LETTER

A Stackelberg Game-Theoretic Solution to Win-Win Situation: A Presale Mechanism in Spectrum Market*

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SUMMARY This paper investigates the spectrum allocation problem. Under the current spectrum management mode, large amount of spectrum resource is wasted due to uncertainty of user's demand. To reduce the impact of uncertainty, a presale mechanism is designed based on spectrum pool. In this mechanism, the spectrum manager provides spectrum resource at a favorable price for presale aiming at sharing with user the risk caused by uncertainty of demand. Because of the hierarchical characteristic, we build a spectrum market Stackelberg game, in which the manager acts as leader and user as follower. Then proof of the uniqueness and optimality of Stackelberg Equilibrium is given. Simulation results show the presale mechanism can promote profits for both sides and reduce temporary scheduling.

key words: *spectrum pool, presale mechanism, Stackelberg game, Stackelberg equilibrium*

1. Introduction

As large chunks of spectral resources are only used sporadically [1], current framework with fixed spectrum resource allocation is a waste of resources. New approaches such as spectrum pooling, which was first mentioned in [2], realize spectrum resource multiplexing and improve spectrum efficiency. Relevant research mainly focused on optimizing profit on the supply side or spectrum allocation on the demand side [3]–[11]. However, comprehensive consideration for both sides is essential for problems with economic backgroud. In conventional spectrum pooling framework, when users have communication demand, they make request and receive service immediately. Though it can meet the requirement of real-time, low utilization of resource is brought about. When supply exceeds demand, extra spectral resource is wasted. When demand exceeds supply, owners cannot benefit to the full.

In this paper, we introduce a presale mechanism to spectrum allocation to avoid low utilization brought by uncertainty of user's demand. The spectrum manager provides spectrum resource at a favorable price for presale in the hope of getting general information of user. The user is willing to estimate his demand and book in advance motivated by a favorable price. In this way, a general estimate of demand is delivered to the manager. The manager accordingly allocates resource in a more precise way. The key difference between previous work [12] and this paper is as follows. In previous work mainly from economic life, the presale mechanism stimulated demand by cutting price and improves profit. But in our paper, the presale mechanism plays a part as implicit interaction to have a general understanding of demand ahead of time, and reduces the impact of uncertainty. To separately show the impact of implicit interaction, users' demand is assumed to be free from price in our paper.

2. System Model

We consider a spectrum market model based on spectrum pool, as shown in Fig. 1. To show the detail of presale mechanism, model with only one user is considered. To receive lower price provided by the manager, the user would book in advance some spectrum resource, which is called booking amount. If booking amount cannot meet demand, he would buy more in real time at normal price. Assume the advance

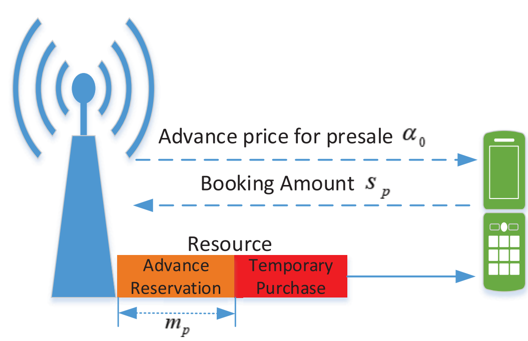

Fig. 1 Spectrum market model and the procedure of presale mechanism.

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Table 1 Summation of used notations

Notations	Explanation
u	profit of user
v	profit of manager
α_0	advance price for user
α_1	normal price for user
β_0	advance cost for manager
β_1	normal cost for manager
γ	unit profit for user
s_p	booking amount
m_p	reservation amount
s_r	actual demand of user

cost of manager to reserve resource for user is β_0, and normal cost of manager to temporarily improvise resource for user is β_1. The advance price for user to book resource is α_0, and normal price for user to temporarily purchase is α_1. Normally we have $\beta_0 < \beta_1$ and $\alpha_0 < \alpha_1$. The user has a unit profit of γ on utilization of spectrum resources. The user predicts its demand of spectrum and makes a decision of booking amount to be s_p. The manager, which has the feature of profit following, makes a decision of reservation amount to be m_p, $s_p \leq m_p$. The actual demand of user is s_r, which is subject to exponential distribution with a fixed parameter λ, probability distribution function to be $P(s_r)$. Considering that advance payment can accelerate turnover of capital, manager's profit from booking amount is additionally multiplied by a coefficient k, which is a little more than one and satisfies $\beta_1 - \beta_0 k > 0$. The profits of user and manager are u and v respectively. For convenience, we list some necessary notations related to this paper in Table 1. Discussion of profits versus s_r is given as follows:

$$u = \begin{cases} \gamma s_r - \alpha_0 s_p, & 0 \leq s_r < s_p, \\ \gamma s_r - \alpha_0 s_p - \alpha_1 (s_r - s_p), & s_p \leq s_r. \end{cases} \tag{1}$$

$$v = \begin{cases} (s_p\alpha_0 - \beta_0 m_p)k, & 0 \leq s_r < s_p, \\ (s_p\alpha_0 - \beta_0 m_p)k + (s_r - s_p)\alpha_1, & s_p \leq s_r < m_p, \\ (s_p\alpha_0 - \beta_0 m_p)k + (m_p - s_p)\alpha_1 \\ +(s_r - m_p)(\alpha_1 - \beta_1), & m_p \leq s_r. \end{cases} \tag{2}$$

We assume both sides know the exact distribution of demand. Mathematical expectation of u and v are given as:

$$E[u] = \int_0^\infty u dP(s_r) = \frac{\gamma}{\lambda} - \alpha_0 s_p - \frac{\alpha_1}{\lambda} e^{-\lambda s_p}. \tag{3}$$

$$\begin{aligned} E[v] &= \textstyle\int_0^\infty v dP(s_r) \\ &= k s_p \alpha_0 - k\beta_0 m_p + \tfrac{\alpha_1}{\lambda} e^{-\lambda s_p} - \tfrac{\beta_1}{\lambda} e^{-\lambda m_p}. \end{aligned} \tag{4}$$

The optimization target is shown as:

$$\begin{cases} \max\limits_{\alpha_0, m_p} E[v] \\ \text{s. t.} \begin{array}{l} m_p \geq s_p^*(\alpha_0) \\ \alpha_1 \geq \alpha_0 \geq 0 \end{array} \\ \quad The\ optimal\ solution : (\alpha_0, s_p^*(\alpha_0)) \\ \quad \begin{cases} \max\limits_{s_p} E[u] \\ \text{s. t.} s_p \geq 0 \end{cases} \end{cases} \tag{5}$$

3. Game Framework for Presale Mechanism and Optimal Strategy

In this section, we first define Stackelberg Equilibrium (SE) of this problem under game framework. Then we prove the uniqueness and optimality of SE. The presale model is a typical hierarchical model which can be analyzed by Stackelberg Game. In the game, the strategy of the manager includes α_0 and m_p, while that of the user includes s_p. The user, which hopes to promote profit, has to decide its strategy based on the choice of α_0 made by the manager. Hence, the manager takes actions first and acts as the leader. The user, which takes actions according to the manager, acts as the follower. The Stackelberg Game is formulated as:

$$G = \{\{manager, user\}, \{A_m, A_u\}, \{v, u\}\}. \tag{6}$$

Here $\{A_m, A_u\}$ denotes the strategy set of the manager and user. Expressions of $\{v, u\}$ is given in (1) and (2). Specifically, the strategy of the manager can be denoted as:

$$A_m = \{\alpha_0, m_p\}. \tag{7}$$

Definition 1: The Stackelberg Equilibrium (SE) [13] $\{m_p^*, \alpha_0^*, s_p^*\}$ of G is defined as:

$$\begin{aligned} E[v(m_p, \alpha_0, s_p^*)] &\leq E[v(m_p^*, \alpha_0^*, s_p^*)], \\ E[u(\alpha_0^*, s_p)] &\leq E[u(\alpha_0^*, s_p^*)]. \end{aligned} \tag{8}$$

The practical meaning of the equilibrium lies in that neither the manager or the user can promote profits by deviating unilaterally.

Lemma 1: The Stackelberg Equilibrium of G is unique and optimal.

A backward induction method will be applied to obtain Stackelberg Equilibrium. We first observe the strategy of follower and then leader.

1) Follower Sub-Game

$$\begin{aligned} & s_p = \arg\ \max\ E[u] \\ & s.t. \quad s_p \geq 0 \end{aligned} \tag{9}$$

Here $E[u]$ is a function of s_p and α_0. The follower chooses s_p according to α_0 decided by the leader. First-order partial derivative of $E[u]$ is derived.

$$\frac{\partial E[u]}{\partial s_p} = (\alpha_1 - \alpha_0) - \alpha_1 P(s_p). \tag{10}$$

The optimal s_p to maximize $E[u]$ is the best choice of the user, that is

$$s_p^* = P^{-1}\left(\frac{\alpha_1 - \alpha_0}{\alpha_1}\right) = \frac{1}{\lambda} \ln\left(\frac{\alpha_1}{\alpha_0}\right). \tag{11}$$

This result of optimal booking amount indicates that user would like to book in advance, though at the risk that booking amount may be beyond actual demand. However, such kind of willingness weakens as presale price increases.

2) Leader Sub-Game

$$\begin{aligned} \{m_p, \alpha_0\} &= \arg\ \max\ E[v] \\ s.t. \quad & m_p \geq s_p^*(\alpha_0) \\ & \alpha_1 \geq \alpha_0 > 0 \end{aligned} \tag{12}$$

Here $E[v]$ is a function of m_p and α_0. The leader chooses m_p and α_0 to maximize $E[v]$, therein m_p is determined according to α_0. Take $s_p^*(\alpha_0)$ from (11) into (4).

$$\frac{\partial E[v]}{\partial m_p} = (\beta_1 - \beta_0 k) - \beta_1 P(m_p). \tag{13}$$

Two cases are discussed:

- For $\alpha_0 \leq \alpha_1\beta_0 k/\beta_1$, (13) satisfies the following two inequalities:

$$(\beta_1 - \beta_0 k) - \beta_1 P(s_p^*(\alpha_0)) \leq 0. \tag{14}$$

$$(\beta_1 - \beta_0 k) - \beta_1 \lim_{x\to+\infty} P(x) < 0. \tag{15}$$

Considering that $P(x)$ is a decreasing function of x, $\frac{\partial E[v]}{\partial m_p}$ is non-positive in the defined area. Therefore manager's optimal m_p^* is given as:

$$m_p^*(\alpha_0)=s_p^*(\alpha_0)=\frac{1}{\lambda}\ln(\frac{\alpha_1}{\alpha_0}). \tag{16}$$

- For $\alpha_0 > \alpha_1\beta_0 k/\beta_1$, similarly, m_p^* is given as:

$$m_p^*(\alpha_0)=\frac{1}{\lambda}\ln(\frac{\beta_1}{\beta_0 k}). \tag{17}$$

Merge (16) and (17), m_p^* is given as:

$$m_p^*(\alpha_0) = \begin{cases} \frac{1}{\lambda}\ln(\frac{\alpha_1}{\alpha_0}), & \alpha_0 \leq \frac{\alpha_1\beta_0 k}{\beta_1}, \\ \frac{1}{\lambda}\ln(\frac{\beta_1}{\beta_0 k}), & \alpha_0 > \frac{\alpha_1\beta_0 k}{\beta_1}. \end{cases} \tag{18}$$

This result of optimal reservation amount indicates that, when presale price is close to normal price, manager would still like to reserve some resource independent of presale price. Thought user may book less as presale price increases, manager keeps reservation amount unchanged because he is sure there is still actual demand anyway.

Take (11) and (18) into (4), full expressions of $\frac{dE[v]}{d\alpha_0}$ is given as:

$$\frac{dE[v]}{d\alpha_0} = \begin{cases} \frac{k}{\lambda}\ln(\frac{\alpha_1}{\alpha_0}) - \frac{k-1}{\lambda} - \frac{k\beta_1}{\lambda\alpha_1} + \frac{\beta_0 k^2}{\lambda\alpha_0}, & \alpha_0 \leq \frac{\alpha_1\beta_0 k}{\beta_1}, \\ \frac{k}{\lambda}\ln(\frac{\alpha_1}{\alpha_0}) - \frac{k-1}{\lambda}, & \alpha_0 > \frac{\alpha_1\beta_0 k}{\beta_1}. \end{cases} \tag{19}$$

It's easy to see that the first-order derivative is continuous and decreasing function of α_0, and $E[v]$ increases firstly and then decreases as α_0 increase. Therefore we can find a unique α_0^* to maximize $E[v]$. Strategy $\{\alpha_0^*, m_p^*\}$ of manager and $\{s_p^*\}$ of user constitute Nash Equilibrium. From the depriving process we can see the optimality.

Based on the analysis above, we can see the effect of presale mechanism. When advance price is low enough, user dares to place a large order. However, manager has to bear the loss. Therefore, manager would fix a reasonable price which attracts booking and also ensures profit coming from advance receipt. In this way, a win-win state is achieved.

4. Simulation Results and Discussions

In this section, we provide some simulation results to illustrate the superiority of presale mechanism. The costs of manager are set as $\beta_0 = 7$, $\beta_1 = 11$. Normal price of spectrum resource is set as α_1=18. Unit profit from utilization of spectrum resource is $\gamma = 20$, coefficient $k = 1.2$ and $\lambda = 1$.

The three-dimensional surface in Fig. 2 shows the impact of manager's strategy on his own profit as (18), given user's optimal strategy. Therein the red curve shows that profit varies with advance price α_0 given the optimal strategy of reservation amount m_p^*. The green curve in the horizontal, which is the vertical projection of the red curve, shows optimal m_p^* varies with advance price α_0, as (18) does. And the blue curve at the top, which is also the horizontal projection of the red curve, confirms the optimality of m_p^*.

Figure 3 shows that optimal booking amount s_p^* gradually diminishes to 0 as the presale price increase, while optimal reservation amount m_p^* decreases first and then remains a positive value. These two variables indirectly reflect the ability of presale mechanism to reduce temporary purchase. It's simple to understand that the user will consistently cut down booking amount s_p^* if incentive is becoming less. m_p^* would remain a fixed positive value in disregard of s_p^* because the manager knows the exact distribution of demand. Though the user may not book in advance, there's still demand irrelevant to price, therefore the manager would prefer to reserve some resource within certain range. Besides, the proportion of temporary purchase demonstrates the ability of risk-sharing.

Figure 4 shows the influence of presale price α_0 on the profit of both sides, given optimal m_p^* and s_p^*. For the user, presale mechanism promotes its profit by cutting cost, and its profit diminishes as the presale price increase. For the

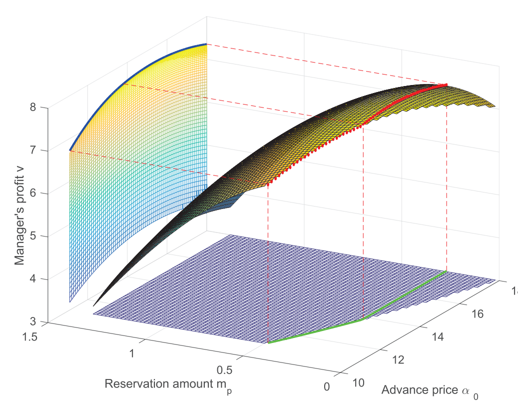

Fig. 2 The influence of manager's strategy on profit.

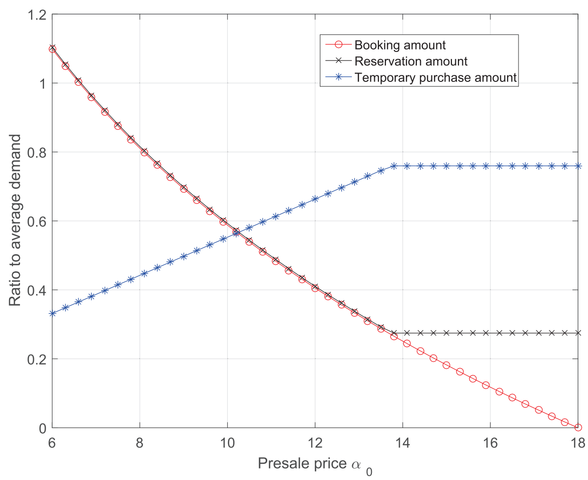

Fig. 3 The relationship between presale price and corresponding optimal strategies.

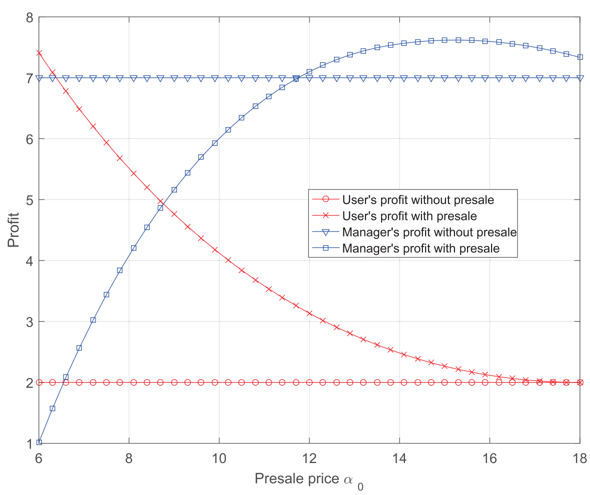

Fig. 4 The comparison of profit with or without presale mechanism.

manager, the impact of presale mechanism is more complicated. Only when α_0 is within certain range will the mechanism benefits manager. It can be seen from the diagram that the profit of manager in Stackelberg Equilibrium is 10 percent higher than without presale mechanism, also true for the user.

5. Conclusions

Dynamic spectrum allocation problem was studied in this paper. We designed a presale mechanism based on spectrum pool. Through presale process, information interaction between manager and user was achieved in an implicit way. The presale mechanism achieved dynamic risk-sharing between manager and user, also solved the problem of resource waste. Simulation results showed the mechanism promotes profits for both sides.

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