LETTER

An Investigation of Learner's Actions in Posing Arithmetic Word Problem on an Interactive Learning Environment

Ahmad Afif SUPIANTO[†a)], ***Nonmember***, **Yusuke HAYASHI**[†], ***and*** **Tsukasa HIRASHIMA**[†], ***Members***

SUMMARY This study investigates whether learners consider constraints while posing arithmetic word problems. Through log data from an interactive learning environment, we analyzed actions of 39 first grade elementary school students and conducted correlation analysis between the frequency of actions and validity of actions. The results show that the learners consider constraints while posing arithmetic word problems.

key words: *problem-posing activity, arithmetic word problems, elementary school students, learning analytics*

1. Introduction

An aim of mathematics education is to develop problem-posing skills for learners [1]. Problem-posing is a demanding process that requires critical thinking, evaluation, and reflection. Problem-posing involves generating new problems aimed at exploring a given situation as well as reformulating a problem during the course of solving a related problem [2]. Monsakun is an interactive learning system that encourages learners in exercising problem-posing of arithmetic word problems [3]. The system asks learners to arrange and integrate five or six presented sentence cards into a problem which is consists of three sentence cards. Based on previous studies, practice in problem-posing also improves problem-solving skills. In this paper, we examine learners' activity during problem-posing through the system's log data.

From the viewpoint of interactive learning, Chang et al. analyzed the effect of the system on the problem-posing abilities by looking at the pre- and post-test scores [4]. Furthermore, Kojima et al. evaluated the learning support through examples by checking the posed problems [5]. There has been considerable work investigating learners' activity throughout the learning process. Fournier-Viger et al. extracted patterns from learners' solutions in problem-solving exercises [6]. Hsieh et al. identified higher and lower engagement patterns to represent students' learning processes in a game-based learning environment in order to perform resource classification ability [7]. However, the central issue in such research is basically limited to problem-solving and does not include problem-posing. In a series of two analyses in a group of learners, we analyze problem-posing processes of Japanese first-grade elementary school students in an actual class.

The basis of Monsakun is triplet structure model [8] that defines the structure of an arithmetic word problem as sentence-integration. This model proposes an arithmetic word problem solved by one arithmetical operation which is an integration of three sentences representing numerical concepts. Previous research in this direction analyze the pre- and post-test scores, it suggests that the practice of problem-posing on the system improves the learners' ability not only in problem-posing but also in problem-solving [9]. Moreover, the system improves the problem-solving ability of low-performing students [10]. Supianto et al. [11] proposed a method to visualize learners' actions from Monsakun log data. In contrast, this study analyzes the problem-posing process based on the Monsakun model. Therefore, this study combines the Monsakun model and Monsakun log data. This study investigates learners' actions based on the Monsakun model focused on satisfying the constraints. Our assumption in this study is that learners consider the constraints while they pose the problems.

2. Problem-Posing in Monsakun

Problem-posing in Monsakun can be described as a series of composition of sentence card(s) in the three available card slots. Learners pose problems by composing given sentences and Monsakun requires that learners continue to pose problems until they can pose the problem satisfying the requirement. Consequently, learners make many compositions of sentence card(s) in the three available card slots.

Monsakun records problem-posing activity as changes in compositions of sentence card(s). We called the composition as a "state". The example of two states in Level 5 Assignment 1 is shown in Fig. 1. An assignment in here is a task for learners to pose a single problem. The requirement of the Level 5 Assignment 1 is to make a story problem about "How many are there overall" that can be solved by "8 – 3," which is an arithmetic word problem with a combination story type. When the state is composed of three sentence cards, then it is called the "posed problem state," which is the card slots is completely arranged (see Fig. 1 (b)). Whereas when the arrangement is not composed of three sentence cards, then it is called the "intermediate state," which is in the process of posing the problem (see Fig. 1 (a)). For the investigation purpose, the sentence cards are encoded with indexing number (see Fig. 1 (c)). When the slot is still empty, index = 0 is implemented. Therefore,

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[†]The authors are with Graduate School of Engineering, Hiroshima University, Higashihiroshima-shi, 739–0046 Japan.
a) E-mail: afif@lel.hiroshima-u.ac.jp
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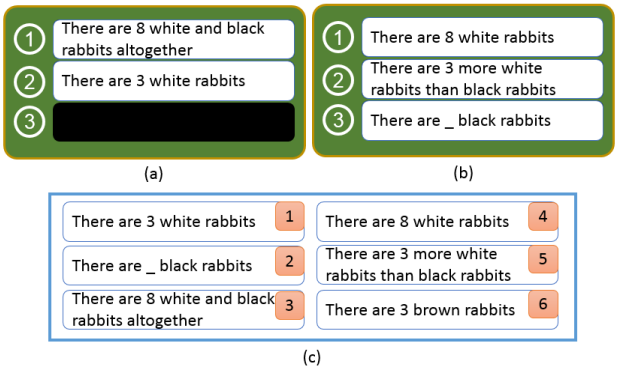

Fig. 1 Example of states in level 5 assignment 1. (a) State 310. (b) State 452. (c) Available sentence cards and their indexes.

according to the indexes, the states in Fig. 1 (a) and Fig. 1 (b) are encoded as State 310 and State 452, respectively.

To complete an assignment, learners continue to make combinations of sentence cards until they pose the required problem. The number of combinations of given five sentence cards is 136 combinations. The combination includes index=0 as empty slot. It can occurs one time such as 310, 201, or 014, and two times such as 300, 010, or 002. As for the given six sentence cards, there are 229 combinations. Therefore, it is almost impossible to complete the required problem at random. If learner's understanding of problem structure is incomplete, the learner may use unnecessary sentence cards. However, such combination might not be totally meaningless if at least one constraint is satisfied. This means the learner knows and can consider the constraint.

3. Analysis of Activity in Term of Constraints

Analyses of problem-posing activity from Monsakun log data of thirty-nine first grade elementary school students is reported. There are five levels in Monsakun. All levels are the same in terms of posing problems from a card set, but they have different requirements. Level 1 – 4 provide the numerical formula of the story, while level 5 is required to consider the unknown number. There are four story types in the level: *combination* story (assignment 1 – 3), *increase* story (assignment 4 – 6), *decrease* story (assignment 7 – 9), and *comparison* story (assignment 10 – 12).

The average of actions and mistake in level 5 were significantly different compared to the others. The mean and the standard deviation of actions of level 1 – 5 are 7.53 (2.28), 6.42 (2.86), 6.92 (2.31), 6.05 (1.62), and 42.01 (28.59), respectively. While the mean and the standard deviation of mistakes of level 1 – 5 are 0.48 (0.33), 0.49 (0.42), 0.64 (0.49), 0.57 (0.23), and 8.16 (6.95), respectively. It shows that level 5 was very challenging for learners. In this study, we investigated learner's actions in the intermediate and the posed problem states at level 5.

Based on the triplet structure model, posing problem can be defined as the task to compose sentences satisfying five constraints [11], they are 1) calculation, which is the numerical expression representing the story type; 2) story type, which is one of the four available story types: combination, increase, decrease, and comparison, and it should be identified in the requirement; 3) number, which is the quantity in the sentence, 4) objects, which is the entity in the sentence; and 5) sentence structure, which is the composition of sentences that must consist of two independent quantity sentences and one relative quantity sentence. When less than five constraints are satisfied, the posed problem is not valid; that is, the problem cannot be solved, or it is not the required one.

Table 1 Average actions and correlation result between frequency of states appearance and validity of states in level 5.

Asg.	Average actions	Pearson's Correlation	p-value
1	50.08	0.087	0.584
2	13.32	0.507 **	0.001
3	9.24	0.581 **	0.002
4	97.18	0.224 **	0.009
5	32.00	0.375 **	7.040E-06
6	60.57	0.270 **	0.001
7	92.39	0.238 **	0.005
8	36.10	0.344 **	4.105E-05
9	54.71	0.384 **	3.878E-06
10	26.88	0.417 *	0.034
11	12.41	0.784 **	8.520E-10
12	19.27	0.631 **	7.617E-06

**: significant difference ($p<0.01$), *: significant difference ($p<0.05$)

The validity of a state is measured based on the number of satisfied constraints. Actually, we can only calculate how many numbers are satisfied when the composition of sentence cards are completely arranged. For instance, in the case of six available sentence cards, we can calculate the validity of State 124 = 1 because of only number constraint is satisfied or the validity of State 134 = 3 because of story type, object, and sentence structure constraints are satisfied. Therefore, in order to cover the measurement of the intermediate states, we measure the validity by calculating the average of their descendant states. For instance, the validity of State 012 is obtained from the average of the validity of State 123, State 124, State 125, and State 126.

3.1 Analysis of Learners' Actions in Level 5

The analysis results of level 5 including the average number of actions to pose problems correctly and the correlation result between the frequency of states appearance and the validity of states are shown in Table 1.

From the average actions, it denotes that level 5 is more difficult than the other levels. Ideally, learners only need three actions to reach the correct answer, because a problem consists of 3 sentence cards. However, in average, they need at least 9 actions (assignment 3) and at most 97 actions (assignment 4).

We conduct a Pearson's correlation test between the frequency of states appearance and the validity of states.

The frequency shows how many numbers of unique states have been reached by learners in order to pose a required problem in one assignment. We assume that the degree of correlation is related to the degree of learners' understanding. Therefore, there is a positive correlation between the validity and the frequency when learners understand the structure and aware of it in problem-posing.

Significant correlation at $p < 0.05$ in eleven out of twelve assignments is found, which shows that many actions performed by learners had an inclination to satisfy as many constraints as possible. Hence, this finding confirms that learners consider thinking about the structure of arithmetic word problem during pose the required problem in Monsakun.

In addition, we pay attention to the difference of degree of correlations among assignments. Basically, significant correlation shows that learners are aware of the structure of arithmetic word problems. However, the correlations in assignments 4 – 9, that are *increase* and *decrease* story types, are weak. Although the minimum actions to get the correct answer is three actions, learners take the average actions ten times or more from the minimum actions. This means their understanding about *increase* and *decrease* story types is worse than *combination* and *comparison* story types. Actually, *increase* and *decrease* story types have severe constraints and require a strict order of sentences in posed problems [11]. We consider that this is a reason why the assignments of *increase* and *decrease* story types have a significant but weak correlation. That is, learners had misunderstood and attempted to pose problems with the misunderstanding for a long time. This finding shows that the learners were trying to satisfy the constraints when they constructed their answer because they produced many mistakes while doing the required problem.

Furthermore, the result of correlation in assignment 1 shows no significant correlation ($p > 0.05$). In order to examine more detail in assignment 1, further analysis is observed in the next sub-section.

3.2 Analysis of Transition in Problem-Posing Activity at Level 5 Assignment 1

In this analysis, we discuss the correlation between the change of satisfied constraints and the transition of states. Here, a transition is an action of learners from one state to another state. We check the difference of the validity of the states. If the difference is positive, it means that learners have a tendency to make more valid compositions of sentence cards.

The result is shown in Fig. 2. The transitions presented by circle shapes and labels. The red dotted lines labeled $\mu + 2\sigma$ indicates the position of double standard deviation from the average, which means that transitions were plotted more than $\mu + 2\sigma$ are the transitions frequently occurred. The red colored dots focus on state frequently arranged by learners. From the result, we can see that three out of five frequent transitions show positive differences of the validity of states.

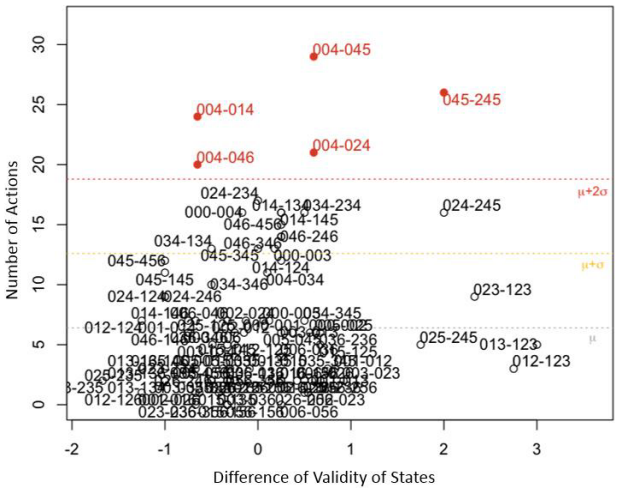

Fig. 2 Scatterplot of correlation of transitions in Level 5 Assignment 1.

Table 2 The validity of several states in assignment 1.

States	All Constraints	C1	C2	C3	C4	C5
004	1.60	0.10	0.00	0.40	0.50	0.60
014	1.00	0.00	0.00	0.25	0.25	0.50
024	2.50	0.25	0.00	0.75	1.00	0.50
034	1.50	0.00	0.00	0.25	0.50	0.75
045	2.00	0.25	0.00	0.50	0.50	0.75
046	1.00	0.00	0.00	0.25	0.25	0.50

C1: calculation, C2: story type, C3: Number, C4: Object, C5: Sentence structure

It indicates that learners tried to improve the validity of their card composition.

Next, we investigate two frequent transitions with negative differences of the validity of states, Transition 004-014 and 004-046. We calculate the validity of the states for each constraint. We show that there is a possibility the learners tried to pose the problem looking at some several constraints, instead of all constraints.

In satisfying the particular constraints, learners attempted to pose more valid intermediate states. Based on the validity of several states listed in Table 2, if the learners have a comprehensive understanding of all constraints, then the validity of State 014 and State 046 is 1.00. Comparing the value with the other states, the validity is relatively low. However, when the learners have a partial understanding, the narrowly focused on some constraints, then the validity becomes different. For example, when a learner takes particular note of "number", "object" and "sentence structure", there is a big difference between these two states and the others. On the other hand, if they mainly focus on "calculation" or "story type", the values are not so different. In such case, it is difficult for learners to distinguish the State 014 and State 046 from the others. Also, the possibility to choose them is not low. This means that the learners' intention in composing State 014 and State 046 is reasonable when it is viewed from several constraints they want to satisfy.

The difficulty of this assignment is that learners are

confused about the gap between the required story type (*combination*) and the numerical expression of subtraction (8 − 3) in the requirement of assignment 1: *Make a word problem about "How many are there overall" that can be solved by "8 − 3"*. Although subtraction implies story type of *decrease* and *comparison*, in this case, learners must pose a problem of *combination* story type. The above assumption that they might mainly focus on "calculation" or "story type" is related to this difficulty. This conflict of "calculation" and "story type" implied from the other can be considered as the cause of the difficulty.

At previous levels, there was no conflict at the required story type and numerical expression. Also, the order of numbers in sentences was the same as the numerical expression. Learners pose the required problem by arranging sentences according to the order of numbers in the numerical expression. However, this is not valid for level 5 because the numerical expression does not express the order of numbers in the required story but the solution is to evaluate unknown number. Learners meet this situation in the first assignment at level 5. This is considered a reason of the unexpected behaviors where there is no significant correlation and there are some frequent transitions with negative differences in the validity. Despite the difficulty encountered in level 5, the analysis shows that the learners had a tendency to enhance the validity of the intermediate states to achieve the correct answers. Therefore, this analysis confirms that learners consider the constraints while they pose arithmetic word problem on Monsakun.

4. Conclusion and Future Works

We conduct an analysis of problem-posing activity from Monsakun log data of elementary school students to investigate their actions while posing arithmetic word problems. The assumption in this study is that they consider the structure of arithmetic word problems as sentence-integration through satisfying as many constraints as possible. The result shows that learners had an inclination to pose the required problem with an awareness of the structure of arithmetic word problems throughout the problem-posing process.

For future research, we plan to analyze in more detail the characteristics of learners' thinking process. Because it is necessary to investigate which constraints learners actually pay attention in a variety of situations.

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