observed LR image Y is used. The blur of the surrounding patches is obtained from the estimated HR image $\hat{X}_k$, and it is subtracted from the LR image Y. An LR patch blurred from mirror symmetric HR patch, $p_{y,k}$, is obtained by adding the blur from mirror symmetry of the center patch itself. Then the LR patch p_l is obtained by high pass filtering as in Eq. (7). The sparse representation vector q is obtained using Eq. (4). Using Eq. (5), the updated HR patch $p_{x,k+1}$ is obtained by multiplying the HR dictionary D_h and the sparse representation vector q. The final HR image $\hat{X}$ is obtained by iterating the above mentioned process using the previous HR image $\hat{X}_k$. The number of iteration is determined by observing the mean square error (MSE) reduction from experiments with many test images. We observe that after two or three iterations the MSE is saturated as shown in Fig. 4.

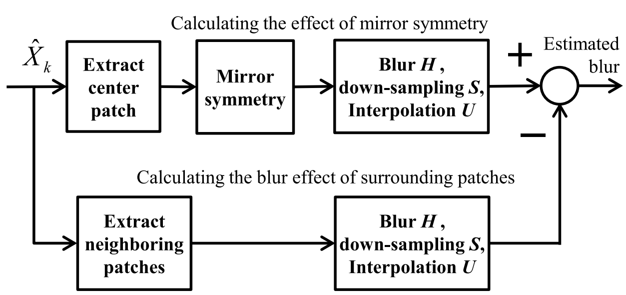

Fig. 3 Details of proposed super-resolution to eliminate boundary effect.

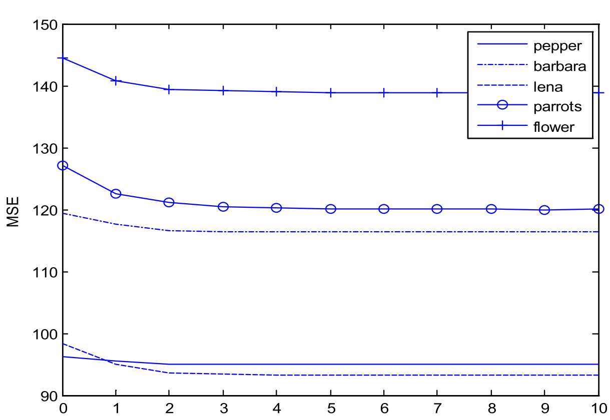

Fig. 4 MSE of SR image after iterations of removing blur from surrounding patches. Improvement of MSE saturates after two or three iterations.

4. Experimental Results and Analysis

We compare the existing SR method based on a sparse representation using PSNR and error images. We use the training set of Yang *et al.* [2]. The LR image is obtained from blurring using the blur PSF and down-sampling a HR image by the factor of three. The patch-pairs of 50,000 are obtained from the HR and LR images randomly, where the size of the patch is 9×9. The dictionary is obtained using the patch-pairs, where the number of the dictionary atom is 1,000.

We compare the estimated SR image quality with Zeyde *et al.* [3] algorithm. Table 1 shows results of seven images using the blur PSF [1 3 5 7 5 3 1]/25. The PSF is applied to vertical and horizontal directions respectively. From Table 1, the PSNR improvement of the proposed method has an average of 0.2 dB and a maximum of 0.4 dB. Fig-

Table 1 PSNR Comparison for blur PSF [1 3 5 7 5 3 1]/25.

Image Name	Bicubic interpolation	Zeyde algorithm [3]	Proposed method
Lena	25.35	28.20	28.43
Man	23.17	25.34	25.47
Boat	23.76	25.57	25.73
Butterfly	20.54	23.64	24.06
Parrots	24.35	27.09	27.33
Girl	29.11	31.26	31.38
Flower	24.18	26.53	26.70
Average	24.35	26.80	27.01

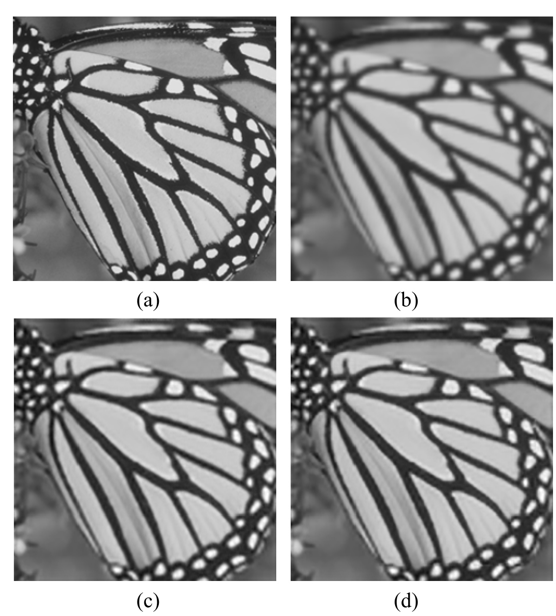

Fig. 5 Result images (a) the original HR image, (b) bicubic interpolation image, (c) Zeyde algorithm [3], (d) the proposed method.

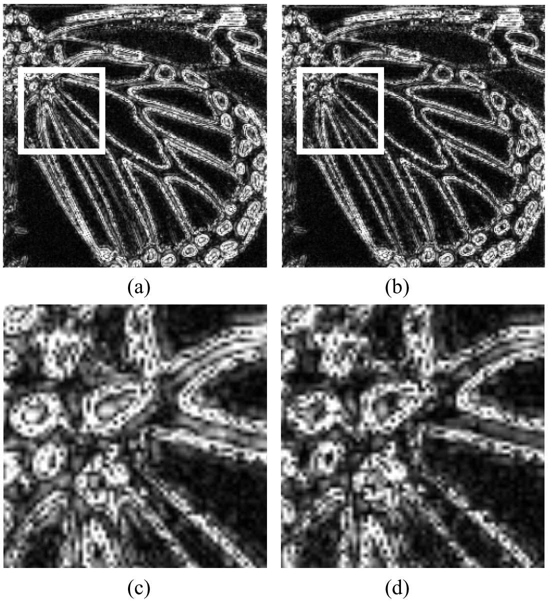

Fig. 6 Error images between the original HR image and the estimation image using (a) Zeyde algorithm [3], (b) proposed method, and (c) and (d) magnification image of the rectangle region of (a) and (b), respectively.

Table 2 PSNR Comparison for blur PSF [2 4 6 7 6 4 2]/31.

Image Name	Bicubic interpolation	Zeyde algorithm [3]	Proposed method
Lena	24.85	27.89	28.17
Man	22.77	24.93	25.19
Boat	23.36	25.26	25.49
Butterfly	20.07	23.17	23.62
Parrots	23.87	26.80	27.20
Girl	28.64	31.10	31.31
Flower	23.76	26.18	26.43
Average	23.90	26.48	26.77

ure 5 compares the result images using the Zeyde algorithm and the proposed method, and Fig. 6 shows the error images between the original and restored HR images. From the error images, we observe that the proposed method estimates the HR image quite accurately. The proposed method eliminates the boundary effect during patch pairing as well as dictionary training, therefore the presented method restores HR patches more accurately. Experimental results verify improved SR restoration performance.

We also present the performance of the proposed method for another PSF. Table 2 shows the results using the PSF [2 4 6 7 6 4 2]/31, which has higher blur at the edges. The PSNR improvement is 0.3 dB on the average with maximum of 0.5 dB, which is better than the results of Table 1. The proposed method estimates HR patches without spurious blur from surrounding patches, therefore our method is more powerful when the blur is severe as in Table 2.

Yu *et al.*'s method will show similar improvements in PSNR, however the HR dictionary training becomes quite formidable because the patch size becomes 24×24 for this example. Processing time is increased by the same factor i.e. $(24/9)^2 \approx 7$.

5. Conclusion

In this paper, we propose to improve a SR algorithm using a sparse representation from a single LR image by removing the blur from surrounding patches. The proposed method trains dictionaries so that the LR patch and HR patch have the same sparse representation vector. The dictionary is crucial to improve the image resolution, therefore we propose to eliminate blur from neighboring patches during LR dictionary training. The proposed method removes the superposition of unwanted blur effect of the surrounding patches from an LR patch during HR image restoration, and the improvements are confirmed from experiments. Our contribution can be applied to most patch-based sparse representation methods other than the Zeyde algorithm. The proposed method shows better performance, especially when the blur is severe.

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