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Accelerating Existing Non-Blind Image Deblurring Techniques through a Strap-On Limited-Memory Switched Broyden Method

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SUMMARY Video surveillance from airborne platforms can suffer from many sources of blur, like vibration, low-end optics, uneven lighting conditions, etc. Many different algorithms have been developed in the past that aim to recover the deblurred image but often incur substantial CPU-time, which is not always available on-board. This paper shows how a "strap-on" quasi-Newton method can accelerate the convergence of existing iterative methods with little extra overhead while keeping the performance of the original algorithm, thus paving the way for (near) real-time applications using on-board processing.

key words: *image deblurring, quasi-Newton, limited-memory, switched Broyden method*

1. Introduction

Computer vision is an ever expanding scientific domain that has become an integral part of everyday life, be it in surveillance, social, medical or artificial intelligence applications. The images that are taken are often part of a continuous video stream and are seldom an end in itself but serve as a tool for abnormal event detection, navigation, diagnosis, etc. Embedded cameras are often of low quality and can suffer from lens distortion or from vibrations that are transmitted by the platform on which the camera is mounted, like a car or an Unmanned Aerial Vehicle (UAV). The images are therefor corrupted by (motion) blur and noise. Recovering the unblurred image is thus a part of the pre-processing steps that are applied to computer vision applications.

Literally hundreds of publications have appeared over the years (e.g. [4], [14], [19], [36], [55], [56] to name but a few). In particular, when the image blur is spatially invariant then the deblurring process can be seen as a deconvolution problem which is generally ill-conditioned [10], [12], [50]. For this reason the algorithms are generally iterative methods that, unfortunately, require a substantial runtime.

In this paper we present a robust acceleration technique that can be "strapped on" existing iterative algorithms, without the need for delicate tuning of preconditioning parameters. Besides robustness, the methods offer a typical gain of 5 to 90% in CPU time compared to a wide range of methods that are known from literature.

This paper is organized as follows. In Sect. 2 we describe the mathematical process behind blurring, while in Sect. 3 we describe existing non-blind iterative algorithms for deblurring images suffering from spatially invariant blur. In Sect. 4 we propose an acceleration technique based on quasi-Newton methods. The convergence speed of the different algorithms is compared in Sect. 5, after which we end with conclusions and ideas for future work.

2. Blurring Model

A simplified linear, spatially invariant model of the discrete (i.e. digital) blurring process of an $m \times n$ pixel image is given by

$$g = K\,f + \eta \tag{1}$$

where $f \in \mathbb{R}^{mn}$ is the original image*, $g \in \mathbb{R}^{mn}$ is the blurred image, $\eta \in \mathbb{R}^{mn}$ is additive (most often white Gaussian) noise and $K \in \mathbb{R}^{mn \times mn}$ represents the point spread function (PSF)** responsible for the actual blurring.

Non-blind image restoration, or deblurring, is the process of finding f based on (approximate) knowledge of K, n and g [33].

The following characteristics make deblurring challenging:

- The problem has a very high dimensionality, which requires matrix-free implementations (cf. Sect. 4.1.4).
- K is, in general, a very ill-conditioned matrix with a cluster of very small singular values.

The effect of the ill-conditioning can be mitigated by the use of regularization to avoid computing solutions that are corrupted by noise. Well-known examples are Tikhonov regularization and Wiener filtering or Total Variation techniques [13], [18], [24], [27], [28], [37], [43], [45], [51]–[54].

Although this approach works well for well-posed problems [3], [23], [46], it does not for ill-posed problems such as image restoration. In this situation, the choice of preconditioner is very sensitive and can result in fast, but

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*Note that we have vectorized the $m \times n$ image by stacking the columns.

**Or "kernel".

erratic, convergence to a poor approximate solution [25], [26], [38]. Furthermore, for these pre-conditioning methods, choosing the best value of the regularization parameter is a nontrivial matter and it may be necessary to solve the problem for many different parameters to determine which is best.

Due to these constraints, iterative image restoration algorithms that do not require preconditioning have become a method of choice, as they have many advantages over simple filtering techniques and can be very efficient for spatially invariant as well as spatially variant blurs [5], [32], [54].

In this paper we focus on the non-blind deconvolution (or deblurring) process and do not take the noise into consideration. We refrain from using pre-conditioning techniques that are based on parameter estimation and in this stage of our research we focus only on spatially invariant blurs.

Our aim is to reduce the cost of an iterative algorithm, which is the product of the CPU time required per iteration and the number of iterations needed to obtain the required quality of the deblurred image.

3. Deblurring Algorithms

Preliminary note: for all the algorithms mentioned in this section, we use the implementation found in [4].

Note that these algorithms do not take η into account. If (an estimate of) η is known it can be catered for by replacing g with $g - \eta$.

3.1 Landweber Iteration

The basic Landweber iteration [18], [27], [28], [54], in the context of image restoration, is given by

Landweber iteration
1. Startup: Take initial value f_1, set $s = 1$.
2. Loop until $\|g - Kf_s\|_2 \leq \epsilon$:
   $f_{s+1} = f_s + \tau K^T(g - Kf_s)$; set $s = s + 1$.

A typical choice for τ is $\frac{1}{\sigma^2_{\max}}$, where $\sigma_{\max}$ is the largest singular value of K, estimated by $\sigma_{\max} = \sqrt{\|K\|_1\|K\|_\infty}$ [22].

3.2 Steepest Descent (SD)

As in the case of the Landweber iteration, the Steepest Descent method takes $K^T(g - Kf_s)$ as a step direction, but with a variable step-size [4].

Steepest descent

1. Startup: Take initial value f_1, set $s = 1$.
2. Loop until $\|g - Kf_s\|_2 \leq \epsilon$:
2.1. $f_{s+1} = f_s + \frac{\|K^T(g-Kf_s)\|_2^2}{\|KK^T(g-Kf_s)\|_2^2}(K^T(g - Kf_s))$;
2.2. Set $s = s + 1$.

3.3 LSQR and CGLS

LSQR and CGLS are related conjugate gradient methods. We refer to the literature for more details [7], [41], [42].

3.4 Hybrid Method (HM)

Hybrid methods combine variational approaches with iterative methods, i.e. an iterative conjugate gradient method is applied to $\min \|Kf - g\|$ and variational regularization is incorporated in the iterations. Again, we refer to the literature for more details [6], [40].

3.5 MRNSD and Richardson-Lucy

As pixels represent perceived intensity values, nonnegativity constraints can be added to the optimisation statement, i.e. we may want to develop algorithms that solve $\min_{f\geq 0} \|Kf - g\|$.

Two common algorithms for image restoration with nonnegativity constraints are MRNSD [2], [30], [39] and Richardson-Lucy [33], [44], [54].

4. Acceleration by Quasi-Newton Methods

The Landweber iteration in Sect. 3.1 can be seen as a fixed-point process, where the creation of f_{s+1} based on f_s is written as $f_{s+1} = H(f_s)$. Then the problem of finding f can be transformed into a root finding problem $H(f) - f = P(f) = 0$. This novel interpretation of deblurring as a root-finding exercise leads us to contemplate the use of a quasi-Newton method as an acceleration method.

Quasi-Newton accelerated Landweber iteration

1. Startup: Take initial value f_1, set $s = 1$.
2. Loop until $\|g - Kf_s\|_2 \leq \epsilon$:
2.1. Compute approximate Jacobian $\hat{P}'_s$ (see below).
2.2. $f_{s+1} = f_s - (\hat{P}'_s)^{-1}\underbrace{\left(K^T(g - Kf_s)\right)}_{P(f_s)}$
2.3. Set $s = s + 1$.

Alternatively a slightly different quasi-Newton step $f_{s+1} = f_s - M'_s P(f_s)$ can be used. Here M'_s serves as an approximation to the inverse of the Jacobian at step s, whereas $\hat{P}'_s$ is an approximation of the Jacobian itself.

The difference between the various quasi-Newton methods that we consider here lies in the choice of $\hat{P}'_s$ (or M'_s).

4.1 Broyden's Methods

We choose Broyden's methods as the quasi-Newton methods that we will use for the acceleration step, mainly because they have been well-studied and allow for a limited-memory implementation later on.

To construct the approximate (inverse) Jacobians in

Broyden's method we first define $\delta f_s = f_{s+1} - f_s$ and $\delta P_s = P(f_{s+1}) - P(f_s)$.

4.1.1 Broyden's Good Method

Broyden's first or good method† (also abbreviated as "BG") [8], [9], [16], [17] is a quasi-Newton method that is part of the family of Least Change Secant Update (LCSU) methods [17], [20], where the approximate Jacobian $\hat{P}'_{s+1}$ is chosen as the solution of the following minimization problem:

$$\min_{\hat{P}'}\{\|\hat{P}' - \hat{P}'_s\|_{Fr}\}, \text{ s.t. } \hat{P}'\delta f_s = \delta P_s. \tag{2}$$

In other words, it gives a new approximate Jacobian that is closest to the previous one in the Frobenius norm and that satisfies the secant equation.

The solution of (2) leads to the following rank-one update:

$$\hat{P}'_{s+1} = \hat{P}'_s + \frac{(\delta P_s - \hat{P}'_s\delta f_s)\delta f_s^T}{\langle\delta f_s, \delta f_s\rangle} \tag{3}$$

$\hat{P}'_1$ is typically set to be equal to $-I$, which means that the first iteration equals a Landweber iteration. Interpreting Broyden's good method differently, we could say that

- $\hat{P}'_{s+1}$ is the projection w.r.t. the Frobenius norm of $\hat{P}'_s$ onto $\{A \in \mathbb{R}^{mn\times mn} : A\delta f_s = \delta P_s\}$;
- no change occurs between $\hat{P}'_{s+1}$ and $\hat{P}'_s$ on the orthogonal complement of δf_s, i.e. $(\hat{P}'_{s+1} - \hat{P}'_s)z = 0$ if $\langle z, \delta f_s\rangle = 0$.

We have the following properties of this method:

1. For linear problems, the method is known to show superlinear convergence [31] and it needs at most $2mn$ iteration to reach the solution (Gay's theorem [21]).
2. No guarantee can be given that the approximate Jacobians are non-singular nor that convergence is monotone.

Using using the Sherman-Morrison theorem [49], Broyden's good method can be written as:

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_s)^{-1} + \frac{(\delta f_s - (\hat{P}'_s)^{-1}\delta P_s)\delta f_s^T(\hat{P}'_s)^{-1}}{\delta f_s^T(\hat{P}'_s)^{-1}\delta P_s}. \tag{4}$$

4.1.2 Broyden's Bad Method

Broyden's second or bad method (also abbreviated as "BB") [8] is a quasi-Newton method that uses an approximation $\hat{M}'$ of the inverse Jacobian. It is also part of the family of LCSU methods [17], [20], where the approximate inverse Jacobian $\hat{M}'_{s+1}$ is chosen as the solution of the following minimization problem:

$$\min_{\hat{M}'}\{\|\hat{M}' - \hat{M}'_s\|_{Fr}\}, \text{ s.t. } \hat{M}'\delta P_s = \delta f_s. \tag{5}$$

i.e. it gives a new approximation of the inverse of the Jacobian that is closest to the previous one in the Frobenius norm and that satisfies the secant equation.

The solution of (5) leads to the following rank- one update

$$\hat{M}'_{s+1} = \hat{M}'_s + \frac{(\delta f_s - \hat{M}'_s\delta P_s)\delta P_s^T}{\langle\delta P_s, \delta P_s\rangle}. \tag{6}$$

Interpreting Broyden's bad method differently, we could say that

- $\hat{M}'_{s+1}$ is the projection w.r.t. the Frobenius norm of $\hat{M}'_s$ onto $\{A \in \mathbb{R}^{mn\times mn} : A\delta P_s = \delta f_s\}$;
- no change occurs between $\hat{M}'_{s+1}$ and $\hat{M}'_s$ on the orthogonal complement of δP_s, i.e. $(\hat{M}'_{s+1} - \hat{M}'_s)z = 0$ if $\langle z, \delta P_s\rangle = 0$.

Broyden himself [8] admitted that this formulation of his algorithm didn't function properly††. The reasons for the "good" or "bad" behavior are not well understood, and it is quite possible that in some instances the bad method outperforms the good method. Indeed, as we will see in Sect. 5.2, BB performs better than BG in this application. We also have the same properties as for Broyden's Good method.

Even though it is the inverse Jacobian that is approximated in this method, we will write $(\hat{P}'_s)^{-1}$ instead of $\hat{M}'_s$ to standardize the notation of the methods in what follows, thus obtaining

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_s)^{-1} + \frac{(\delta f_s - (\hat{P}'_s)^{-1}\delta P_s)\delta P_s^T}{\delta P_s^T\delta P_s}. \tag{7}$$

Again, $\hat{P}'_1$ is typically set to be equal to $-I$, which means that the first iteration equals a Landweber iteration.

4.1.3 Switched Broyden Method

As Broyden's Good method is not always better than Broyden's Bad method, we follow an idea suggested in [34] that avoids the need to choose between the two methods and create a switched version of BG/BB (called "BS") in the following manner. If

$$\frac{|\delta f_s^T\delta f_{s-1}|}{|\delta f_s^T(\hat{P}'_s)^{-1}\delta P_s|} < \frac{|\delta P_s^T\delta P_{s-1}|}{\delta P_s^T\delta P_s} \tag{8}$$

then the rank-one update of BG is used, otherwise the update of BB is used. To our knowledge, this variant of Broyden's method has not been given the attention that it deserves, even though we will see that it lies at the basis of the best performing algorithm in Sect. 5.

4.1.4 Matrix-Free and Limited Memory Implementation of Broyden's Algorithms

All the methods that we have mentioned so far require

†Most often simply called *Broyden's method*.

††This is the reason the method is called "bad".

matrix-vector multiplications with K and/or K^T. As the size of the matrices and vectors in the test-cases are typically very large, a matrix-free implementation of the algorithms is necessary. We first develop Broyden's good method as

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_1)^{-1} + \sum_{i=1}^{s} \frac{\delta f_i - (\hat{P}'_i)^{-1}\delta P_i}{\delta f_i^T (\hat{P}'_i)^{-1}\delta P_i} \delta f_i^T (\hat{P}'_i)^{-1},$$

while for Broyden's bad method we get

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_1)^{-1} + \sum_{i=1}^{s} \frac{(\delta f_i - (\hat{P}'_i)^{-1}\delta P_i)}{\delta P_i^T \delta P_i} \delta P_i^T$$

In summary, both methods can be written as

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_1)^{-1} + \sum_{i=1}^{s} w_i v_i^T$$

where $w_i = \delta f_i - (\hat{P}'_i)^{-1}\delta P_i$ and

	BG	BB
v_i	$\frac{\delta f_i^T (\hat{P}'_i)^{-1}}{\delta f_i^T (\hat{P}'_i)^{-1}\delta P_i}$	$\frac{\delta P_i^T}{\delta P_i^T \delta P_i}$

Obviously, when a lot of iterations are required, the Broyden methods become both memory heavy and computationally expensive. In this paper we propose a limited memory version of the algorithms, where only the last κ (w_i, v_i) pairs are kept. We will call the resulting methods BG(κ), BB(κ) and BS(κ) and where the inverse Jacobian is given by

$$(\hat{P}'_{s+1})^{-1} = (\hat{P}'_1)^{-1} + \sum_{i=\max(1,s-\kappa)}^{s} w_i v_i^T.$$

Admittedly, the choice of the value κ is subject to further research. The only firm conclusion for the moment (as the experiments in the next section will show) is that it cannot be too small (otherwise the method performs badly) nor too high (otherwise the method becomes too computationally heavy).

5. Performance Comparisons

5.1 Test-Cases

5.1.1 Spatially Invariant Gaussian Blur

Gaussian PSFs are typically used to evaluate the performance of deblurring algorithms and can be written as $K = [k_{ij}]$ with

$$k_{ij} = \frac{1}{2\pi\sqrt{\gamma}} e^{-\frac{1}{2}([i\ j]C^{-1}[i\ j]^T)} \tag{9}$$

where

$$C = \begin{bmatrix} \alpha_1^2 & \rho^2 \\ \rho^2 & \alpha_2^2 \end{bmatrix} \tag{10}$$

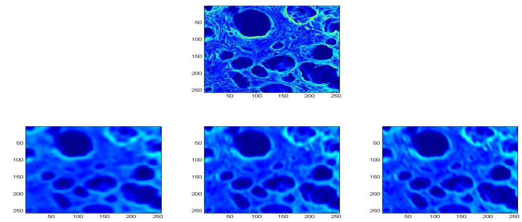

Fig. 1 Gaussian blur. Unblurred image (top), blurred images (bottom), from left to right: G1, G2, G3. $m = n = 256$.

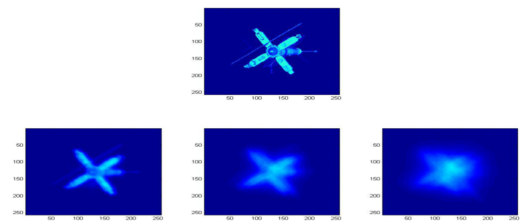

Fig. 2 Atmospheric blur. Unblurred image (top), blurred images (bottom), from left to right: A1, A2, A3. $m = n = 256$.

$$\gamma = (\alpha_1\alpha_2)^2 - \rho^4 > 0 \tag{11}$$

We select three examples of spatially invariant Gaussian blurs from [4] and which are illustrated in Fig. 1:

1. G1: $(\alpha_1, \alpha_2, \rho) = (4, 4, 0)$;
2. G2: $(\alpha_1, \alpha_2, \rho) = (4, 2, 0)$;
3. G3: $(\alpha_1, \alpha_2, \rho) = (4, 2, 2)$.

5.1.2 Spatially Invariant Atmospheric Turbulence Blur

When viewing objects through a telescope, a quantification of the observation conditions can be given by $\frac{d}{r_o}$, where d is diameter of the telescope and r_o is the Fried parameter, which is a statistical measure of atmospheric turbulence [29]. A low value of $\frac{d}{r_o}$ corresponds to good conditions.

We use three models, illustrated in Fig. 2, corresponding to $\frac{d}{r_o} = 10$ (A1), 30 (A2) and 50 (A3) respectively.

5.2 Comparison between Broyden Variants

We first compare the Landweber iteration with BG, BB and BS. As a convergence criterion $\epsilon = mn \cdot 10^{-3}$ is used. As the experiments will show, this is a rather stringent convergence criterion, corresponding to a high quality of the restored image (e.g. Fig. 3 for G1).

For all six test-cases the same behavior was noted:

- All methods had monotonous convergence, except for Broyden's good method.
- Convergence for Landweber's method was significantly slower than for BB and BS. The performance of the latter two methods was very similar over the range of test-cases.

The convergence history for test-case G1 is given in

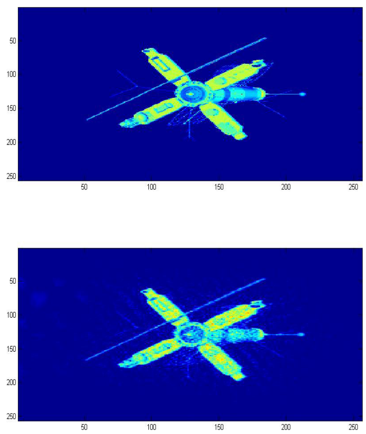

Fig. 3 Atmospheric blur A2. Unblurred image (top), restored image with BS (bottom).

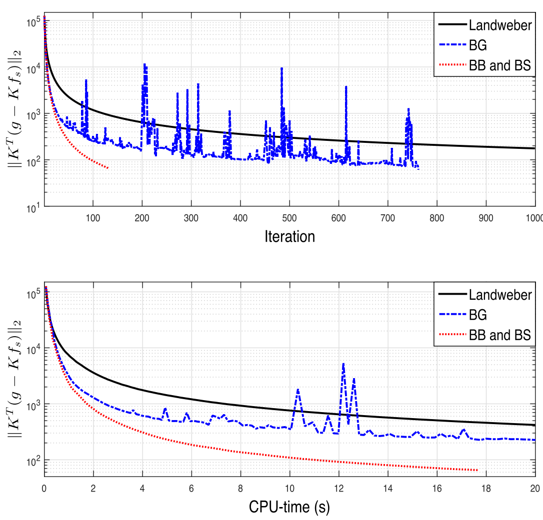

Fig. 4 Convergence history against iteration count and CPU-time for test-case G1 using Landweber, BG, BB and BS.

Fig. 4 (both as a function of iteration count and as a function of CPU time) and in Fig. 5 for test-case A2. Results for BB and BS are indistinguishable on the figures. (Results for test-cases G2 and G3 are very similar to G1, while those for A1 and A3 are very similar to A2.) The gain in CPU time (measured on a 3.3GHz Intel Core i3-2120 with 4GB RAM) for BS, compared to Landweber iteration, was in the order of 90% for all test-cases. Typically the order of convergence (approximated by $\frac{\log|\frac{f_{s+1}-f_s}{f_s-f_{s-1}}|}{\log|\frac{f_s-f_{s-1}}{f_{s-1}-f_{s-2}}|}$, for a sufficiently big value of s) of the Landweber method was 0.8 to 0.95, while that of BS was 1.2 to 1.3.

Looking at these results, we chose BS as the best method of these four.

When we compare BS with BS(κ), we note that for values of $\kappa > 10$ (Fig. 6), divergence is possible, although it must be added that for $\kappa = 11$ or 12, this is just barely, as the algorithm comes within a whisker of the (admittedly stringent) stopping criterion before diverging. The source

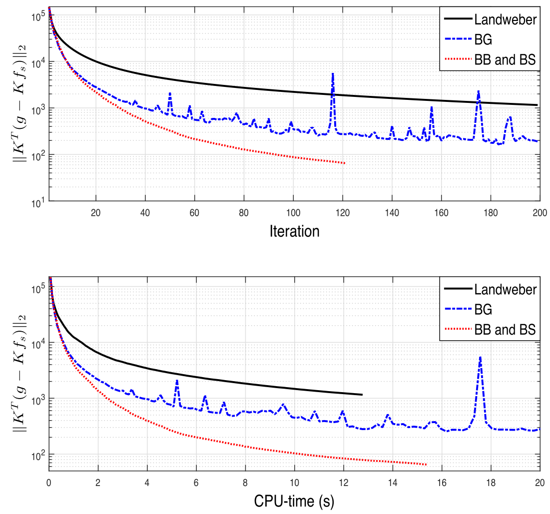

Fig. 5 Convergence history against iteration count and CPU-time for test-case A2 using Landweber, BG, BB and BS.

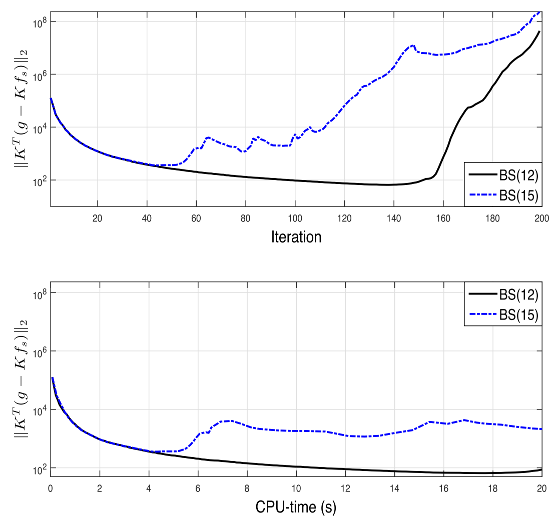

Fig. 6 Convergence history against iteration count and CPU-time for test-case G1 using BS(12) and BS(15).

of this phenomenon is the numerical instability of the chosen limited-memory algorithm. While this might warrant further investigation, it needs to be emphasized that there is no point in trying to keep the algorithm stable for very high values of κ as this would go against the idea of a limited-memory method.

When we compare BS with BS(κ), we further note that

- using $\kappa \in [2, 10]$ does not significantly change the convergence behavior when plotted against iteration number (Fig. 7), with $\kappa = 8$ giving slightly better results. In particular, Fig. 7 shows that BS(10) is slightly better than BS(8) when comparing the iteration count, but slightly worse when comparing the CPU-time. A value of $\kappa = 8$ is therefore the one around which the gain in CPU-time per iteration starts to outweigh the gain in total number of iterations.

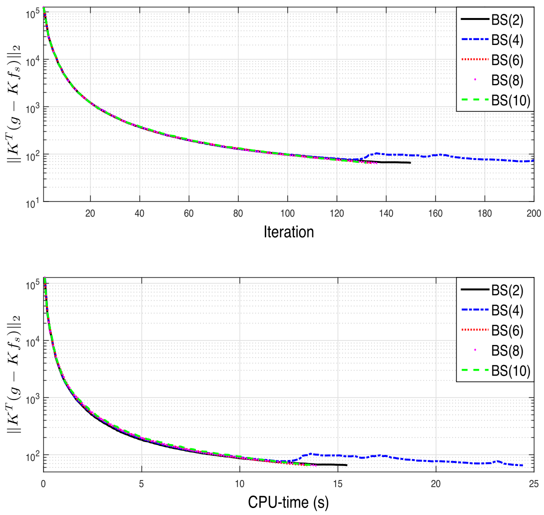

Fig. 7 Convergence history against iteration count and CPU-time for test-case G1 using BS(2), BS(4), BS(6), BS(8) and BS(10).

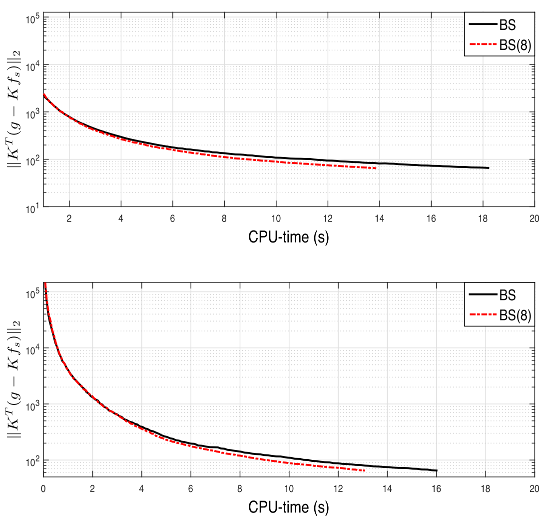

Fig. 8 Convergence history for test-cases G1 (top) and A2 (bottom) using BS and BS(8).

- When plotted against CPU-time BS(8) yielded a gain of roughly 25% compared to BS on test-case G2 (Fig. 7) and 20% compared to BS on test-case A2 (Fig. 8). For other blurs the results were similar.

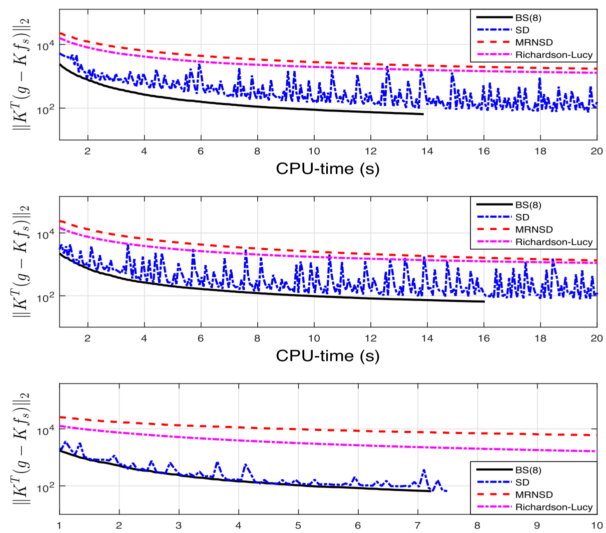

Fig. 9 Convergence history for test-cases G1 (top), G2 (middle) and A1 (bottom) using BS(8), SD, MRNSD and Richardson-Lucy.

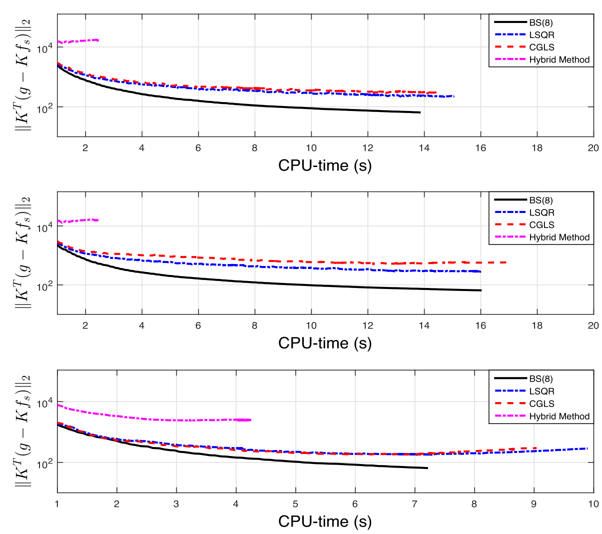

Fig. 10 Convergence history for test-cases G1 (top), G2 (middle) and A1 (bottom) using BS(8), LSQR, CGLS and the Hybrid Method.

5.3 Comparison with Other Methods

Having selected the best-performing Broyden variant (i.e. BS(8)), we turn our attention to the more sophisticated iterative methods for non-blind deblurring described in [4] and using the implementations therein. These are "Steepest Descent" (SD), "LSQR", "CGLS", the "Hybrid Method" (HM), MRNSD and Richardson-Lucy. The convergence history for test-cases G1, G2 and A1 are given in Figs. 9 and 10; those for G3, A2 and A3 (not shown) exhibit similar behavior. The best performing of these methods is SD. For the test-cases with Gaussian blur it still required around 50% to 70% more CPU time than BS(8), while for the atmospheric blurring it only required around 5 to 10% more. Furthermore, convergence of SD is erratic, while BS(8) has monotone convergence. All the other methods initially had a monotone, but excessively slow convergence, which sometimes resulted in stagnation (typical for the Hybrid Method) or in slow divergence before the tolerance was reached. In none of the cases, the performance came close to that of BS(8).

6. Conclusions

We have shown how a switched variant of Broyden's quasi-Newton method, and in particular a matrix-free, limited memory version of this algorithm, offers faster convergence than the most common iterative methods used for non-blind image deblurring. Compared to the most competitive of the other methods, the gain is still in the order of 5 to 40% in CPU time for the test-cases that we used. Furthermore the new methods offers monotone convergence and is easy to implement. We would like to point out that at this point the new quasi-Newton method has not yet been computationally optimized and that further gains are possible when tweaked accordingly. This will form the basis of future work.

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