

LETTER

Blind Image Deblurring Using Weighted Sum of Gaussian Kernels for Point Spread Function Estimation

Hong LIU[†], Student Member and BenYong LIU^{†a)}, Nonmember

SUMMARY Point spread function (PSF) estimation plays a paramount role in image deblurring processing, and traditionally it is solved by parameter estimation of a certain preassumed PSF shape model. In real life, the PSF shape is generally arbitrary and complicated, and thus it is assumed in this manuscript that a PSF may be decomposed as a weighted sum of a certain number of Gaussian kernels, with weight coefficients estimated in an alternating manner, and an l_1 norm-based total variation (TV $_{l_1}$) algorithm is adopted to recover the latent image. Experiments show that the proposed method can achieve satisfactory performance on synthetic and realistic blurred images.

key words: point spread function, blind deblurring, Gaussian kernel, total variation

1. Introduction

The relative motion between a camera and an original scene would lead to a blurred image. If the blurring process is simplified to be shift-invariant, the degraded procedure may be represented by convolution as $B(x, y) = L(x, y) \otimes k(x, y) + \varepsilon(x, y)$, where $B(x, y)$ is the blurred image, $L(x, y)$ is the latent image, $k(x, y)$ is the point spread function (PSF) (also called blurring kernel), $\varepsilon(x, y)$ is the additive noise, and \otimes denotes two dimensional convolution. The main purpose of image deblurring is to restore the latent image $L(x, y)$ from the blurred version $B(x, y)$.

As an ill-posed problem, image deblurring may be categorized to non-blind and blind cases. In non-blind deblurring (NBD), the PSF is assumed to be known or computed in a certain manner. On the contrary, in blind deblurring (BD) the PSF is unknown and thus it is a much more challenging problem. In early work, many researchers focused on some special types of blurring process such as defocus process and linear motion process, wherein the PSF is generally approximated through a simple parametric model which is characterized by one or a few parameters. However, in practical applications, the PSF of a blurring process is generally arbitrary and complicated, and thus it is necessary to develop some more practical approaches to solve this problem. Fergus *et al.* [1] exploited a Gaussian mixture model to fit the heavy-tailed natural image gradients prior, which is solved by a variational Bayesian framework. Shan *et al.* [2] introduced a new smoothness constraint on the latent im-

age, which is very effective in suppressing ringing artefacts. Xu and Jia [3] proposed a novel two-phase kernel estimation algorithm, and exploited an l_1 norm-based total variation (TV $_{l_1}$) algorithm to recover a latent image.

In this letter, a weighted sum of Gaussian kernels is proposed to represent a PSF which holds an arbitrary shape, with the weighted coefficient set estimated in an alternating manner, and a TV $_{l_1}$ algorithm is adopted to restore the latent image.

The framework of the blind deblurring algorithm in this letter is similar to that in [3]. However, in [3], the blurring process was represented as a traditional convolution model, in which the PSF was a general two dimensional function. In our proposed method, the PSF is represented as a weighted sum of Gaussian kernels, and thus as a result, the blurred image is regarded as a weighted sum of Gaussian blurred images, which can more accurately approximate the really blurring process. For this reason, the proposed deblurring algorithm provides more satisfactory performance based on the new model.

2. Weighted Gaussian Kernel Model

In the view of function approximation, an unknown PSF could be represented by $k(x, y) = \sum_{j=1}^N \lambda_j h_j(x, y)$, where $\{h_j(x, y)\}_{j=1}^N$ is a certain family of known functions and $\{\lambda_j\}_{j=1}^N$ is an approximation coefficient set which needs to be estimated.

In our study, we adopt the Gaussian kernel function to approximate the PSF since, in one reason, in most optical imaging systems there always exists diffraction-limited blurring effect, which is a fundamental resolution limitation due to diffraction. In addition, in most commonly used cameras, an anti-aliasing filter is adopted in the lens system to remove the high-frequency components which are beyond the Nyquist limit of the digital camera sensors, wherein the anti-aliasing blurring effect is generally embedded. Therefore, generally there exists blurring effect in the imaging process, even though any other effects, such as relative motion between the camera and the scene and incorrect lens setting, are absent. Furthermore, the Gaussian kernel is a lowpass filter and generally so is a PSF, any shape of a PSF may be approximated by a suitable linear combination of Gaussian kernels. Finally, the Gaussian kernel hosts most of other properties of a PSF, for examples, the value of the Gaussian kernel is non-negative and the integral of the Gaussian ker-

Manuscript received May 26, 2015.

Manuscript revised July 17, 2015.

Manuscript publicized August 5, 2015.

[†]The authors are with Guizhou University, GuiYang, 550025 China.

a) E-mail: csc.byliu@gzu.edu.cn

DOI: 10.1587/transinf.2015EDL8121

nel equals to 1. Based on the above reasons, the degraded procedure may be represented as follows:

$$B(x, y) = L(x, y) \otimes \sum_{j=1}^N \lambda_j h_j(x, y) + \varepsilon(x, y) \quad (1)$$

where $\{h_j(x, y)\}_{j=1}^N$ is a two-dimensional Gaussian kernel function set. Using the properties of the convolution, this leads to the following form:

$$B(x, y) = \sum_{j=1}^N \lambda_j \{L(x, y) \otimes h_j(x, y)\} + \varepsilon(x, y) \quad (2)$$

In (2), the blurred image is regarded as a weighted sum of Gaussian blurred images. However, in [3], the blurring process was represented as a traditional convolution model:

$$B(x, y) = L(x, y) \otimes k(x, y) + \varepsilon(x, y) \quad (3)$$

As mentioned above, generally there exists blurring effect in the imaging process, even though under the ideal imaging condition. Therefore, the new model (1) can more accurately approximate the really blurring process.

3. Coefficient Set Estimation

The coefficient set $\{\lambda_j\}_{j=1}^N$ is estimated using a two-step manner similar to the method in [3].

3.1 Step One

In the first step, like other BD methods, $\{\lambda_j\}_{j=1}^N$ is alternately estimated in a multi-scale framework as follows.

First, in order to reconstruct the salient sharp edges, which can guide the kernel initialization, we use Gaussian filter to pre-smooth the image and then enhance the edges using a shock filter.

Next, we choose useful edges for coefficient estimation using the criterion in [3]. Let $N(x)$ be a neighborhood of pixel x . A metric to measure the usefulness of gradient information, is defined as:

$$r(x) = \left\| \sum_{y \in N(x)} \nabla B(y) \right\| \left/ \left(\sum_{y \in N(x)} \|\nabla B(y)\| + 0.5 \right) \right. \quad (4)$$

We then select pixels belonging to big r -values using a mask $M = S(r - \tau_r)$, where $S(\cdot)$ is the Heaviside step function, τ_r is a threshold. The final selected edges for coefficients estimation are determined by

$$\nabla L^s = \nabla \tilde{L} \circ S(M \circ \|\nabla \tilde{L}\|_2 - \tau_s) \quad (5)$$

where \circ denotes element-wise multiplication, \tilde{L} is the shock filtered image and τ_s is a threshold of the gradient magnitude. In order to include more and more edges, the values of τ_r and τ_s are decreased iteratively (say, divided by 1.1 in each iteration).

Finally, we estimate the coefficient set $\{\lambda_j\}_{j=1}^N$ in gradient domain. Let l and b be the vector form of the latent image and the blurred image, respectively, and b_h and b_v are differences of B in horizontal and vertical directions respectively (and similarly for l_h, l_v, l_h^s, l_v^s). By writing convolution as matrix multiplication, the coefficients estimation problem is written as:

$$\min_{\lambda} \left\{ \|D\lambda - b_d\|_2^2 + \gamma \|\lambda\|_2^2 \right\} \quad (6)$$

where $\lambda = [\lambda_1, \dots, \lambda_N]$, $b_d = [b_h; b_v]$, γ is parameter for balancing the regularization term and the fidelity term. Let $\text{col}_j(D) = [H_j \mathbf{0}; \mathbf{0} H_j][l_h^s; l_v^s]$, where $\text{col}_j(\cdot)$ fetches the j -th column of matrix D , H_j is a block circulant matrix representing a Gaussian kernel function, and $\mathbf{0}$ is an all-zero matrix with the same size of H_j . Taking derivative of (6) and setting it to zero yields:

$$\lambda = (D^T b_d) / (D^T D + \gamma) \quad (7)$$

Then we use the predicted sharp edge gradient $l_d^s = [l_h^s; l_v^s]$ to guide the recovery of a coarse version of the latent image. Therefore, the objective function is defined as:

$$\min_l \left\{ \|Kl - b\|_2^2 + \eta \|Gl - l_d^s\|_2^2 \right\} \quad (8)$$

where $K = \sum_{j=1}^N \lambda_j H_j$, $G = [G^{(1)}; G^{(2)}]$, with $G^{(1)}$ and $G^{(2)}$ the first-order forward finite difference matrices in the horizontal and vertical directions, respectively. In addition, the normal equations of (8) can be written as:

$$(K^T K + \eta G^T G)l = K^T b + \eta \left[(G^{(1)})^T l_h^s + (G^{(2)})^T l_v^s \right] \quad (9)$$

Using the convolution theorem of Fourier transforms, the solution to (8) can be obtained as following:

$$l = F^{-1} \left\{ \frac{\overline{F(K)} \circ F(b) + \eta \left[\overline{F(G^{(1)})} \circ F(l_h^s) + \overline{F(G^{(2)})} \circ F(l_v^s) \right]}{\overline{F(K)} \circ F(K) + \eta \left[\overline{F(G^{(1)})} \circ F(G^{(1)}) + \overline{F(G^{(2)})} \circ F(G^{(2)}) \right]} \right\} \quad (10)$$

With a slight abuse of notation, we have used $F(K)$ for the 2D FFT of the function represented by K in the convolution Kl , and similarly for $G^{(1)}$ and $G^{(2)}$. In the above representation, $F^{-1}(\cdot)$ denotes the inverse FFT, and $\overline{(\cdot)}$ is the complex conjugate operator. The division is performed element-wise.

3.2 Step Two

In the second step, to refine the PSF estimate, the iterative support detection (ISD) method in [4] is adopted to apply only on the highest resolution, instead of in a multi-scale framework used in step one. ISD is an iterative method, whose idea is to iteratively retain the large value elements in the PSF by relaxing the regularization penalty. In each iteration, previously estimated PSF elements are divided two sets: elements that are greater than one threshold ε^i are put into a set S^{i+1} and all other belong to the set $\overline{S^{i+1}}$. The ε^i is determined by applying the ‘‘first significant jump’’

rule. We sort all elements of PSF in an ascending order to form a vector ko ($ko_{[j]}$ denotes the j th largest component of ko). $\varepsilon^i = ko_{[s_j]}^i$, where $s_j = \min_j \{ko_{[j+1]}^i - ko_{[j]}^i > \tau^i\}$, $\tau^i = \|ko^i\|_\infty / (2p \cdot i)$, p is the PSF width. A refined PSF is gotten by solving

$$\min_k \frac{1}{2} \|Ak - b_d\|^2 + \sum_{j \in \bar{\mathcal{S}}_{i+1}} |k_j| \quad (11)$$

where A is gotten by writing convolution as matrix. The solution of (11) could be gotten by solving

$$\left[A^T A + \text{diag}(v_{\bar{\mathcal{S}}} \Psi^{-1}) \right] k^t = A^T b_d \quad (12)$$

where $v_{\bar{\mathcal{S}}}$ is the vector form of $\bar{\mathcal{S}}$, Ψ is defined as $\Psi = \max(|k^{t-1}|, 1e^{-5})$, which is the weight related to the PSF estimate from the previous iteration. $\text{diag}(\cdot)$ produces a diagonal matrix from the input vector.

4. Non-Blind Deblurring

Finally, one NBD algorithm based on TVl_1 model is employed to recover the latent image. Given the finally estimated PSF and blurred image, the latent image can be computed from the following model:

$$\min_l \left\{ \|Kl - b\|_1 + \xi \sum_i \|G_i l\|_1 \right\} \quad (13)$$

The efficient alternating minimization method in [5] is used to solve (13) as follows. Two auxiliary variables v and w_i are introduced for each pixel, which are approximations of $(Kl - b)$ and $G_i l$, respectively. Then the approximation model to (13) is modified as

$$\min_{w,v,l} \left\{ \frac{\alpha}{2} \|Kl - b - v\|_2^2 + \|v\|_1 + \sum_i \xi \|w_i\|_1 + \frac{\beta}{2} \|w_i - G_i l\|_2^2 \right\} \quad (14)$$

As $\alpha \rightarrow \infty$, $\beta \rightarrow \infty$ the solution to (14) converges to that to (13). In the following experiments, the values of α_0 , β_0 are set to 1, and $\alpha_{\max} = \beta_{\max} = 100$. For a fixed l , $w_i = \max\{\|G_i l\|_2 - \xi/\alpha, 0\}(G_i l / \|G_i l\|_2)$, where the convention $0 \cdot (0/0) = 0$ is followed, and the minimization with respect to v is given by $v = \max\{|Kl - b| - \beta^{-1}, 0\} \circ \text{sgn}(Kl - b)$. All operations are done element-wise. On the other hand, for fixed w and v , the minimization with respect to l is a least squared problem, which can be solved in spectral domain similar to (8).

5. Experiment

In our experiments, the initial values of τ_r and τ_s are adaptively set like the manner in [3]. In coefficient set estimation, the value of γ is set to $2e^{-4}$, and that of η is set to $2e^{-3}$. The number of Gaussian kernels is decided by the size of the PSF in each scale, and the variance of each Gaussian kernel is set to 0.9. In the final Non-blind image deblurring, ξ

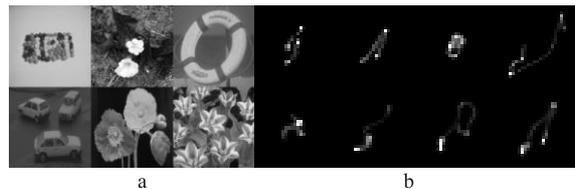


Fig. 1 Set of images (Fig. 1 a) and PSFs (Fig. 1 b) used for synthetic experiment.

Table 1 PSNR comparison.

Method	Average PSNR on various PSFs (dB)							
	#1	#2	#3	#4	#5	#6	#7	#8
[1]	27.44	24.95	28.86	22.30	28.97	26.76	24.84	23.78
[2]	27.08	29.38	28.94	24.96	27.02	26.84	26.21	26.24
[3]	29.77	31.09	30.27	29.00	29.50	29.23	29.74	28.82
Proposed	30.34	31.62	33.08	27.54	32.85	31.41	27.28	29.42

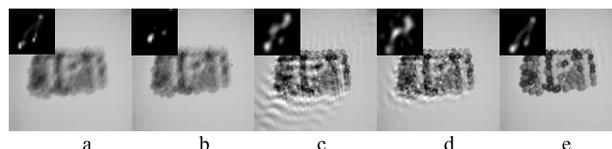


Fig. 2 Visual comparison on synthetic image (Fig. 2 a: Blurred image and ground truth PSF. Fig. 2 b–e: Deblurred results and estimated PSFs of methods in [1]–[3] and proposed method).

is set to $2e^{-2}$. The proposed method is compared to other state-of-the-art methods in [1], [2] and [3], wherein the authors' implementations are adopted. For fairness, we make our best to adjust the parameter of the above three methods, but run all test images with equal parameters. In synthetic experiment, 48 synthetic blurred images are obtained using the 6 original images and the 8 PSFs provided in [6], as shown in Fig. 1.

The average peak signal-to-noise ratios (PSNRs) of the deblurred images obtained by different methods are shown in Table 1. The best results for each PSF are shown in bold. Table 1 clearly indicates that the proposed method outperforms the methods of [1] and [2]. The method in [3] only outperforms ours in the images produced with PSF #4 and #7. Figure 2 a shows one sample from the 48 blurred images used in this experiment, and the experimental results are compared in Fig. 2 b–e. The deblurred result of the proposed, as shown in Fig. 2 e, has more details and sharp edges.

We also compare the proposed method to the three methods mentioned above on real-life photograph provided in [7], as shown in Fig. 3 a. Figure 3 b is the close-up of a region marked as a red rectangle in Fig. 3 a. The close-ups in Fig. 3 c–f illustrate that the proposed method preserves more image details, meanwhile produces less ringing artefacts near edges in comparison to others. In Fig. 3 c–f, the estimated PSFs are depicted in the top left corners. It shows that PSF estimation plays a vital role in deblurring performance.

Once again, experimental results on synthetic and real-life blurred images validate that the proposed model can

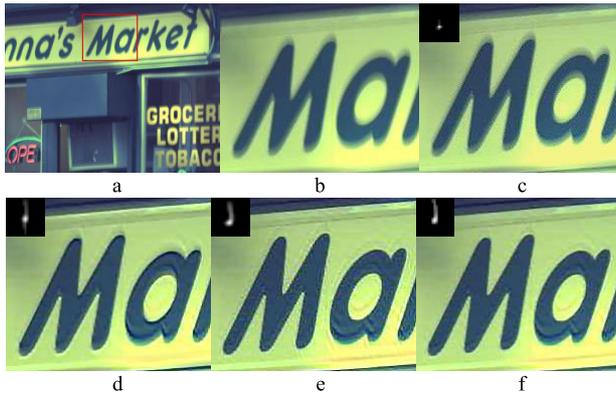


Fig. 3 Visual comparison on real-life photograph (Fig. 3 a: Real-life photograph. Fig. 3 b: Close-up of a region marked as red rectangle in Fig. 3 a. Fig. 3 c–f: Close-ups and estimated PSFs of methods in [1]–[3] and proposed method).

more accurately approximate the really blurring process, as analyzed previously.

6. Conclusion

A blind image deblurring method is proposed, wherein the PSF is represented using weighted sum of Gaussian kernels, and the coefficient set is estimated in the space domain, which can improve the accuracy of the PSF estimation. Deblurring results on some synthetic and real-life blurred images show satisfactory performance of the proposed method. However, deblurring experiments with very large blurring kernels is not performed since the coefficient set's estimation process in space domain is computationally intensive, whereas the method in [3] could deal with very large blurring kernels since the estimation of the PSF was performed

in frequency domain, and extending our deblurring method in similar manner to surmount the above limitation will be part of our future work.

Acknowledgments

This work was supported in part by grants of the Natural Science Foundation of China (60862003), Project of Int'l S&T Cooperation Program of China (2009DFR10530), and The Ph.D. Programs Foundation of Ministry of Education of China (20095201110002).

References

- [1] R. Fergus, B. Singh, A. Hertzmann, S.T. Roweis, and W.T. Freeman, "Removing camera shake from a single photograph," *ACM Trans. Graph.*, vol.25, no.3, pp.787–794, 2006.
- [2] Q. Shan, J. Jia, and A. Agarwala, "High-quality motion deblurring from a single image," *ACM Trans. Graph.*, vol.27, no.3, pp.1–10, 2008.
- [3] L. Xu and J. Jia, "Two-phase kernel estimation for robust motion deblurring," *Proc. European Conf. on Comput. Vision (ECCV)*, Heraklion, pp.157–170, Sept. 2010.
- [4] Y. Wang and W. Yin, "Sparse signal reconstruction via iterative support detection," *SIAM Journal on Imaging Sciences*, vol.3, no.3, pp.462–491, 2010.
- [5] J. Yang, Y. Zhang, and W. Yin, "An efficient TVL1 algorithm for deblurring multichannel images corrupted by impulsive noise," *SIAM Journal on Science Computer*, vol.31, no.4, pp.2842–2865, 2009.
- [6] A. Levin, Y. Weiss, F. Durand, and W.T. Freeman, "Understanding and evaluating blind deconvolution algorithms," *Proc. IEEE Conf. Comput. Vision and Pattern Recognit. (CVPR)*, Miami, pp.1964–1971, June 2009.
- [7] T.S. Cho, S. Paris, B.K.P. Horn, and W.T. Freeman, "Blur kernel estimation using the radon transform," *Proc. IEEE Conf. Comput. Vision and Pattern Recognit. (CVPR)*, Colorado Springs, pp.241–248, June 2011.