

LETTER

Facial Expression Recognition via Regression-Based Robust Locality Preserving Projections*

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SUMMARY In this paper, we present a novel regression-based robust locality preserving projections (RRLPP) method to effectively deal with the issue of noise and occlusion in facial expression recognition. Similar to robust principal component analysis (RPCA) and robust regression (RR) approach, the basic idea of the presented RRLPP approach is also to lead in the low-rank term and the sparse term of facial expression image sample matrix to simultaneously overcome the shortcoming of the locality preserving projections (LPP) method and enhance the robustness of facial expression recognition. However, RRLPP is a nonlinear robust subspace method which can effectively describe the local structure of facial expression images. The test results on the Multi-PIE facial expression database indicate that the RRLPP method can effectively eliminate the noise and the occlusion problem of facial expression images, and it also can achieve better or comparative facial expression recognition rate compared to the non-robust and robust subspace methods meantime.

key words: facial expression recognition, regression-based robust locality preserving projections (RRLPP), augmented Lagrangian multiplier

1. Introduction

Facial expression recognition is a significant research area which has widespread applications such as children's social emotion ability identification, human-computer interaction (HCI) and film-making [1]. In facial expression recognition, there are some pivotal issues including facial expression data collection, emotion feature extraction, illumination variation, noise and occlusion which are worth studying to improve the facial expression recognition rate. Among

those pivotal issues, the noise and the occlusion are two frequently encountered problems and often influence the performance of facial expression recognition seriously. For example, due to the influence of data collection equipment, illumination, hair, glasses and other occlusions, the collected facial expression images may carry a part of noise and distort more or less [2]–[6]. Therefore, the study of the noise and occlusion problem is very significant for facial expression recognition and it will improve the practicability in actual environment.

In recent years, some researchers present some valid robust subspace methods to degree the influence of the noise and occlusion problem in the application of bimodal emotion recognition [7], face recognition [2], [8], gesture estimation [3], [4], motion segmentation [9], [10] and action unit alignment [5]. Among those robust subspace methods, robust principal component analysis (RPCA) [9], [10] and robust regression (RR) [3], [4] are the two most representative approaches which are both based on the low-rank sparse modal [6]. RPCA is proposed by Wright et al. [9], [10] which can effectively deal with the noise and occlusion problem by decomposing the original data sample matrix into the term of low-rank and sparse. To effectively make use of the category information, Huang et al. [3], [4] propose the RR method on the base of the RPCA method and the regression modal and obtain the better recognition result. The RPCA and the RR method are both linear robust subspace method and they can not describe the local manifold structure as the locality preserving projections (LPP) method [1], [11].

Aroused by the RPCA [9], [10], RR [3], [4] and other method [5]–[8], [12], we present a novel regression-based robust locality preserving projections (RRLPP) method to effectively deal with the issue of noise and occlusion in facial expression recognition on the base of the regression-based locality preserving projections method [13]. Similar to the RPCA and the RR approach, the basic idea of the presented RRLPP approach is also to lead in the low-rank term and the sparse term [3], [4], [9], [10] of facial expression image sample matrix to simultaneously overcome the shortcoming of the LPP method and enhance the robustness of facial expression recognition. However, the RRLPP approach can effectively describe the local manifold structure [11] of facial expression images compared to the RPCA and the RR approach.

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2. Regression-Based Robust Locality Preserving Projections

According to the literature of [1] and [13], the conventional LPP approach [11] can be expressed in minimizing the regression modal of

$$\|[\psi(\mathbf{M}) - \mathbf{B}\mathbf{E}^T\mathbf{M}]\mathbf{W}^{\frac{1}{2}}\|_F^2, \quad (1)$$

where $\mathbf{M} \in \mathbf{R}^{p \times N}$ denotes the facial image matrix, $\psi(\mathbf{M}) \in \mathbf{R}^{p^\phi \times N}$ denotes the mapped facial image matrix, $\mathbf{E} \in \mathbf{R}^{p \times c}$ is the projection matrix of the LPP method, $\mathbf{B} \in \mathbf{R}^{p^\phi \times c}$ is the projection matrix of $\psi(\mathbf{M})$, p and p^ϕ denote the dimension of the facial image matrix and the mapped facial image matrix respectively, N and c denote the number of the facial image and projection vectors respectively [1], [14], $\mathbf{W}_{ii} = \sum_j \mathbf{D}_{ij}$, where \mathbf{D}_{ij} is denoted as [1], [11], [13], [15]

$$\mathbf{D}_{ij} = \begin{cases} e^{-\frac{\|\mathbf{m}_i - \mathbf{m}_j\|_2^2}{\sigma}}, & \text{if } \mathbf{m}_i \text{ and } \mathbf{m}_j \text{ are among KNN} \\ & \text{of each other} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Aroused by the RPCA [9], [10], RR [3], [4] and other method [5]–[8], [12], [13], the RRLPP method is expressed in minimizing the regression modal of

$$\begin{aligned} \arg \min_{\mathbf{M}_L, \mathbf{M}_S, \mathbf{E}, \mathbf{B}} & \|[\psi(\mathbf{M}) - \mathbf{B}\mathbf{E}^T\mathbf{M}]\mathbf{W}^{\frac{1}{2}}\|_F^2 + \text{rank}(\mathbf{M}_L) \\ & + \alpha_{MS} \|\mathbf{M}_S\|_0, \\ \text{s.t. } & \mathbf{M} = \mathbf{M}_L + \mathbf{M}_S \end{aligned} \quad (3)$$

where \mathbf{M}_L and \mathbf{M}_S is denoted as the low-rank term and the sparse term of the facial image matrix \mathbf{M} respectively [3], [4], [8]–[10], α_{MS} is the sparse parameter of the sparse term \mathbf{M}_S [1], [14]. The same to the RPCA [9], [10] and the RR [3], [4] method, the formula of (3) also can be written as the regression modal of (4)

$$\begin{aligned} \arg \min_{\mathbf{M}_L, \mathbf{M}_S, \mathbf{E}, \mathbf{B}} & \|[\psi(\mathbf{M}) - \mathbf{B}\mathbf{E}^T\mathbf{M}]\mathbf{W}^{\frac{1}{2}}\|_F^2 + \|\mathbf{M}_L\|_* \\ & + \alpha_{MS} \|\mathbf{M}_S\|_1. \\ \text{s.t. } & \mathbf{M} = \mathbf{M}_L + \mathbf{M}_S \end{aligned} \quad (4)$$

As the method used in the literature of [1], [3], [4], [14], [16] and [17], we set $\hat{\mathbf{M}}_L = \mathbf{M}_L$ and then obtain the following formula of (5)

$$\begin{aligned} \arg \min_{\mathbf{M}_L, \hat{\mathbf{M}}_L, \mathbf{M}_S, \mathbf{E}, \mathbf{B}} & \|[\psi(\mathbf{M}) - \mathbf{B}\mathbf{E}^T\mathbf{M}]\mathbf{W}^{\frac{1}{2}}\|_F^2 + \|\mathbf{M}_L\|_* \\ & + \alpha_{MS} \|\mathbf{M}_S\|_1 + \text{tr}[\zeta_{ML}^T (\hat{\mathbf{M}}_L - \mathbf{M}_L)] \\ & + \text{tr}[\zeta_{MS}^T (\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)] \\ & + \frac{\beta_{MS}}{2} \|\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S\|_F^2 \\ & + \frac{\beta_{ML}}{2} \|\hat{\mathbf{M}}_L - \mathbf{M}_L\|_F^2, \end{aligned} \quad (5)$$

where $\beta_{ML} > 0$, $\beta_{MS} > 0$, ζ_{MS} and ζ_{ML} all denote the Lagrange multiplier of the corresponding matrix term [1],

[3], [4], [8], [14].

We can rewrite \mathbf{B} as $\mathbf{B} = \psi(\mathbf{M})\mathbf{A}$ on the basis of [1], [13], [14], and substitute it into the formula of (5), then we obtain the following formula of (6)

$$\begin{aligned} \arg \min_{\mathbf{M}_L, \hat{\mathbf{M}}_L, \mathbf{M}_S, \mathbf{E}, \mathbf{A}} & \|[\psi(\mathbf{M}) - \psi(\mathbf{M})\mathbf{A}\mathbf{E}^T\mathbf{M}]\mathbf{W}^{\frac{1}{2}}\|_F^2 + \|\mathbf{M}_L\|_* \\ & + \alpha_{MS} \|\mathbf{M}_S\|_1 + \text{tr}[\zeta_{ML}^T (\hat{\mathbf{M}}_L - \mathbf{M}_L)] \\ & + \text{tr}[\zeta_{MS}^T (\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)] \\ & + \frac{\beta_{MS}}{2} \|\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S\|_F^2 \\ & + \frac{\beta_{ML}}{2} \|\hat{\mathbf{M}}_L - \mathbf{M}_L\|_F^2, \end{aligned} \quad (6)$$

then the formula of (6) can be expressed as the following formula of (7)

$$\begin{aligned} \arg \min_{\mathbf{M}_L, \hat{\mathbf{M}}_L, \mathbf{M}_S, \mathbf{E}, \mathbf{A}} & \mathbf{J} = \text{tr}(\mathbf{W}\mathbf{K}) + \text{tr}(\mathbf{W}\hat{\mathbf{M}}_L^T \mathbf{E}^T \mathbf{K} \mathbf{A} \mathbf{E}^T \hat{\mathbf{M}}_L) \\ & - 2\text{tr}(\mathbf{W}\hat{\mathbf{M}}_L^T \mathbf{E}^T \mathbf{K}) + \|\mathbf{M}_L\|_* \\ & + \alpha_{MS} \|\mathbf{M}_S\|_1 + \text{tr}[\zeta_{ML}^T (\hat{\mathbf{M}}_L - \mathbf{M}_L)] \\ & + \text{tr}[\zeta_{MS}^T (\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)] + \frac{\beta_{MS}}{2} \\ & \times \text{tr}[(\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)(\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)^T] \\ & + \frac{\beta_{ML}}{2} \text{tr}[(\hat{\mathbf{M}}_L - \mathbf{M}_L)(\hat{\mathbf{M}}_L - \mathbf{M}_L)^T], \end{aligned} \quad (7)$$

where $\mathbf{K} = \psi(\mathbf{M})^T \psi(\mathbf{M})$.

Then we could get the next three formulas of (8), (9) and (10)

$$\begin{aligned} \frac{\partial \mathbf{J}}{\partial \hat{\mathbf{M}}_L} & = -2\mathbf{E} \mathbf{A}^T \mathbf{K} \mathbf{W} + 2\mathbf{E} \mathbf{A}^T \mathbf{K} \mathbf{A} \mathbf{E}^T \hat{\mathbf{M}}_L \mathbf{W} \\ & + \zeta_{ML} + \beta_{ML} (\hat{\mathbf{M}}_L - \mathbf{M}_L) = 0, \end{aligned} \quad (8)$$

$$\frac{\partial \mathbf{J}}{\partial \mathbf{A}} = -2\mathbf{K} \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E} + 2\mathbf{K} \mathbf{A} \mathbf{E}^T \hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E} = 0, \quad (9)$$

$$\frac{\partial \mathbf{J}}{\partial \mathbf{E}} = -2\hat{\mathbf{M}}_L \mathbf{W} \mathbf{K} \mathbf{A} + 2\hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E} \mathbf{A}^T \mathbf{K} \mathbf{A} = 0. \quad (10)$$

Suppose $\mathbf{E}^T \hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E}$, $\hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T$ and $\mathbf{A}^T \mathbf{K} \mathbf{A}$ are all invertible, we could get $\hat{\mathbf{M}}_L$, \mathbf{A} and \mathbf{E} as the following formula of (11), (12) and (13)

$$\begin{aligned} \hat{\mathbf{M}}_L & = \mathbf{M}_L + \frac{1}{\beta_{ML}} (2\mathbf{E} \mathbf{A}^T \mathbf{K} \mathbf{W} - 2\mathbf{E} \mathbf{A}^T \mathbf{K} \mathbf{A} \mathbf{E}^T \hat{\mathbf{M}}_L \mathbf{W} \\ & - \zeta_{ML}), \end{aligned} \quad (11)$$

$$\mathbf{A} = \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E} (\mathbf{E}^T \hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T \mathbf{E})^{-1}, \quad (12)$$

$$\mathbf{E} = (\hat{\mathbf{M}}_L \mathbf{W} \hat{\mathbf{M}}_L^T)^{-1} \hat{\mathbf{M}}_L \mathbf{W} \mathbf{K} \mathbf{A} (\mathbf{A}^T \mathbf{K} \mathbf{A})^{-1}. \quad (13)$$

The same to RPCA [9], [10], RR [3], [4] and other method [1], [5]–[8], [14], [17], we can acquire \mathbf{M}_S by minimizing

$$\begin{aligned} \arg \min_{\mathbf{M}_S} & \frac{1}{2} \|\mathbf{M}_S - (\mathbf{M} - \mathbf{M}_L + \frac{\zeta_{MS}}{\beta_{MS}})\|_F^2 \\ & + \frac{\alpha_{MS}}{\beta_{MS}} \|\mathbf{M}_S\|_1. \end{aligned} \quad (14)$$

According to the formula and conclusion introduced in the literature of [1], [7], [8], [12], [14], [16] and [17], we can

acquire \mathbf{M}_S as

$$\mathbf{M}_S = \xi_{\frac{\alpha_{MS}}{\beta_{MS}}} [\mathbf{M} - \mathbf{M}_L + \frac{\zeta_{MS}}{\beta_{MS}}]. \quad (15)$$

Moreover, we can acquire the low-rank term \mathbf{M}_L by minimizing the formula of (16)

$$\begin{aligned} \arg \min_{\mathbf{M}_L} & \|\mathbf{M}_L\|_* + tr\|\zeta_{ML}^T(\hat{\mathbf{M}}_L - \mathbf{M}_L)\| \\ & + tr\|\zeta_{MS}^T(\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)\| + \frac{\beta_{MS}}{2} \\ & \times tr\|(\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)(\mathbf{M} - \mathbf{M}_L - \mathbf{M}_S)^T\| \\ & + \frac{\beta_{ML}}{2} tr\|(\hat{\mathbf{M}}_L - \mathbf{M}_L)(\hat{\mathbf{M}}_L - \mathbf{M}_L)^T\|, \end{aligned} \quad (16)$$

and the formula of (16) is equal to minimizing

$$\begin{aligned} \arg \min_{\mathbf{M}_L} & \frac{1}{2} \|\mathbf{M}_L - \frac{1}{\beta_{MS} + \beta_{ML}} [\beta_{MS}(\mathbf{M} - \mathbf{M}_S) + \beta_{ML}\hat{\mathbf{M}}_L \\ & + \zeta_{ML} + \zeta_{MS}]\|_F^2 + \frac{1}{\beta_{MS} + \beta_{ML}} \|\mathbf{M}_L\|_*. \end{aligned} \quad (17)$$

Similar to [3], [4], we can acquire the low-rank term \mathbf{M}_L by utilizing the formula of (18) [5]–[10], [12], [16]

$$\begin{aligned} \mathbf{M}_L &= \mathbf{U}\xi_\eta[\xi]\mathbf{V}^T \\ &= \arg \min_{\mathbf{M}_L} \eta \|\mathbf{M}_L\|_* + \frac{1}{2} \|\mathbf{M}_L - \mathbf{X}\|_F^2, \end{aligned} \quad (18)$$

where $\mathbf{X} = \frac{1}{\beta_{MS} + \beta_{ML}} [\beta_{MS}(\mathbf{M} - \mathbf{M}_S) + \beta_{ML}\hat{\mathbf{M}}_L + \zeta_{ML} + \zeta_{MS}]$, $\mathbf{U}\xi\mathbf{V}^T$ is the SVD of \mathbf{X} , and [1], [3], [4], [7], [8], [12], [14], [16], [17]

$$\xi_\eta[\xi_{ij}] = \begin{cases} \xi_{ij} + \eta, & \text{if } \xi_{ij} < -\eta \\ \xi_{ij} - \eta, & \text{if } \xi_{ij} > \eta \\ 0, & \text{otherwise} \end{cases}$$

where $\eta = \frac{1}{\beta_{MS} + \beta_{ML}}$. At last, we can acquire the algorithm of RRLPP as follows.

3. Experiments

Since the RRLPP, RR [3], [4] and RPCA [9], [10] method are only used in the condition that the number of facial expression sample is more than the number of dimension, so we can not adopt those facial expression database which only has a few facial expression samples, such as JAFFE and POFA [1], [18]–[20]. In our experiment, we select the Multi-PIE facial expression database [17], [21], [22] which has a lot of samples as the test data for our RRLPP method and other four methods including PCA, LPP, RPCA+PCA and RR. The Multi-PIE facial expression database is build on the year of 2008 by Gross et al, and is the most widely used non-frontal facial expression database which has 755370 front and non-frontal images from 337 experiment subjects in all and six emotions containing surprise, disgust, squint, smile, scream and neutral [3], [4], [17], [21], [22].

In our experiment, we select 4400 front facial expression images containing five emotions (surprise, disgust, smile, scream and neutral) from the original Multi-PIE fa-

Regression-based Robust Locality Preserving Projections (RRLPP)

Input: facial expression sample matrix \mathbf{M} , β_{MS} , β_{ML} , ζ_{MS} , ζ_{ML} , α_{MS} , ε , μ .

1. Compute $\mathbf{K} = \psi(\mathbf{M})^T \psi(\mathbf{M}) = \mathbf{W}^{-1} \mathbf{D} \mathbf{W}^{-1}$.
2. Give $\mathbf{M}_S = \mathbf{I}_{p \times N}$, $\mathbf{M}_L = \mathbf{M} - \mathbf{M}_S$, $\hat{\mathbf{M}}_L = \mathbf{M}$, $\mathbf{E} = \mathbf{I}_{p \times c}$, $\zeta_{MS} = \mathbf{0}_{N \times N}$, $\zeta_{ML} = \mathbf{0}_{N \times N}$, where \mathbf{I} and $\mathbf{0}$ are identity matrix and zero matrix respectively,

While $\frac{\|\hat{\mathbf{M}}_L - \mathbf{M}_L\|_F}{\|\mathbf{M}_L\|_F} > \varepsilon$ **Do**

1. $\mathbf{M}_S^{k+1} = \xi_{\frac{\alpha_{MS}}{\beta_{MS}}} [\mathbf{M} - \mathbf{M}_L^k + \frac{\zeta_{MS}^k}{\beta_{MS}}]$,
2. $\mathbf{X}^{k+1} = \frac{1}{\beta_{MS}^k + \beta_{ML}^k} [\beta_{MS}^k (\mathbf{M} - \mathbf{M}_S^k) + \beta_{ML}^k \hat{\mathbf{M}}_L^k + \zeta_{ML}^k + \zeta_{MS}^k]$,
3. $\eta^{k+1} = \frac{1}{\beta_{MS}^k + \beta_{ML}^k}$,
4. $\mathbf{U}\xi\mathbf{V}^T = \mathbf{X}^{k+1}$,
5. $\mathbf{M}_L^{k+1} = \mathbf{U}\xi_\eta^{k+1}[\xi]\mathbf{V}^T$,
6. $\mathbf{A}^{k+1} = \mathbf{W}(\hat{\mathbf{M}}_L^k)^T \mathbf{E}^k [(\mathbf{E}^k)^T \hat{\mathbf{M}}_L^k \mathbf{W}(\hat{\mathbf{M}}_L^k)^T \mathbf{E}^k]^{-1}$,
7. $\mathbf{E}^{k+1} = [\hat{\mathbf{M}}_L^k \mathbf{W}(\hat{\mathbf{M}}_L^k)^T]^{-1} \hat{\mathbf{M}}_L^k \mathbf{W} \mathbf{K} \mathbf{A}^k [(\mathbf{A}^{k+1})^T \mathbf{K} \mathbf{A}^{k+1}]^{-1}$,
8. $\hat{\mathbf{M}}_L^{k+1} = \mathbf{M}_L^{k+1} + \frac{1}{\beta_{ML}^k} [2\mathbf{E}^{k+1} (\mathbf{A}^{k+1})^T \mathbf{K} \mathbf{W} - 2\mathbf{E}^{k+1} (\mathbf{A}^{k+1})^T \mathbf{K} \mathbf{A}^{k+1} (\mathbf{E}^{k+1})^T \hat{\mathbf{M}}_L^k \mathbf{W} - \zeta_{ML}^k]$,
9. $\zeta_{ML}^{k+1} = \zeta_{ML}^k + \beta_{ML}^k (\hat{\mathbf{M}}_L^{k+1} - \mathbf{M}_L^{k+1})$,
10. $\zeta_{MS}^{k+1} = \zeta_{MS}^k + \beta_{MS}^k (\mathbf{M} - \mathbf{M}_L^{k+1} - \mathbf{M}_S^{k+1})$,
11. $\beta_{ML}^{k+1} = \mu \beta_{ML}^k$,
12. $\beta_{MS}^{k+1} = \mu \beta_{MS}^k$.

End while

Output:

- The low-rank term \mathbf{M}_L and the projection matrix \mathbf{E} .

cial expression database, and each emotion has 880 images and their size is 60×60 [3], [4], [17], [21], [22]. We utilize the ten fold cross validation and the KNN classifier to implement experiment on the Multi-PIE facial expression database [14], [18], [23]. Moreover, similar to [3], [4], [9] and [10], we add Gaussian noise or occlusion to each facial expression images for reflecting the robustness effect of the RRLPP method better.

Figure 1 is the experiment affect of the RRLPP method after getting ride of Gaussian noise and occlusion. In Fig. 1, the first row and the second row are the experiment affect of getting ride of occlusion and Gaussian noise respectively, the first column, the second column and the third column are the facial expression images after adding occlusion or Gaussian noise, the low-rank term of facial expression images and the sparse term of facial expression images respectively [3], [4], [9], [10]. From Fig. 1, we can see that the RRLPP method can separate the occlusion and Gaussian noise from the original facial expression images in general.

Tables 1 and 2 are the experiment results of adding the occlusion and the Gaussian noise to facial expression images on the Multi-PIE facial expression database respectively. From Tables 1 and 2, we can see that the robust method including RRLPP, RR and RPCA+PCA are all better than the non-robust methods such as PCA and LPP, and our RRLPP method is better than the RPCA+PCA method and is approximate to the RR method in general.

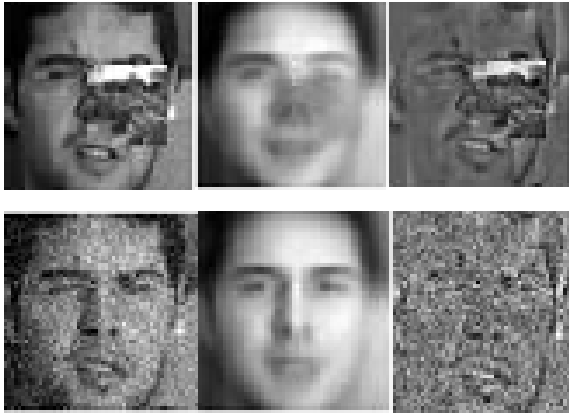


Fig. 1 The experiment affect of the RRLPP method after getting ride of Gaussian noise and occlusion.

Table 1 The experiment results of adding the occlusion to facial expression images.

Method	PCA	LPP	RPCA+PCA	RR	RRLPP
disgust	73.30	73.30	73.30	61.59	74.89
neutral	61.14	61.14	61.36	54.43	49.55
scream	61.70	61.70	61.48	58.64	74.66
smile	41.02	41.02	41.25	59.43	41.02
surprise	35.23	35.23	35.68	53.18	59.77
average	54.48	54.48	54.61	57.45	59.98

Table 2 The experiment results of adding the Gaussian noise to facial expression images.

Method	PCA	LPP	RPCA+PCA	RR	RRLPP
disgust	69.32	71.02	72.84	59.48	70.45
neutral	65.57	64.43	66.82	56.02	67.16
scream	63.07	62.50	62.73	70.68	72.95
smile	38.75	42.05	45.68	57.61	45.11
surprise	37.39	39.77	39.09	62.27	43.84
average	54.82	55.95	57.43	61.20	59.91

4. Conclusions

In this paper, we present a novel regression-based robust locality preserving projections (RRLPP) method to effectively deal with the issue of noise and occlusion in facial expression recognition. The test results on the Multi-PIE facial expression database indicate that the RRLPP method can effectively eliminate the noise and the occlusion problem of facial expression images, and it also can achieve better or comparative facial expression recognition rate compared to the non-robust and robust subspace methods meantime. Compared to the RR method, our RRLPP method is a unsupervised robust method and we can study the supervised RRLPP method in future.

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