LETTER

The Controllability of Power Grids in Comparison with Classical Complex Network Models

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SUMMARY The controllability of complex networks has attracted increasing attention within various scientific fields. Many power grids are complex networks with some common topological characteristics such as small-world and scale-free features. This Letter investigate the controllability of some real power grids in comparison with classical complex network models with the same number of nodes. Several conclusions are drawn after detailed analyses using several real power grids together with Erdös-Rényi (ER) random networks, Wattz-Strogatz (WS) small-world networks, Barabási-Albert (BA) scale-free networks and configuration model (CM) networks. The main conclusion is that most driver nodes of power grids are hub-free nodes with low nodal degree values of 1 or 2. The controllability of power grids is determined by degree distribution and heterogeneity, and power grids are harder to control than WS networks and CM networks while easier than BA networks. Some power grids are relatively difficult to control because they require a far higher ratio of driver nodes than ER networks, while other power grids are easier to control for they require a driver node ratio less than or equal to ER random networks.
key words: *power grids, complex networks, controllability, driver nodes*

1. Introduction

Complex network theory and its application have attracted many scholars to research since the small-world [1] and scale-free [2] features were found in 1998 and 1999, respectively. Complex networks can be abstracted from many systems in our everyday life, e.g., traffic networks, power grids, and Internet. Some scholars are engaged in the models and static properties of complex networks [3]. Some authors are interested in the dynamic behaviors and properties in complex networks [4]. Some researchers are devoted to important node mining and community detection problems in complex networks [5]. Among all topics, controlling is one of the most challenging problems, and thus significant efforts have been devoted to understanding the controllability of the complex network recently [6]–[8]. Controllability is characterized by the minimum number of driver nodes which can offer full control over the network.

Large-scale grid interconnection of electrical energy can realize long-distance transmission and make the power distribution economically reasonable. However, it not only brings considerable economic benefits, but also brings greater uncertainty in grid operation and makes the analysis of operating status and dynamic properties of the power grid more difficult. In recent years, many researchers have paid much attention to analyzing power grids based on the complex network theory [9] in three aspects: to reveal the topology characteristics [10]; to reveal the inherent vulnerabilities and weaknesses [11]; to analyze the mechanism of cascading failures [12]. Since there are relatively few research works on the controllability of power grids, this Letter focuses on this topic according to the controllability research fruits in complex networks.

2. Exact Controllability Theory

A complex network is controllable if imposing appropriate external signals on a subset of its nodes, the system can be driven from any initial state to any final state in finite time. The minimal set of driver nodes required to control a network is called the minimum driver node set (MDNS). The minimum number of driver nodes is denoted by N_D and the controllability of a network is defined as the ratio of the number of driver nodes (N_D) to the total number of nodes in the network (N), i.e., $n_D = N_D/N$. A controllable complex network with N nodes can be described by the following linear ordinary differential equation

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_N)^{\mathrm{T}}$ denotes the states of N nodes, and $\mathbf{A} \in \mathbf{R}^{N\times N}$ is the coupling matrix of the network, in which a_{ij} represents the weight of a directed link from node j to i (for undirected networks, $a_{ij} = a_{ji}$). $\boldsymbol{u}$ is the controlling vector with m controllers, i.e., $\boldsymbol{u} = (u_1, u_2, \ldots, u_m)^{\mathrm{T}}$, and $\mathbf{B} \in \mathbf{R}^{N\times m}$ is the control matrix. The classic Kalman rank condition stipulates that Eq. (1) can be controlled from any initial state to any final state in finite time if and only if $\mathrm{rank}[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \ldots, \mathbf{A}^{N-1}\mathbf{B}] = N$. Thus, in order to fully control the complex network, we should choose an appropriate $\mathbf{B}$ and $\boldsymbol{u}$. The central goal is to find a matrix $\mathbf{B}$ corresponding to the minimum number N_D of independent drivers or controllers required to control the whole network. According to the results of Liu et al. [6] and Yuan et al. [7], we have

$$N_D = \min\{\mathrm{rank}(\mathbf{B})\} \tag{2}$$

Considering the PBH rank condition [13], for an arbitrary network, Yuan et al. [7] proved that N_D is determined

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by the maximum geometric multiplicity $\mu(\lambda_i)$ of the eigenvalue λ_i of $\mathbf{A}$, i.e.

$$N_D = \max_i\{\mu(\lambda_i)\} \tag{3}$$

where $\mu(\lambda_i) = \dim \mathrm{V}_{\lambda i} = N - \mathrm{rank}(\lambda_i \mathbf{I}_N - \mathbf{A})$ and λ_i ($i = 1, 2, \ldots, l$) are the distinct eigenvalues of $\mathbf{A}$. For a symmetric coupling matrix $\mathbf{A}$, its geometric multiplicity equals to the algebraic multiplicity. Thus, for an undirected networks, N_D is determined by the maximum algebraic multiplicity $\delta(\lambda_i)$ of λ_i [7], i.e.

$$N_D = \max_i\{\delta(\lambda_i)\} \tag{4}$$

Thus, we can calculate the N_D value of an arbitrary network topology using Eqs. (2)-(4). Then, the measure of controllability n_D can be calculated as follows [6]:

$$n_D = N_D/N \tag{5}$$

3. Controllability of Power Grids in Comparison with Classical Complex Network Models

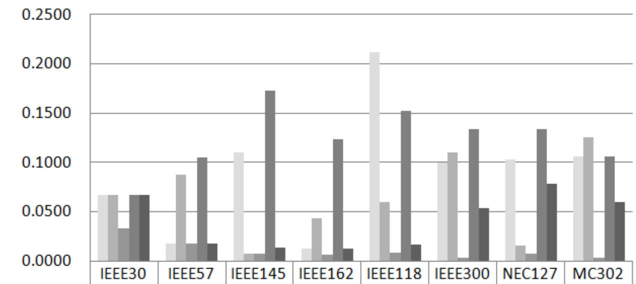
In this Section, we focus on testing the exact controllability values of eight power grids, which are all modeled as undirected and unweighted networks and the number of drivers (N_D) is calculated based on Eq. (3) throughout the paper. The six IEEE power grids are with 30, 57, 118, 145, 162 and 300 nodes respectively. One Chinese power grid is the North-East China (NEC) power grid with 127 nodes, and the other is the Middle China (MC) power grid with 302 nodes. For comparison, we also generate 8 ER random models, 8 WS small-world models, 8 BA scale-free models and 8 CM networks [14] that have the same degree distributions as the power grids.

Firstly, we show some basic topological features of all these models in Table 1. These topological features include the number of nodes N, the number of links M, the average degree $\langle k\rangle$, the clustering coefficient C, the diameter D and the average path length L. In Table 1, INF means infinite if the generated network is not connected. From this table, we can see that most power grids are small-world networks, since their average path lengths are comparable to those of corresponding WS models and their clustering coefficients are far larger than those of corresponding ER models. Secondly, we calculate the exact controllability values of the networks in Table 1, the results are shown in Fig. 1. We can see that IEEE118, IEEE145 and NEC127 have far higher n_D values than their corresponding ER networks, thus they are hard to control. On the contrary, other IEEE power grids together with the MC302 network have n_D values no larger than their corresponding ER networks, thus they are easy to control. Most corresponding BA networks are harder to control than power grids except IEEE118, while WS networks are easier to control than power grids. In addition, the n_D values of power grids are larger than or equal to their corresponding CMs with the same degree distributions, which may imply that degree distribution is the main factor but not the only one that determines networks' controllability. The difference may be attributed to that the average path length of power grids is larger than that of corresponding CMs because real power grids are designed under some spatial constraints such as the positions of power stations and the length of high-voltage transmission lines. Thirdly, we show for each real-world power grid, which nodes are selected as the driver nodes. Here, we only show the results of three networks, i.e., IEEE30, IEEE118 and MC302, as given in Figs. 2-4 respectively. In each figure, all nodes are drawn in sizes in proportion to their nodal degree values, and the square nodes stand for the controllers, while round ones are regular nodes.

Table 1 The topological features of power grids and corresponding ER, WS, BA and CM networks with the same number of nodes.

Network	N	M	<k>	C	D	L
IEEE30	30	41	2.733	0.261	6	3.306
IEEE57	57	78	2.737	0.124	12	4.954
IEEE118	118	179	3.034	0.175	14	6.309
IEEE145	145	422	5.821	0.581	11	4.391
IEEE162	162	280	3.457	0.106	12	5.657
IEEE300	300	409	2.727	0.111	24	9.935
NEC127	127	163	2.567	0.071	16	7.112
MC302	302	396	2.623	0.167	33	12.908
ER30	30	41	2.733	0.133	9	3.175
ER57	57	78	2.737	0.028	12	4.479
ER118	118	181	3.068	0.008	9	4.323
ER145	145	412	5.683	0.0345	6	3.026
ER162	162	283	3.494	0.0108	INF	INF
ER300	300	406	2.707	0.003	INF	INF
ER127	127	178	2.803	0.009	10	4.785
ER302	302	393	2.603	0.008	INF	INF
SW30	30	60	4.000	0.363	5	2.871
SW57	57	114	4.000	0.253	7	3.365
SW118	118	236	4.000	0.262	9	4.441
SW145	145	290	4.000	0.387	11	5.562
SW162	162	324	4.000	0.341	10	5.287
SW300	300	600	4.000	0.251	10	5.306
SW127	127	254	4.000	0.292	9	4.664
SW302	302	604	4.000	0.286	11	5.573
BA30	30	56	3.733	0.172	5	2.506
BA57	57	110	3.860	0.182	5	2.718
BA118	118	232	3.932	0.152	5	2.929
BA145	145	286	3.945	0.0966	5	3.1156
BA162	162	320	3.951	0.0843	6	3.284
BA300	300	596	3.973	0.076	7	3.602
BA127	127	250	3.937	0.127	6	3.110
BA302	302	600	3.974	0.056	7	3.507
CM30	30	41	2.733	0.0222	9	3.526
CM57	57	78	2.737	0.045	10	4.135
CM118	118	179	3.034	0.209	10	4.556
CM145	145	422	5.821	0.0956	7	3.081
CM162	162	280	3.457	0.0155	9	4.312
CM300	300	409	2.727	0.0095	13	5.969
CM127	127	163	2.567	0.0037	13	5.574
CM302	302	396	2.623	0.0169	15	6.004

From Fig. 2, we can see that, for IEEE30, the number of driver nodes is 2, i.e., Node 8 and Node 30, and their degree values are 2 both. Thus the controllability value is $n_D = 2/30 = 0.0667$, and the average degree value of the driver nodes $\langle k\rangle_D = 2$, which is less than the overall average degree value $\langle k\rangle = 2.733$. From Fig. 3, we can see that, for



	IEEE30	IEEE57	IEEE145	IEEE162	IEEE118	IEEE300	NEC127	MC302
n_D	0.0667	0.0175	0.1103	0.0123	0.2119	0.1000	0.1024	0.1060
ER's n_D	0.0667	0.0877	0.0069	0.0432	0.0593	0.1100	0.0157	0.1258
WS's n_D	0.0333	0.0175	0.0069	0.0062	0.0085	0.0033	0.0079	0.0033
BA's n_D	0.0667	0.1052	0.1724	0.1234	0.1525	0.1333	0.1339	0.1060
CM's n_D	0.0667	0.0175	0.0138	0.0123	0.0169	0.0533	0.0787	0.0596

Fig. 1 Comparisons of the controllability values between all power grids and corresponding ER, WS, BA and CM network models.

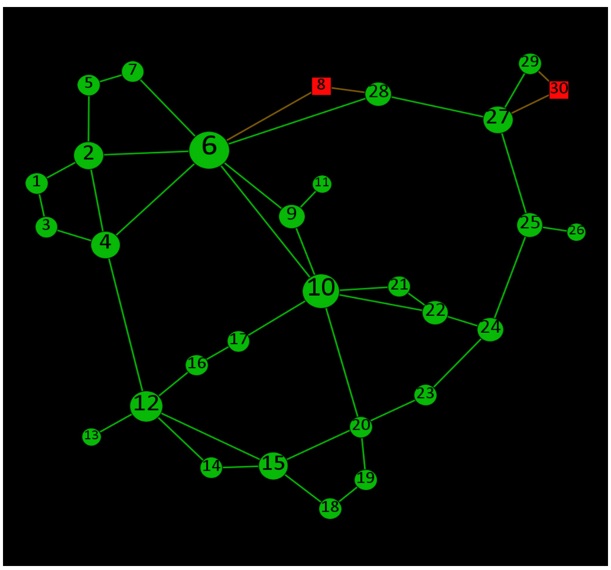


Fig. 2 The topology of the IEEE30 power grid, where squared nodes denote the controllers.

IEEE118, the number of driver nodes is 25 and $\langle k \rangle_D = 2.32$ that is less than $\langle k \rangle = 3.03$, and thus $n_D = 25/118 = 0.2119$. From Fig. 4, we can see that, for MC302, $N_D = 32$ and $\langle k \rangle_D = 1.47$ that is less than $\langle k \rangle = 2.62$, and thus $n_D = 32/302 = 0.106$.

4. Analysis

From Sect. 3, we can see that the IEEE118 power grid requires about 20 percent of the nodes to control the whole network, while NEC127 and MC302 power grids only requires about 10 percent of the nodes. Thus, the IEEE118 power grid is harder to control than Chinese power grids. Furthermore, from Figs. 2-4, we can see that the driver nodes of power grids tend to be low degree nodes but adjacent to high degree nodes. To verify our hypothesis, we compare the average degree of driver nodes, all nodes, neighbors of all nodes, and neighbors of driver nodes as shown in Fig. 5. The results demonstrate $\langle k \rangle_D < \langle k \rangle < \langle k \rangle_{\text{neighbors}} < \langle k \rangle_{\text{neighbors},D}$ for all power grids except IEEE118, revealing that the driver nodes are inclined to be Hub-free nodes, which corresponds to Liu et al.'s conclusion, but to be adjacent to high degree nodes, which has not been noticed before our work. To further characterize the driver nodes, we also investigate the betweenness centrality (BC) and closeness centrality (CC) of driver nodes in Table 2. It shows that the mean BC value of driver nodes is much less than that of overall nodes whereas the mean CC value is comparable to that of overall nodes, which indicates that the driver nodes tend to avoid bottleneck nodes whereas the CC value has no obvious relationship with the controllability.

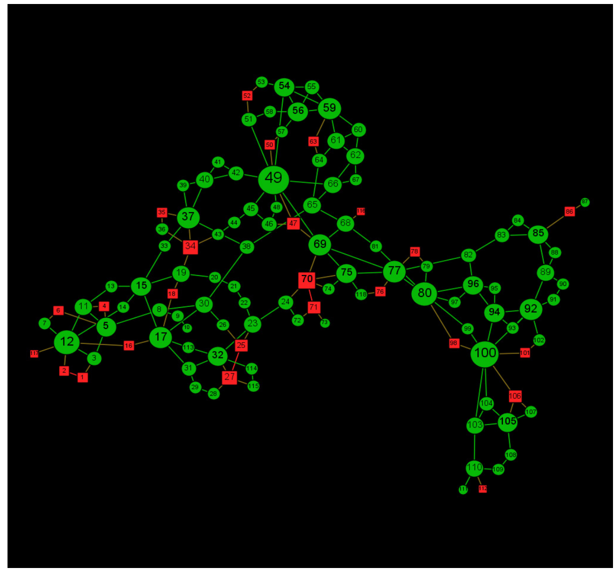


Fig. 3 The topology of the IEEE118 power grid, where squared nodes denote the controllers.

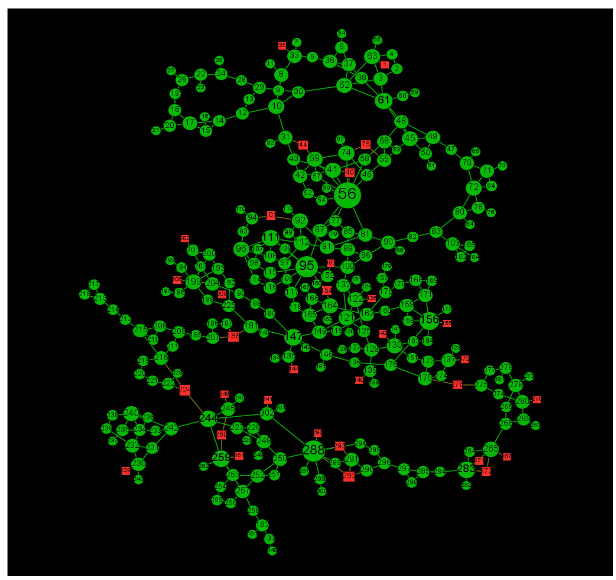


Fig. 4 The topology of the Middle China power grid (MC302), where squared nodes denote the controllers.

Liu et al. have concluded that the major factors that

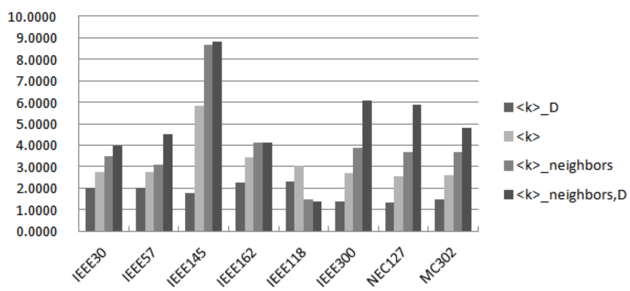


Fig. 5 The comparison of the average degree of driver nodes, all nodes, neighbors of all nodes, and neighbors of driver nodes.

Table 2 The mean betweenness centrality and closeness centrality of driver nodes and the whole network.

	IEEE30	IEEE57	IEEE118	IEEE300	NEC127	MC302
$\langle BC\rangle_D$	0.0000	0.0063	0.0070	0.0049	0.0075	0.0324
$\langle BC\rangle$	0.0823	0.0719	0.0178	0.0299	0.0489	0.0397
$\langle CC\rangle_D$	0.2846	0.1879	0.0541	0.0964	0.1409	0.0777
$\langle CC\rangle$	0.3097	0.2074	0.0517	0.1037	0.1443	0.0801

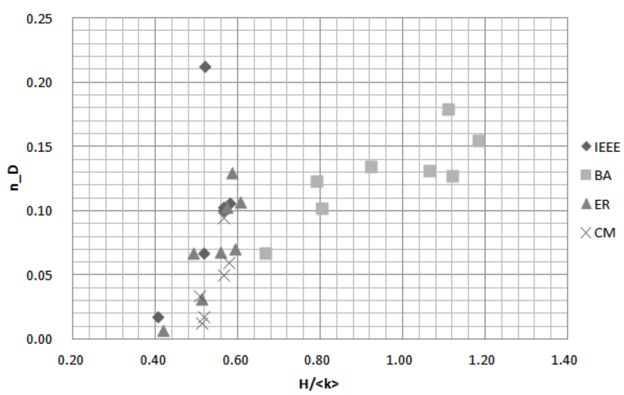


Fig. 6 Plot of n_D as a function of $H/\langle k\rangle$ for power grids and their corresponding ER, BA and CM networks.

influence the controllability are degree distribution and heterogeneity, where the latter can be defined as follows:

$$H = \sqrt{\frac{\sum_i (k_i - \langle k\rangle)^2}{N}} \tag{6}$$

Based on Eq. (6), we can calculate the heterogeneity values for six power grids shown in Table 2. We can see that IEEE118 has the largest heterogeneity, followed by NEC127 and MC302. According to Liu et al.'s conclusion, the controllability of a network is determined by both average degree and heterogeneity, that is, the less average degree a network has, or the more heterogeneity it has, the larger n_D value it has, i.e.

$$n_D \propto \frac{H}{\langle k\rangle} \tag{7}$$

Based on Eq. (7), we can calculate the ratio $H/\langle k\rangle$ as given in Fig. 6. It clearly show the trend that n_D increases with $H/\langle k\rangle$, which well agrees with Liu et al.'s conclusion. However, IEEE118 stands out as an outlier once again; the reason is still under research.

5. Conclusion

This Letter investigates the exact controllability of power grids. We find that most power grids have a nearly power-law degree distribution, showing scale-free properties. For IEEE118, IEEE145 and NEC127 power grids, their n_D values are far higher than those of the corresponding ER networks, and thus they are relatively hard to control, while other IEEE power grids and the MC302 power grid are relatively easy to control since they have n_D values not larger than ER networks. For each power grid, the average degree and betweenness centrality of driver nodes are far less than those of the whole network, which indicates that the driver nodes are inclined to be Hub-free and avoid bottleneck nodes. Finally, the controllability of a network is determined by both average degree and heterogeneity, and sparse heterogeneous power grids are hardest to control.

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