PAPER Special Section on Foundations of Computer Science—Developments of the Theory of Algorithms and Computation—

Visibility Problems for Manhattan Towers*

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SUMMARY A Manhattan tower is a monotone orthogonal polyhedron lying in the halfspace $z \geq 0$ such that (i) its intersection with the xy-plane is a simply connected orthogonal polygon, and (ii) the horizontal cross section at higher levels is nested in that for lower levels. Here, a monotone polyhedron meets each vertical line in a single segment or not at all. We study the computational complexity of finding the minimum number of guards which can observe the side and upper surfaces of a Manhattan tower. It is shown that the vertex-guarding, edge-guarding, and face-guarding problems for Manhattan towers are NP-hard.

key words: *guarding problem, Manhattan towers, NP-hard*

1. Introduction

The art gallery problem is to determine the minimum number of guards who can observe the interior of a gallery. Chvátal [6] proved that $\lfloor n/3 \rfloor$ guards are the lower and upper bounds for this problem; namely, $\lfloor n/3 \rfloor$ guards are always sufficient and sometimes necessary for observing the interior of an n-vertex simple polygon. Lee and Lin [12] studied the computational complexity of the guarding problem. They proved the NP-hardness of finding the minimum number of guards in a given polygon.

An interesting variant of the art gallery problem is the fortress problem, which determines the minimum number of guards who can observe the *exterior* of a polygon. It is known that the lower and upper bounds of vertex guards are $\lceil n/3 \rceil$ for n-vertex polygons [13]. Here, a *vertex guard* is a guard that is only allowed to be placed at the vertices of a polygon. Also, it is known that $\lceil n/4 \rceil + 1$ is both the lower and upper bounds of the vertex guards for n-vertex *orthogonal* polygons [1], [13].

In three dimensions, a similar visibility problem has been considered for n-vertex triangulated polyhedral terrains. It is known that $\lfloor n/2 \rfloor$ is both the lower bound [4] and the upper bound [3] of vertex guards of a polyhedral terrain. Also, the minimum vertex-guard problem is known to be NP-hard [7].

An *edge guard* is a guard that is only allowed to be placed on the edges of a terrain, and the edge guard can move between the endpoints of the edge. For the edge guarding problem for n-vertex triangulated polyhedral terrains, it is known that the lower bound is $\lfloor (4n-4)/13 \rfloor$ [4], the upper bound is $\lfloor n/3 \rfloor$ [3], and the minimum edge-guard problem is NP-hard [2].

A *face guard* is a guard that is allowed to be placed on the faces of a terrain, and the face guard can walk around only on the allocated face. It is known that $\lfloor (2n-5)/7 \rfloor$ is the lower bound and $\lfloor n/3 \rfloor$ is the upper bound for the number of face guards of an n-vertex triangulated polyhedral terrain [10]. After that, both the lower and upper bounds are shown to be $\lfloor (n-1)/3 \rfloor$ [11].

In this paper, we study the computational complexity of the guarding problem for *Manhattan towers* (see Fig. 1). A Manhattan tower is a monotone orthogonal polyhedron lying in the halfspace $z \geq 0$ such that (i) its intersection with the xy-plane is a simply connected orthogonal polygon, and (ii) the horizontal cross section at higher levels is nested in that for lower levels (see Fig. 2). Here, a monotone polyhedron meets each vertical line in a single segment or not at all. We will prove that the vertex-guarding, edge-guarding,

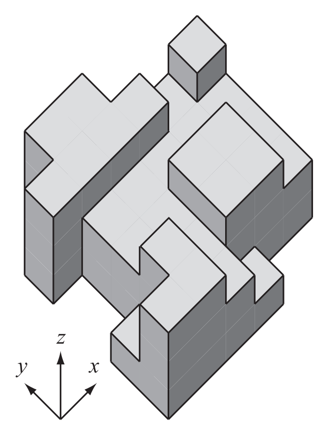

Fig. 1 Manhattan tower T of height 3.

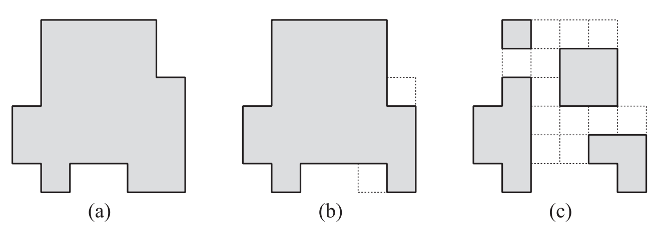

Fig. 2 Horizontal cross sections of Manhattan tower T at height z, where (a) $0 \leq z \leq 1$, (b) $1 < z \leq 2$, and (c) $2 < z \leq 3$.

Manuscript received March 26, 2015.
Manuscript revised June 4, 2015.
Manuscript publicized December 16, 2015.

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*This research was supported in part by Scientific Research Grant, Ministry of Japan.

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DOI: 10.1587/transinf.2015FCP0007

and face-guarding problems for Manhattan towers are NP-hard. A previous work on Manhattan towers is reported by Damian, Flatland, and O'Rourke [8]; they constructed an algorithm for unfolding the surface of a Manhattan tower to a nonoverlapping planar orthogonal polygon.

2. Definitions

The definitions of Manhattan towers and visibility are mostly from [8] and [4], respectively.

Let Z_k be the plane $\{z = k\}$ for $k \geq 0$. Consider an orthogonal polyhedron T. Here, the term polyhedron is used to denote the union of the boundary and of the interior. The orthogonal polyhedron T is said to be a *Manhattan tower* if the following two conditions are satisfied: (i) T lies in the halfspace $z \geq 0$, and its intersection with Z_0 is a simply connected orthogonal polygon. (ii) For $0 \leq k < j$, $T \cap Z_k \supseteq T \cap Z_j$ (namely, the cross section at higher levels is nested in that for lower levels).

Manhattan towers are orthogonal polyhedrons in that they meet each vertical line (parallel to the z-axis) in a single segment or not at all. Thus, they are monotone with respect to the z-axis. Manhattan towers may not be monotone with respect to the x-axis or y-axis. In general, $T \cap Z_k$ has several connected components for $k > 0$ (see Fig. 2(c)), but the base layer of T is simply connected.

In this paper, we assume that each vertex of any Manhattan tower has integral coordinates. Therefore, a Manhattan tower can be illustrated as a *polycube* (see Fig. 3), which is a solid figure formed by joining one or more unit cubes face-to-face.

Two points p and q on the surface of T are said to be *visible* if the line segment pq does not intersect any point strictly inside T. A *vertex guard* is a guard that is only allowed to be placed at the vertices of T.

An *edge guard* is a guard that is only allowed to be placed on the edges of T, and the edge guard can move between the endpoints of the edge. A point p on the surface of T is said to be visible from an edge if there exists a point q on the edge such that p and q are visible.

A *face guard* is a guard that is allowed to be placed

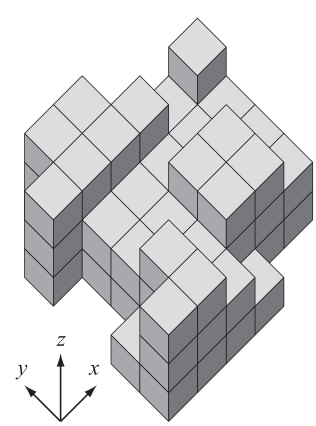


Fig. 3 Manhattan tower illustrated as a polycube.

on a face of T's side and upper surfaces, and the face guard can walk around only on the allocated face. A point p on the surface of T is said to be visible from a face if there exists a point q on the face such that p and q are visible. Thus, the visible region from a face guard always contains the allocated face and its adjacent faces. Here, two faces are said to be adjacent if they share a vertex.

A set of guards is said to *cover* a Manhattan tower T if every point on T's upper and side surfaces is visible from at least one guard in the set. The instance of a guarding problem is a Manhattan tower T and a positive integer k. The vertex (resp. edge, face) guarding problem asks whether there exists a set of vertex (resp. edge, face) guards of size k that covers T.

In Sects. 3, 4, and 5, we will show that the vertex-guarding, edge-guarding, and face-guarding problems for Manhattan towers are NP-hard, respectively.

3. Vertex-Guarding Problem for Manhattan Towers

In this section, we will prove the following theorem.

Theorem 1: The vertex-guarding problem for Manhattan towers is NP-hard.

In Sect. 3.2, we will present a polynomial-time transformation from the instance C of PLANAR 3SAT problem to Manhattan tower T_1 and integer k_1 such that C is satisfiable if and only if there exists a vertex set G_1 of size k_1 that covers T_1.

3.1 PLANAR 3SAT Problem

The definition of PLANAR 3SAT is mostly from [LO1] of [9]. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a set of Boolean *variables*. Boolean variables take on values 0 (false) and 1 (true). If x is a variable in U, then x and $\overline{x}$ are *literals* over U. The value of $\overline{x}$ is 1 (true) if and only if x is 0 (false). A *clause* over U is a set of literals over U, such as $\{\overline{x_1}, x_3, x_4\}$. It represents the disjunction of those literals and is *satisfied* by a truth assignment if and only if at least one of its members is true under that assignment.

An instance of PLANAR 3SAT is a collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses over U such that (i) $|c_j| \leq 3$ for each $c_j \in C$ and (ii) the bipartite graph $G = (V, E)$, where $V = U \cup C$ and E contains exactly those pairs $\{x, c\}$ such that either literal x or $\overline{x}$ belongs to the clause c, is planar.

The PLANAR 3SAT problem asks whether there exists some truth assignment for U that simultaneously satisfies all the clauses in C. This problem is known to be NP-complete. For example, $U = \{x_1, x_2, x_3, x_4\}$, $C = \{c_1, c_2, c_3, c_4\}$, and $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$ provide an instance of PLANAR 3SAT. In this instance, the answer is "yes," since there is a truth assignment $(x_1, x_2, x_3, x_4) = (1, 0, 1, 1)$ satisfying all clauses. It is known that PLANAR 3SAT is NP-complete even if each variable occurs exactly once positively and exactly twice negatively in C [5].

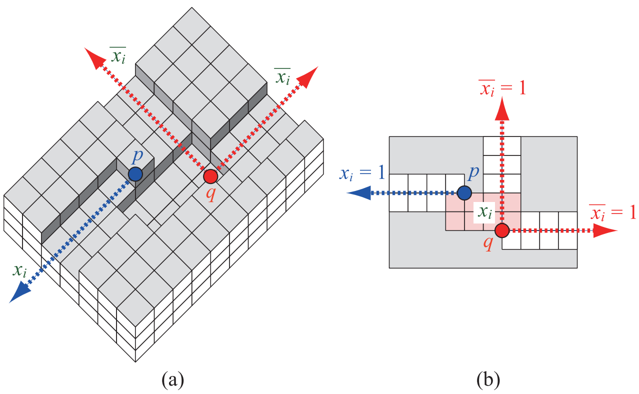


Fig. 4 (a) Variable gadget for x_i. (b) Simplified illustration of (a). If a guard is placed at p (resp. q), then the 2×4 hole and the left channel (resp. upper and right channels) can be observed.

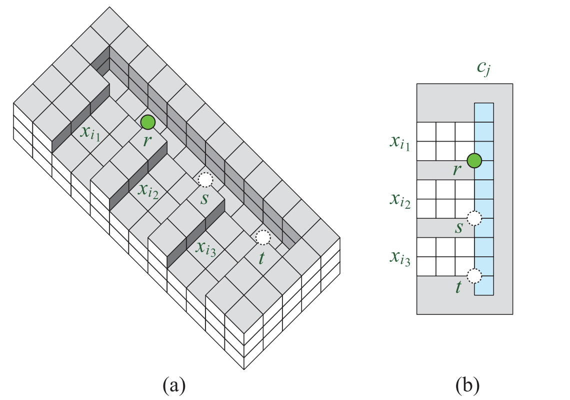


Fig. 5 (a) Clause gadget for $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. (b) Simplified illustration of (a). If a guard is placed at one of the three positions r, s, and t, then the 10×1 hole is observed.

3.2 Transformation from 3SAT-instance to Manhattan Tower

Each variable $x_i \in \{x_1, x_2, \ldots, x_n\}$ is transformed to a *variable gadget* shown in Fig. 4(a). Figure 4(b) is the simplified illustration of Fig. 4(a). This gadget has a 2×4 hole of depth 2, which are connected with two horizontal and one vertical *channels*. Each channel has depth 1 and width 2. The left horizontal channel (see the blue arrow) in Fig. 4 corresponds to literal x_i, and the remaining two channels (with red arrows) correspond to literal $\overline{x_i}$. Note that the instances of 3SAT considered in this paper have the restriction explained just before Sect. 3.2.

In order to observe the interior of the 2×4 hole, we need at least one vertex guard on this gadget. If a guard is placed at blue vertex p (resp. red vertex q) of Fig. 4, then the hole and the left channel (resp. upper and right channels) can be observed. Later, one can see that a blue guard on p (resp. red guard on q) implies $x_i = 1$ (resp. $\overline{x_i} = 1$).

Each clause $c_j \in \{c_1, c_2, \ldots, c_m\}$ is transformed to a *clause gadget* shown in Fig. 5. This gadget has a 10×1 hole of depth 2, which is connected with three horizontal channels. Horizontal channels are labeled with x_{i_1}, x_{i_2}, and x_{i_3} if c_j is composed of those literals.

In order to observe the interior of the 10×1 hole, we need at least one vertex guard on this gadget. If a guard is placed at one of the three positions r, s, and t, then the hole is observed. (If clause c_j consists of two literals (resp. one literal), then the corresponding clause gadget has a 7×1 hole (resp. 4×1 hole) of depth 2, which is connected with two horizontal channels (resp. one horizontal channel).)

Figure 6 is called a *right-and-left turn gadget*. From the shape of the gadget, one can see that two guards are necessary and sufficient for observing the interior of the channel. For example, guards on vertices u and v can observe the channel. Note that if there is a guard g which observes the channel from the right side of the gadget (see Fig. 6(c)), then the two guards can be placed at positions u' and v'. A left-and-right turn gadget is defined similarly. (Yellow cells

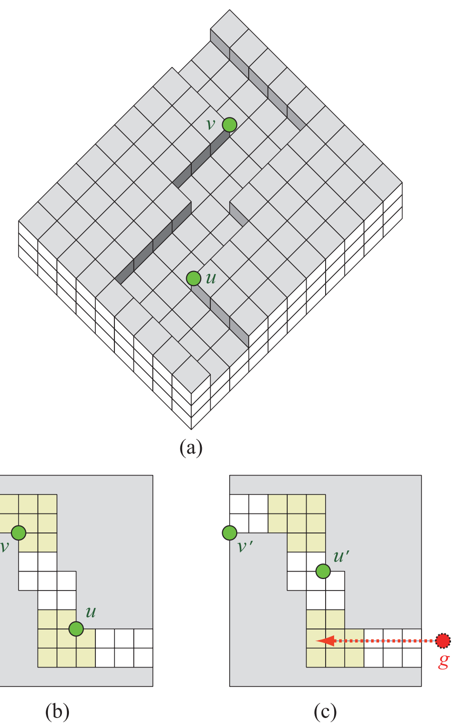
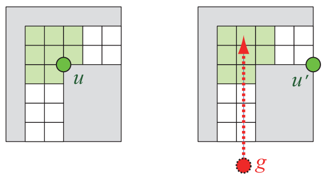


Fig. 6 (a) Right-and-left turn gadget. (b) Simplified illustration of (a). Guard set $\{v, u\}$ can observe the interior of the channel. (c) If there is a guard g which is observing the channel from the right side of the gadget, then the two guards can be placed at positions u' and v'.

of Figs. 6(b) and 6(c) are used later in this section.)

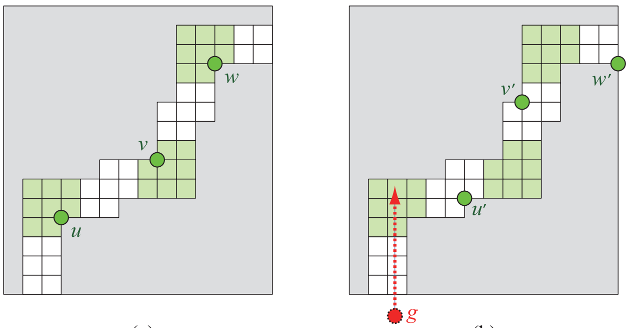
The right-and-left turn gadgets are used for connecting the *horizontal* channels of a variable gadget to clause gadgets. (For example, in Fig. 9, x_4 is connected to c_3 with a right-and-left turn gadget, and to c_2 with a left-and-right turn gadget.) There are several variant forms of the left-and-right turn gadget. (For example, one can find the "right-and-right turn gadget" from x_3 to c_3 in Fig. 9.) In order to connect a variable gadget to a clause gadget, each turn gadget can be stretched.

Figures 7 and 8 are the *right turn* and *right-left-right*



(a) (b)

Fig. 7 Right turn gadget.



(a) (b)

Fig. 8 Right-left-right turn gadget.

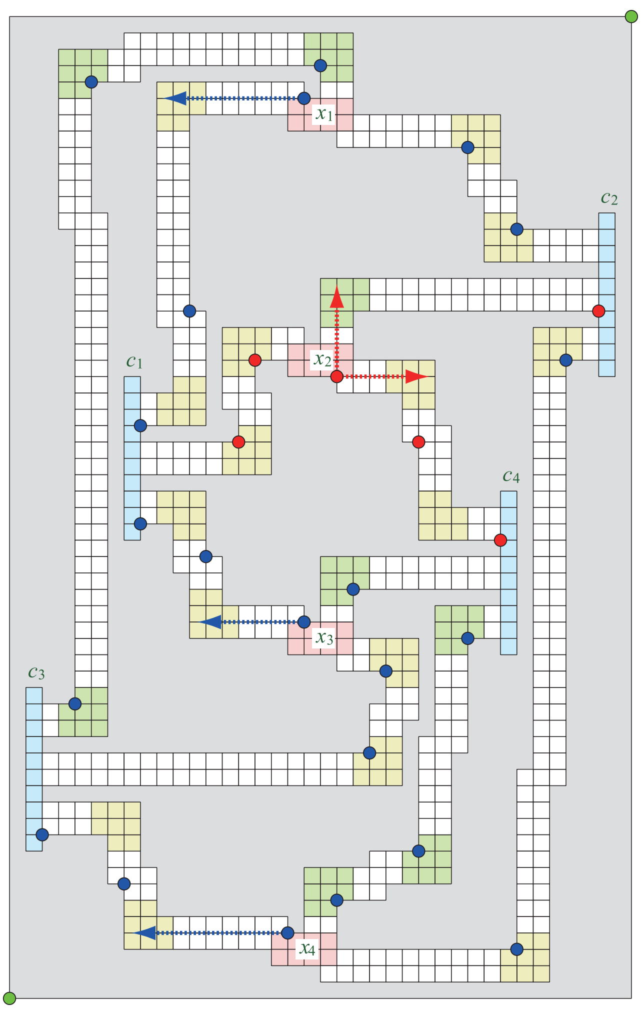


Fig. 9 Manhattan tower T_1 transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. C is satisfiable if and only if Manhattan tower T_1 is covered by $k_1 = 30$ vertex guards.

turn gadgets, which are used for connecting the *vertical* channels of variable gadgets to clause gadgets. (For example, a right turn (resp. right-left-right turn) gadgets can be found between x_3 and c_4 (resp. between x_4 and c_4) in Fig. 9.)

Finally, let $k_1 = n + t_1 + 2$, where n is the number of variables, t_1 is the number of "turns" of the turn gadgets (see green and yellow areas), and the last 2 is the number of guards observing the outer walls (see green vertices at the right-upper and left-lower corners of Fig. 9). From this construction, C is satisfiable if and only if Manhattan tower T_1 is covered by k_1 vertex guards.

Note that k_1 vertex guards can observe all of n red areas (see a variable gadget of Fig. 4(b)), all of t_1 yellow/green areas (see turn gadgets of Figs. 6(b) and 8(a)), and all outer walls. If C is satisfiable, every clause c_j is satisfied by at least one of c_j's literals. Thus, every blue area (see a clause gadget of Fig. 5(b)) is observed by at least one vertex guard. On the other hand, if Manhattan tower T_1 is covered by k_1 vertex guards, then positions of the n vertex guards observing red areas indicate the truth assignment for $(x_1, x_2, \ldots, x_n)$ which satisfies all clauses $\{c_1, c_2, \ldots, c_m\}$.

Figure 9 is a Manhattan tower T_1 transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. One can see that T_1 can be observed by k_1 guards, where $k_1 = n+t_1+2 = 4+24+2 = 30$. From the positions of the four guards observing 2×4 holes, one can see that $(x_1, x_2, x_3, x_4) = (1, 0, 1, 1)$ satisfies all the clauses.

4. Edge-Guarding Problem for Manhattan Towers

An *edge guard* is a guard that is only allowed to be placed on the edges of a terrain, and the edge guard can move between the endpoints of the edge.

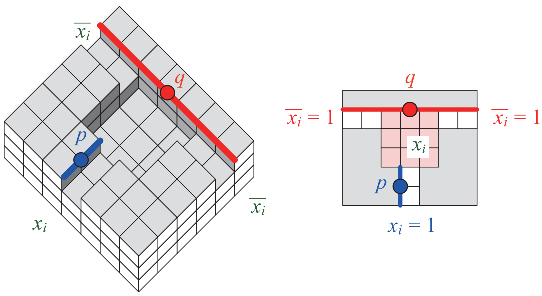
Theorem 2: The edge-guarding problem for Manhattan towers is NP-hard.

In the following, we will present a polynomial-time transformation from the instance C of PLANAR 3SAT problem to Manhattan tower T_2 and integer k_2 such that C is satisfiable if and only if there exists an edge set G_2 of size k_2 that covers T_2.

The outline of the proof is similar to the previous section. Each variable $x_i \in \{x_1, x_2, \ldots, x_n\}$ is transformed to a *variable gadget* shown in Fig. 10. This gadget has a 3×3 hole of depth 2, which are connected with two horizontal and one vertical channels. Channels are of depth 1 and width 1. The vertical channel corresponds to literal x_i, and the two horizontal channels correspond to literal $\overline{x_i}$.

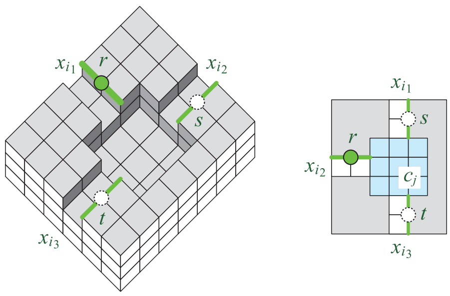
In order to observe the interior of the 3×3 hole, we need at least one edge guard on this gadget. If a guard is placed on blue edge p (resp. red edge q) of Fig. 10, then the hole and the vertical channel (resp. horizontal channels) can be observed.

Each clause $c_j \in \{c_1, c_2, \ldots, c_m\}$ is transformed to a *clause gadget* shown in Fig. 11. This gadget has a 3×3 hole of depth 2, which is connected with three channels. In order



(a) (b)

Fig. 10 (a) Variable gadget for x_i. (b) Simplified illustration of (a).



(a) (b)

Fig. 11 (a) Clause gadget for $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. (b) Simplified illustration of (a). If a guard is placed on one of the three edges r, s, and t, then the 3×3 hole is observed.

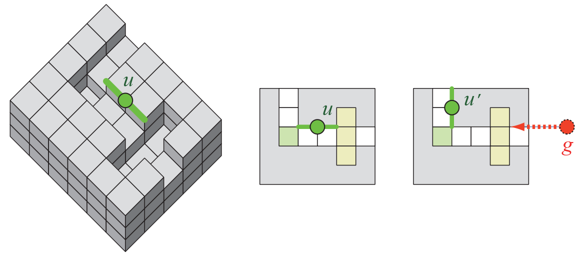
to observe the interior of the 3×3 hole, we need at least one edge guard on this gadget. If a guard is placed on one of the three edges r, s, and t, then the hole is observed. (If clause c_j consists of two literals (resp. one literal), then the 3×3 hole of depth 2 is connected with two channels (resp. one channel).)

Figure 12 is a right-turn gadget, which has a 3×1 hole and a 1×1 hole of depth 2. A left turn gadget is defined similarly. Suppose that literal x_i appears in c_j, and $\overline{x_i}$ appears in c_k and c_l (see Fig. 13). We connect variable gadget x_i to three clause gadgets c_j, c_k, and c_l with right-turn and left-turn gadgets. In order to connect a variable gadget to a clause gadget, each turn gadget can be stretched (see Fig. 14).

In Fig. 13(a), if a guard is placed on edge p for observing the 3×3 hole x_i, then two guards can be placed on edges r and s. The guard on edge s can observe the hole c_j. In this case, two guards must be placed on edges t and u; thus hole c_k or c_l cannot be observed by any guard in the figure.

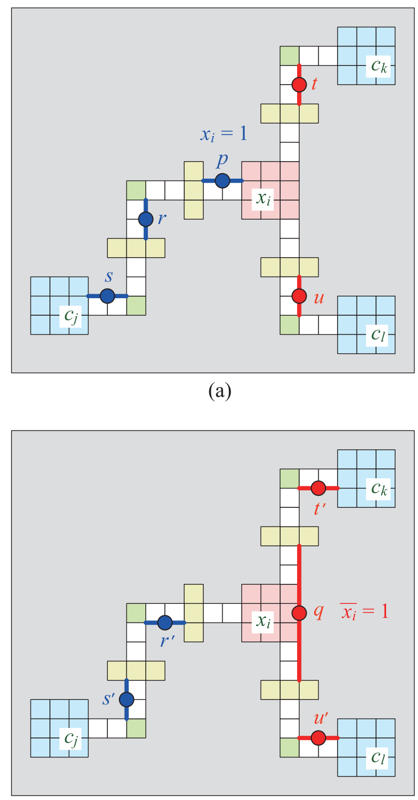
In Fig. 13(b), on the other hand, if a guard is placed on edge q, then two guards can be placed on edges t' and u'. Thus, those two guards can observe holes c_k and c_l. In this case, hole c_j cannot be observed by any guard in the figure.

Finally, let $k_2 = n + t_2 + 2$, where n is the number of variables, t_2 is the number of "turns" of the left-turn and right-turn gadgets, and the last 2 is the number of guards which observe the outer walls (see green edges at the top and bottom of Fig. 14). From this construction, C is satisfiable



(a) (b) (c)

Fig. 12 (a) Right turn gadget. (b) Simplified illustration of (a). This gadget can be observed by one guard u. (c) If there is a guard g which can move to the border of the 3×1 hole (to observe its interior), then a guard can be placed on edge u'. Left turn gadget is defined similarly.



(a)

(b)

Fig. 13 Suppose literal x_i appears in c_j, and $\overline{x_i}$ appears in c_k and c_l. (a) If a guard is placed on edge p for observing the 3×3 hole, then guard s can observe hole c_j. (b) If a guard is placed on edge q, then guards t' and u' can observe holes c_k and c_l, respectively.

if and only if Manhattan tower T_2 is covered by k_2 edge guards.

Note that k_2 edge guards can observe all of n red areas (see a variable gadget of Fig. 10(b)), all of t_2 green holes (see turn gadgets of Fig. 12(b)), and all outer walls. If C is satisfiable, every clause c_j is satisfied by at least one of c_j's literals. Thus, every blue area (see a clause gadget of Figs. 11(b) and 13) is observed by at least one edge guard. On the other hand, if Manhattan tower T_2 is covered by k_2 edge guards, then positions of the n edge guards observing red areas indicate the truth assignment for $(x_1, x_2, \ldots, x_n)$ which satisfies all clauses $\{c_1, c_2, \ldots, c_m\}$.

Figure 14 is a Manhattan tower T_2 transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. One can see that T_2 can be observed by k_2 guards, where $k_2 = n + t_2 + 2 = 4 + 18 + 2 = 24$.

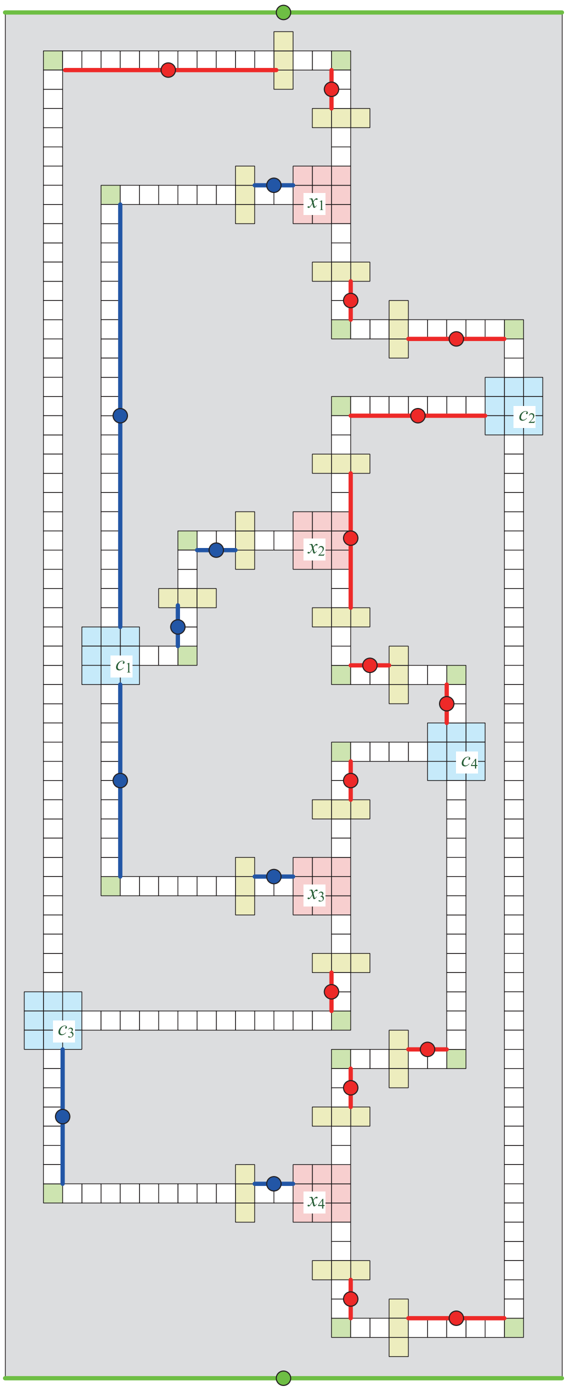


Fig. 14 Manhattan tower T_2 transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. C is satisfiable if and only if Manhattan tower T_2 is covered by $k_2 = 24$ edge guards.

5. Face-Guarding Problem for Manhattan Towers

A *face guard* is a guard that is allowed to be placed on the faces of a terrain, and the face guard can walk around only on the allocated face. The visible region from a face guard always contains the allocated face and its adjacent faces, where two faces are said to be adjacent if they share a vertex.

Theorem 3: The face-guarding problem for Manhattan towers is NP-hard.

In the following, we will present a polynomial-time transformation from the instance C of PLANAR 3SAT problem to Manhattan tower T_3 and integer k_3 such that C is satisfiable if and only if there exists a face set G_3 of size k_3 that covers T_3.

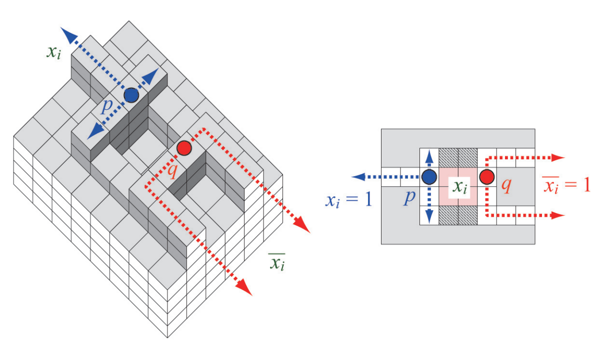
Each variable $x_i \in \{x_1, x_2, \ldots, x_n\}$ is transformed to a *variable gadget* shown in Fig. 15. This gadget has a *T-shaped wall* and a *U-shaped wall* of height 2 (see blue and red arrows in the figure), which are connected with two 1×2 walls of height 1. This gadget also has a 2×2 holes (see the red area labeled with x_i of Fig. 15(b)).

In order to observe the interior of the 2×2 hole, we need at least one face guard on this gadget. If a guard is placed at the blue node p or red node q of Fig. 15, then the 2×2 hole can be observed.

Each clause $c_j \in \{c_1, c_2, \ldots, c_m\}$ is transformed to a *clause gadget* shown in Fig. 16. This gadget has a 4×4 *square-shaped wall* of height 1 and three walls of height 2. In the square-shaped wall, there is a 2×2 hole of depth 2. If a face guard is placed at one of the three nodes r, s, and t, then the 2×2 hole is observed. (If clause c_j consists of two literals (resp. one literal), then the 4×4 square-shaped wall of height 1 has two walls (resp. one wall) of height 2.)

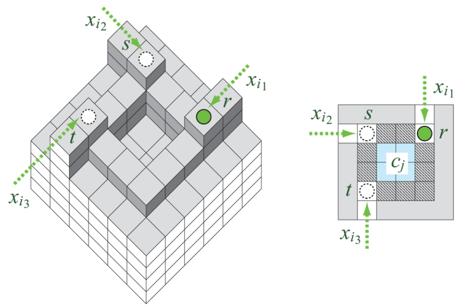
Variable gadgets are connected to clause gadgets with walls of height 2 (see white cells of Fig. 17). Here a simple but important property is that connections between variable and clause gadgets can be turned arbitrarily based on the property of the face guards.

For simplicity, we explain the construction of Manhattan tower T_3 by using the following 3SAT-instance $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$,



(a) (b)

Fig. 15 (a) Variable gadget for x_i. (b) Simplified illustration of (a).



(a) (b)

Fig. 16 (a) Clause gadget for $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. (b) Simplified illustration of (a). If a guard is placed on one of the three positions r, s, and t, then the 2×2 hole of depth 2 is observed.

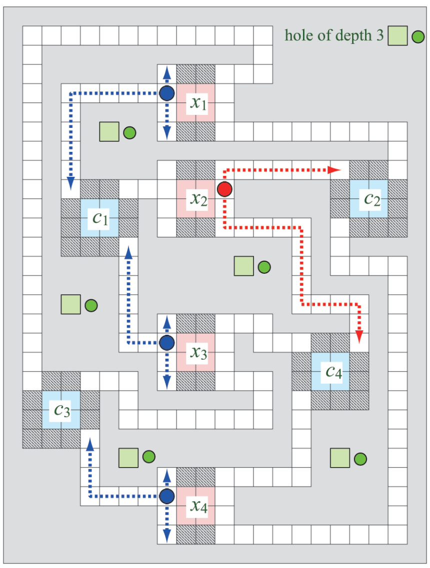


Fig. 17 Manhattan tower T_3 transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. Each of the six gray areas has a green square hole of depth 3. Six green nodes are guards which observe the interior of the green square holes and all side faces of walls. C is satisfiable if and only if Manhattan tower T_3 is covered by $k_3 = 10$ face guards.

$c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$.

Figure 17 is a Manhattan tower T_3 transformed from C. In each of the six gray areas of the figure, there are a *green square hole* of depth 3 and a *green guard*. Note that the bottom of the green square hole of depth 3 cannot be observed by any guard on the walls. The six green guards observe the interior of the green square holes and all side faces of walls. One of the green guards (see the upper right green guard of Fig. 17) observes the four side faces of the outer gray area.

Finally, let $k_3 = n + t_3$ $(= 4 + 6 = 10)$, where n is the number of variables, and t_3 is the number of green square holes. From this construction, C is satisfiable if and only if Manhattan tower T_3 is covered by k_3 face guards.

Note that k_3 vertex guards can observe all of n red areas (see a variable gadget of Fig. 15(b)) and all of t_3 green holes (see Fig. 17). If C is satisfiable, every clause c_j is satisfied by at least one of c_j's literals. Thus, every blue area (see a clause gadget of Fig. 16(b)) is observed by at least one guard. On the other hand, if Manhattan tower T_3 is covered by k_3 face guards, then positions of the n face guards observing red areas indicate the truth assignment for $(x_1, x_2, \ldots, x_n)$ which satisfies all clauses $\{c_1, c_2, \ldots, c_m\}$.

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