LETTER

Research on Analytical Solution Tensor Voting

Hongbin LIN[†a)], ***Member***, **Zheng WU**[†], **Dong LEI**[†], **Wei WANG**[†], ***and*** **Xiuping PENG**[††], ***Nonmembers***

SUMMARY This letter presents a novel tensor voting mechanism — analytic tensor voting (ATV), to get rid of the difficulties in original tensor voting, especially the efficiency. One of the main advantages is its explicit voting formulations, which benefit the completion of tensor voting theory and computational efficiency. Firstly, new decaying function was designed following the basic spirit of decaying function in original tensor voting (OTV). Secondly, analytic stick tensor voting (ASTV) was formulated using the new decaying function. Thirdly, analytic plate and ball tensor voting (APTV, ABTV) were formulated through controllable stick tensor construction and tensorial integration. These make the each voting of tensor can be computed by several non-iterative matrix operations, improving the efficiency of tensor voting remarkably. Experimental results validate the effectiveness of proposed method.

key words: *perceptual grouping, tensor voting, analytic tensor voting, salient structure inference, structural information propagation*

1. Introduction

Tensor voting is a computational framework that addresses the problem of perceptual organization. It was originally proposed by Medioni and colleagues [1], to convey human perception principles into a unified framework that can be adapted to extract visually salient elements from possibly noisy or corrupted data. In the past 20 years, tensor voting has been proven versatile since its successfully adaptation to various problems like contour and surface inferences, stereo-matching and image processing, and so on.

Despite its effectiveness, tensor voting has not been widely used in applications where efficiency is an issue, due to the high computational cost of its traditional implementation. In the past 20 years, many efforts have been made to improve its efficiency, e.g. [2] discard part of the votes for the sake of efficiency. Moreno et al [3] proposed two alternative formulations of tensor voting based on numerical approximations of the votes, to reduce the high computational complexity while keeping the same perceptual meaning of the original tensor voting. [4] attempt to propose a closed-form solution to tensor voting (CFTV), which does not require numerical integration. But, as pointed in [3], [5], their methods yield very different values from those obtained through the original tensor voting.

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[†]The authors are with the School of Electrical Engineering, Yanshan University, Qinhuangdao 066004, China.
[††]The author is with the School of Information Science and Engineering, Yanshan University, Qinhuangdao 066004, China.
a) E-mail: honphin@ysu.edu.cn
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This letter presents a novel analytic tensor voting mechanism, deriving explicit formulations for stick, plate and ball tensor voting. Rest of this letter is organized as follows: Sect. 2 presents a new decaying function following basic spirit of original decaying function. Section 3 provides and proves the analytic stick, plate and ball tensor voting (ASTV, APTV and ABTV). Section 4 shows an experimental comparison between the popular tensor voting methods and the proposed method. Finally, Sect. 5 concludes the obtained results and makes some final remarks.

2. Tensor Voting

The voting of 2-order symmetric tensor from point $\mathbf{q}$ to neighboring point $\mathbf{p}$ is expressed by:

$$\mathrm{TV}(\mathbf{p}) = \sum_{\mathbf{q}\in neigh(\mathbf{p})} \left(\mathrm{SV}(\mathbf{v},\mathbf{S}_q) + \mathrm{PV}(\mathbf{v},\mathbf{P}_q) + \mathrm{BV}(\mathbf{v},\mathbf{B}_q)\right) \tag{1}$$

where

$$\begin{cases} \mathbf{S}_q = \lambda_{Sq}\mathbf{e}_1\mathbf{e}_1^T \\ \mathbf{P}_q = \lambda_{Pq}\left(\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_2\mathbf{e}_2^T\right) \\ \mathbf{B}_q = \lambda_{Bq}\left(\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_2\mathbf{e}_2^T + \mathbf{e}_3\mathbf{e}_3^T\right) \end{cases} \tag{2}$$

are the stick, plate, and ball tensor components respectively, $\lambda_{Sq} = \lambda_1 - \lambda_2, \lambda_{Pq} = \lambda_2 - \lambda_3$ and $\lambda_{Bq} = \lambda_3$, are the corresponding saliencies of tensor at the voter $\mathbf{q}$, $\mathbf{v} = \mathbf{p} - \mathbf{q}$, SV, PV and BV represent the stick, plate and ball voting of corresponding tensor components. λ_i and $\mathbf{e}_i$ are the ith largest eigenvalue and its corresponding eigenvector of the tensor at the voter, respectively.

2.1 Stick Tensor Voting

Stick tensors are used to encode the orientation information of the surface normal at a specific point. Tensor voting propagates surfaceness in a neighborhood by voting of a stick tensor based on Gestalt psychology [1]. Basic principle of stick tensor voting is shown in Fig. 1. Let $\mathbf{S}_q$ be a stick tensor at voter $\mathbf{q}$, the tensor received at votee $\mathbf{p}$ can be computed by

$$\mathrm{SV}(\mathbf{S}_q,\mathbf{v}) = \mathrm{DF}(\mathbf{S}_q,\mathbf{v},\sigma)\mathbf{R}_{2\theta}\mathbf{S}_q\mathbf{R}_{2\theta}^T \tag{3}$$

where $\mathrm{DF}(\mathbf{S}_q,\mathbf{v},\sigma)$ is the decaying function, $\mathbf{R}_{2\theta}$ is the rotation, θ is the angle shown in Fig. 1.

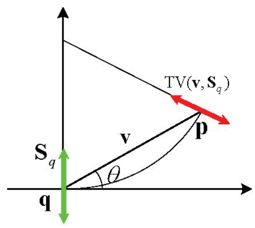


Fig. 1 Principle of stick tensor voting.

2.2 Plate Tensor Voting

Plate tensors are used to encode the orientation information of the curve tangent at a specific point. Tensor voting propagates curveness in a neighborhood by voting of a plate tensor. Unlikely from stick tensor voting, plate votes are computed in a constructive way. Let $\mathbf{P}_q$ be a plate tensor at voter $\mathbf{q}$, the tensor received at votee $\mathbf{p}$ can be computed by:

$$\mathrm{PV}(\mathbf{P}_q, \mathbf{v}) = \frac{\lambda_{Pq}}{\pi} \int_0^{2\pi} \mathrm{SV}(\mathbf{v}, \mathbf{S}_{P_q}(\alpha)) \mathrm{d}\alpha \tag{4}$$

where λ_{Pq} is the saliency of plate components at voter, $\mathbf{S}_{P_q}(\alpha)$ is a stick tensor constructed by $\mathbf{P}_q$.

2.3 Ball Tensor Voting

Ball tensors are used to encode junctionness or noiseness. Ball tensor voting is also defined in a constructive way. Let $\mathbf{B}_q$ be a ball tensor at voter $\mathbf{q}$, the tensor received at votee $\mathbf{p}$ can be computed by:

$$\mathrm{BV}(\mathbf{B}_q, \mathbf{v}) = \frac{3\lambda_{1b}}{4\pi} \int_\Omega \mathrm{SV}(\mathbf{v}, \mathbf{S}_B(\beta, \gamma)) \mathrm{d}\Omega \tag{5}$$

where Ω is the surface of the unitary sphere, $\mathbf{S}_{B_q}(\beta, \gamma)$ is a stick tensor constructed by $\mathbf{B}_q$.

Original tensor voting is highly time consuming since (4) and (5) cannot be analytically simplified in original mechanism. This letter presents the analytical tensor voting formulations based on novel tensor voting mechanisms.

3. The Decaying Function

In original tensor voting, the decaying function used to reduce the strength of the vote, which was defined by

$$\mathrm{DF}(\mathbf{S}_q, \mathbf{v}, \sigma) = \mathrm{e}^{-\frac{s^2 + b\kappa^2}{\sigma^2}} \tag{6}$$

where $s = \frac{\theta \|\mathbf{v}\|}{\sin\theta}$ is the arc length, $\kappa = \frac{2\sin\theta}{\|\mathbf{v}\|}$ is the curvature, b is a user defined parameter to weight the curvature, and σ is the scale parameter, which determines the effective neighborhood size.

In fact, $\mathrm{DF}(\mathbf{S}_q, \mathbf{v}, \sigma)$ was defined to penalize votes by both distance and curvature. Following this spirit, [3] used normalized curvature $\overline{\kappa} = \sin\theta$ instead of curvature κ. Following this spirit, we redefine the decaying function as

$$\mathrm{DF}_2(\mathbf{S}_q, \mathbf{v}, \sigma) = f \cdot \left(1 - \frac{\mathbf{v}^T \mathbf{S}_q \mathbf{v}}{\lambda_{S_q} \mathbf{v}^T \mathbf{v}}\right). \tag{7}$$

where $f = \mathrm{e}^{-\frac{\mathbf{v}^T\mathbf{v}}{\sigma^2}}$, $\frac{\mathbf{v}^T \mathbf{S}_q \mathbf{v}}{\lambda_{S_q} \mathbf{v}^T \mathbf{v}} = \sin^2\theta$ is the square normalized curvature $\overline{\kappa}$. (7) means penalizing votes by both distance and square normalized curvature.

4. Analytic Tensor Voting

The complexity of stick tensor voting mainly stems from the computation of angle θ and the decaying function. Despite the efficiency of stick tensor voting, it is rather difficult to derive analytic solution to plate and ball tensor voting using original definition of decaying function. Thus, we employ the newly defined decaying function in (7) to formulate the analytic stick, plate and ball tensor voting.

4.1 Analytic Stick Tensor Voting

Let $\mathbf{S}_q = \lambda_{Sq} \mathbf{e}_1 \mathbf{e}_1^{\mathrm{T}}$ be a stick tensor at $\mathbf{q}$ (where $\mathbf{e}_1$ is a unitary vector), according to (3), stick tensor vote comprised of computation of decaying function and tensor rotation. In original tensor voting, the tensor rotation is computed by $\mathbf{R}_{2\theta} \mathbf{S}_q \mathbf{R}_{2\theta}^T$, which involves a time consuming computation of angle θ. In this letter, a geometrical method is employed. Let's re-examine Fig. 1 from geometric view, the rotation in (3) could be expressed as:

$$\mathbf{R}_{2\theta} = \mathbf{R} = \mathbf{I} - 2\frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}} \tag{8}$$

Thus the analytic solution to stick tensor voting, which is named as ASTV(analytic stick tensor voting) would be:

$$\mathrm{ASTV}(\mathbf{v}, \mathbf{S}_q) = \mathrm{DF}_2(\mathbf{S}_q, \mathbf{v}, \sigma) \mathbf{R} \mathbf{S}_q \mathbf{R}^T = f \mathbf{R} \mathbf{H}_{\mathbf{S}_q} \mathbf{R}^T \tag{9}$$

and $\mathbf{H}_{\mathbf{S}_q}$ is obtained by simple substitution, which is

$$\mathbf{H}_{\mathbf{S}_q} = \mathbf{S}_q - \frac{\mathbf{v}^{\mathrm{T}} \mathbf{S_q} \mathbf{v} \mathbf{S_q}}{\lambda_{S_q} \mathbf{v}^{\mathrm{T}} \mathbf{v}} \tag{10}$$

It can be concluded that (9) and (10) hold the same properties as that in original stick tensor voting defined in (3) on the aspects of decaying function and direction transformation.

4.2 Analytic Plate Tensor Voting

Let $\mathbf{P}_q = \lambda_{Pq}\left(\mathbf{e}_1\mathbf{e}_1^{\mathrm{T}} + \mathbf{e}_2\mathbf{e}_2^{\mathrm{T}}\right)$ be a plate tensor at point $\mathbf{q}$ (where $\mathbf{e}_1$ and $\mathbf{e}_2$ be orthogonal unitary vectors). Following the constructive way, a unitary vector $\mathbf{e}_\theta$ was constructed. The construction should fulfill constraints below:

(1) $\mathbf{e}_\theta$ should lay on the plane spanned by $\mathbf{e}_1$ and $\mathbf{e}_2$;
(2) $\mathbf{P}_q$ could be yielded through mathematical expectation of stick tensor $\mathbf{S}_{Pq}(\theta) = \mathbf{e}_\theta \mathbf{e}_\theta^T$;
(3) $\mathbf{e}_\theta$ should be rotatable around the circle.

In this letter, $\mathbf{e}_\theta$ was constructed simply by

$$\mathbf{e}_\theta = \mathbf{e}_1 \sin\theta + \mathbf{e}_2 \cos\theta. \tag{11}$$

where θ be uniform distributions with $\theta \in [0, 2\pi]$. Hence,

$$\mathbf{P}_q = \lambda'_{Pq} \int_0^{2\pi} \mathbf{S}_q(\theta) f(\theta) d\theta = \frac{\lambda'_{Pq}}{2} \left(\mathbf{e}_1\mathbf{e}_1^{\mathrm{T}} + \mathbf{e}_2\mathbf{e}_2^{\mathrm{T}} \right) \tag{12}$$

where $\lambda'_{Pq} = 2\lambda_{Pq}, \mathbf{S}_q(\theta) = \mathbf{e}_\theta\mathbf{e}_\theta^T, f(\theta) = \frac{1}{2\pi}$.

Thus, the analytic solution to plate tensor voting, which is named-as APTV(analytic plate tensor voting) would be

$$\begin{aligned} \mathrm{APTV}(\mathbf{P}_q, \mathbf{v}) &= 2\lambda_{Pq} \int_0^{2\pi} \mathrm{ASTV}(\mathbf{S}_q(\theta), \mathbf{v}) f(\theta) d\theta \\ &= c\mathbf{R}\mathbf{H}_{Pq}\mathbf{R}^T \end{aligned} \tag{13}$$

where

$$\begin{aligned} \mathbf{H}_{Pq} &= 2\lambda_{Pq} \int_0^{2\pi} \left[\mathbf{S}_q(\theta) - \frac{\mathbf{v}^T\mathbf{S}_q(\theta)\mathbf{v}\mathbf{S}_q(\theta)}{\lambda_{SPq}(\theta)\mathbf{v}^T\mathbf{v}} \right] f(\theta) d\theta \\ &= \mathbf{P}_q - \frac{1}{4\lambda_{Pq}\mathbf{v}^T\mathbf{v}} \left(\mathbf{v}^T\mathbf{P}_q\mathbf{v}\mathbf{P}_q + 2\mathbf{P}_q\mathbf{v}\mathbf{v}^T\mathbf{P}_q \right). \end{aligned} \tag{14}$$

It can be concluded that (13) and (14) hold the same properties as that in original plate tensor voting defined in (4).

4.3 Analytic Ball Tensor Voting

Let $\mathbf{B}_q = \lambda_{Bq} \left(\mathbf{e}_1\mathbf{e}_1^{\mathrm{T}} + \mathbf{e}_2\mathbf{e}_2^{\mathrm{T}} + +\mathbf{e}_3\mathbf{e}_3^{\mathrm{T}} \right)$ be a ball tensor at $\mathbf{q}$ (where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ be orthogonal unitary vectors). Following the constructive way, a unitary vector $\mathbf{e}_{u,\theta}$ was constructed. The construction must fulfill constraints below:

(1) $\mathbf{e}_{u,\theta}$ must be spanned by $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$;
(2) $\mathbf{B}_q$ could be yield through mathematical expectation of stick tensor $\mathbf{S}_{Bq}(\theta) = \mathbf{e}_{u,\theta}\mathbf{e}_{u,\theta}^T$;
(3) $\mathbf{e}_{u,\theta}$ must be rotatable around the unitary sphere.

Let's express above constraints in form below:

$$\mathbf{e}_{u,\theta} = x(u,\theta)\mathbf{e}_1 + y(u,\theta)\mathbf{e}_2 + z(u,\theta)\mathbf{e}_3 \tag{15}$$

where $x(u,\theta), y(u,\theta)$ and $z(u,\theta)$ fulfill:

$$\left\{ \begin{array}{l} x^2(u,\theta) + y^2(u,\theta) + z^2(u,\theta) = 1 \\ \int_{u,\theta} x^2(u,\theta) f(u,\theta) du d\theta = c \\ \int_{u,\theta} y^2(u,\theta) f(u,\theta) du d\theta = c \\ \int_{u,\theta} z^2(u,\theta) f(u,\theta) du d\theta = c \\ \int_{u,\theta} x(u,\theta) y(u,\theta) f(u,\theta) du d\theta = 0 \\ \int_{u,\theta} y(u,\theta) z(u,\theta) f(u,\theta) du d\theta = 0 \\ \int_{u,\theta} z(u,\theta) x(u,\theta) f(u,\theta) du d\theta = 0 \end{array} \right. \tag{16}$$

where c is a constant, $f(u,\theta)$ is the joint probability density function of random variable u and θ.

We construct $x(u,\theta), y(u,\theta)$ and $z(u,\theta)$ by

$$\left\{ \begin{array}{l} x(u,\theta) = \sqrt{1-u^2}\cos\theta \\ y(u,\theta) = \sqrt{1-u^2}\sin\theta \\ z(u,\theta) = u \end{array} \right. \tag{17}$$

where u and θ are independent uniform distribution, with $u \in [-1, 1]$ and $\theta \in [0, 2\pi]$. It can be validated that (17) fulfills constraints in (16), with $c = \frac{1}{3}$. Hence

$$\begin{aligned} \mathbf{B}_q &= \lambda'_{Bq} \int_{u,\theta} \mathbf{S}_{Bq}(u,\theta) f(u,\theta) du d\theta \\ &= \frac{\lambda'_{Bq}}{3} \left(\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_2\mathbf{e}_2^T + \mathbf{e}_3\mathbf{e}_3^T \right) \end{aligned} \tag{18}$$

where $\lambda'_{Bq} = 3\lambda_{Bq}, \mathbf{S}_{Bq}(u,\theta) = \mathbf{e}_{u,\theta}\mathbf{e}_{u,\theta}^T, f(u,\theta) = \frac{1}{4\pi}$.

Thus, the analytic solution to ball tensor voting, named as ABTV(analytic ball tensor voting) would be

$$\begin{aligned} &\mathrm{ABTV}(\mathbf{B}_q, \mathbf{v}) \\ &= 3\lambda_{Bq} \int_{u=-1}^{1} \int_{\theta=0}^{2\pi} \mathrm{ASTV}(\mathbf{S}_{Bq}(u,\theta), \mathbf{v}) f(u,\theta) du d\theta \\ &= c\mathbf{R}\mathbf{H}_{Bq}\mathbf{R}^T \end{aligned} \tag{19}$$

where

$$\begin{aligned} \mathbf{H}_{Bq} &= 3\lambda_{Bq} \int_{u=-1}^{1} \int_{\theta=0}^{2\pi} \left[\mathbf{S}_{Bq}(u,\theta) - \frac{\mathbf{v}^T\mathbf{S}_{Bq}(u,\theta)\mathbf{v}\mathbf{S}_{Bq}(u,\theta)}{\lambda_{SBq}(u,\theta)\mathbf{v}^T\mathbf{v}} \right] f(u,\theta) du d\theta \\ &= \mathbf{B}_q - \frac{1}{5\lambda_{Bq}\mathbf{v}^T\mathbf{v}} \left(\mathbf{v}^T\mathbf{B}_q\mathbf{v}\mathbf{B}_q + 2\mathbf{B}_q\mathbf{v}\mathbf{v}^T\mathbf{B}_q \right). \end{aligned} \tag{20}$$

It can be concluded that (19) and (20) hold the same properties as that in original plate tensor voting defined in (5).

As a result, ASTV (defined in (9) and (10)), APTV (defined in (13) and (14)) and ABTV (defined in (19) and (20)) guarantee each voting of tensor could be compute through several matrix operations, avoiding the time consuming numerical integration in original plate or ball tensor voting.

5. Experimental Results

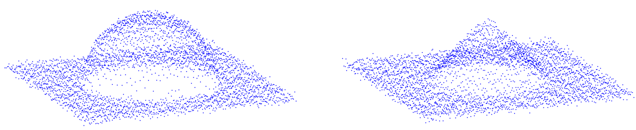
5.1 Efficiency

Formulations were coded in Matlab on Intel Core 2 Quad Q6600 with a 4GB RAM. Efficiencies of ATV are examined and compared with efficient (ETV in [3]), simplified tensor voting (STV in [3]) and MM (in [6]) for their better efficiency respect to OTV.

Averaged running time of ETV, STV, MM and proposed ATV were examined and summarized in Table 1. An observation with respect to this table is ETV, STV and MM yield similar performance, taking nearly 0.05, 0.17, and 0.04 milliseconds for per stick, plate, and ball vote, respectively. While, the proposed ATV takes nearly 0.01, 0.02 and 0.005 milliseconds for every stick, plate, and ball vote, respectively. Especially, the proposed APTV yields significant improvement in efficiency with respect to comparing methods,

Table 1 Speed comparison of tensor voting methods (in ms per vote)

Methods	Stick Votes	Plate Votes	Ball Votes
ETV	0.0524	0.1845	0.0537
STV	0.0511	0.1493	0.0392
MM	—	0.2036	0.0397
ATV	0.0094	0.0216	0.0047



(a) (b)

Fig. 2 Point cloud used in the experiments (each with 3,721 points and Gaussian noise with standard deviation of 0.2). (a) Point-sampled surfaces of semisphere. (b) Point-sampled surfaces of cone.

Table 2 Mean angular error of $\mathbf{e}_1$ and $\mathbf{e}_3$ in degrees for the data sets of Fig. 2

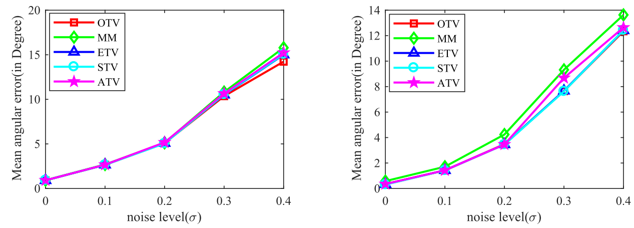
Methods	Semisphere		Cone	
	$\mathbf{e}_1$	$\mathbf{e}_3$	$\mathbf{e}_1$	$\mathbf{e}_3$
OTV	5.02	2.54	5.09	3.48
MM	5.09	3.20	5.12	4.24
ETV	5.01	2.53	5.09	3.46
STV	**4.96**	**2.51**	**5.05**	**3.45**
ATV	5.15	2.72	5.17	3.46

which guarantees more efficient curved structure inference in structural analysis. The efficiency of ATV mainly benefits from the analytic tensor voting formulation, which guaranteed each voting of tensor could be compute through several vectors, matrix and exponential operations, avoiding the time consuming computation of arcsine for angle θ, as well as that of s'_{ip}, in ETV and STV, which is usually fitted by exponential functions using precomputed results by OTV.

5.2 Accuracy

In order to examine the accuracy of normal and tangent estimation for surface and curve points respectively, different tensor voting techniques were applied to synthetic data sets shown in Fig. 2. Accuracies were examined by comparing the groundtruth with the results of obtained by different tensor voting techniques.

For evaluating the accuracy of normal and curve tangent estimation of different tensor voting techniques, mean angular error between $\mathbf{e}_1$ and ideal normals on surface points, $\mathbf{e}_3$ and ideal tangent at edge points have been used to measure the accuracy, respectively. Tables 2 summarize the results for data sets in Fig. 2, which shows that all methods yielded similar accuracy for used noisy data sets. In addition, different noise level of data sets were used to examine the accuracies of the tensor voting techniques, experimental results were shown in Fig. 3. As can be seen, all tested methods performed similar accuracy descendence as noise get larger. The proposed performed better than MM, but slightly worse than OTV, ETV and STV (no more than 1 degree larger mean angular error than OTV for normal and tangent estimation respectively). This accuracy loss partially comes the proposed decaying function in (7), which is not identical to that used in OTV, although they follows same spirit. Especially, for ATV, the $\frac{\pi}{4}$ cut-off, which is of-



(a) (b)

Fig. 3 Mean angular error of $\mathbf{e}_1$ and $\mathbf{e}_3$ for different noise level of cone. (a) Mean angular error of $\mathbf{e}_1$ for different noise level of cone. (b) Mean angular error of $\mathbf{e}_3$ for different noise level of cone.

ten used in most tensor voting techniques, is omitted for analytic derivation. The authors think this level of accuracy loss is affordable for most structural inference applications.

6. Concluding Remarks

In this letter, a novel tensor voting method — analytic tensor voting was proposed. A new decaying function was designed and explicit formulations for voting of stick, plate and ball tensor were derived mathematically. These make the tensor voting process can be accomplished through several non-iterative matrix operations, which improves the tensor voting efficiency notably. Experimental results on synthetic data sets validated that the proposed ATV can yield promising performance on efficiency with affordable accuracy loss. Extra efforts can be made on the extension of proposed ATV to higher dimensions.

Acknowledgments

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