PAPER

How Centrality of Driver Nodes Affects Controllability of Complex Networks

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SUMMARY Recently, the controllability of complex networks has become a hot topic in the field of network science, where the driver nodes play a key and central role. Therefore, studying their structural characteristics is of great significance to understand the underlying mechanism of network controllability. In this paper, we systematically investigate the nodal centrality of driver nodes in controlling complex networks, we find that the driver nodes tend to be low in-degree but high out-degree nodes, and most of driver nodes tend to have low betweenness centrality but relatively high closeness centrality. We also find that the tendencies of driver nodes towards eigenvector centrality and Katz centrality show very similar behaviors, both high eigenvector centrality and high Katz centrality are avoided by driver nodes. Finally, we find that the driver nodes towards PageRank centrality demonstrate a polarized distribution, i.e., the vast majority of driver nodes tend to be low PageRank nodes whereas only few driver nodes tend to be high PageRank nodes.
key words: *complex networks, network controllability, driver nodes, nodal centrality*

1. Introduction

Complex networks are ubiquitous in natural, social, and man-made systems [1], the dynamics taking place on them has been deeply studied to understand their principles and underlying mechanisms [2]. However, as Barabási said, the ultimate proof of our understanding of complex networks is reflected in our ability to control them [3]. That is, with a suitable choice of inputs, driving the networked system from any initial state to any desired final state within finite time [4]. Although great effort [5]–[9] has been devoted to understand the controllability of complex networks, the progress is less than satisfactory. The challenge lies in how to efficiently determine the minimal number of controllers to make sure the network is fully controllable. Until recently, Liu et al. [3] made a breakthrough that they developed a minimum input theory to efficiently characterize the structural controllability of directed networks, allowing a minimum set of nodes (called driver nodes) to be identified to achieve full control. In particular, the authors proved that the structural controllability problem can be converted into an equivalent maximum matching problem [10], where external control is necessary for every unmatched node. Liu et al.'s work caused a huge sensation and triggered an avalanche of research on network controllability. Several basic issues have been carefully addressed, such as linear edge dynamics [11], exact controllability of networks [12], upper and lower bounds of energy required for control [13], robustness of controllability [14]–[16], optimization of controllability [17], [18], and so on.

Besides, it is also important to understand how network's structural characteristics affect its controllability. In this direction, there have been many good attempts. Liu *et al.* [3] first pointed out that the controllability of a network is mainly determined by its degree distribution, and thus sparse and heterogeneous networks are the most difficult to control. Banerjee and Roy [19] argued that despite degree distribution, the distance based measures such as betweenness and closeness also affect network's controllability. Pósfai *et al.* [20] studied the effects of clustering, modularity and degree correlations on network controllability, found that clustering and modularity have no discernible impact on controllability, but degree correlations show a robust effect, whose magnitude and direction depends on the type of correlation. Menichetti *et al.* [21] claimed that the structural controllability of a network depends strongly on the fraction of nodes with in-degree and out-degree equal to one and two.

Recently, some studies start to pay attention to the intrinsic properties of driver nodes themselves. The driver node, applying external inputs to which can bring network under full control, is in the central position of network controllability. Therefore, it is necessary and fundamental to first investigate the properties of driver nodes themselves before studying the overall structural impact on controllability. In fact, Liu *et al.* [3] have showed that the driver nodes tend to avoid high degree nodes (hub nodes). There also have been many related works on how to choose driver nodes focusing on centrality measures. Moradiamani *et al.* [22] proposed a new centrality measure and compared with the cases with the driver nodes with the proposed measure and the cases with other centrality measures. Jalili *et al.* [23] also proposed an optimal placement strategy and discussed the difference between effects of the chosen nodes from the proposed method and the nodes with different network centrality on the controllability of networks.

Since degree is just one metric of nodal centrality, and nodal centrality has been shown to be crucial to many network dynamics such as synchronization [24] and robustness [25], therefore, it is natural and interesting to ask: how is nodal centrality of driver nodes? In this paper, we are dedicated to studying the nodal centrality of driver nodes

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in controlling complex networks. Seven popular metrics of nodal centrality, including in-degree, out-degree, betweenness, closeness, eigenvector, Katz and PageRank, are studied to show the tendencies of driver nodes towards various nodal centralities. Many previous valuable works are dedicated to proposing new centrality measures [22], finding driver nodes or key nodes [9], designing an optimal placement strategy [23] or designing attacking strategies [26], but our paper focuses on studying the properties of driver nodes in terms of their centralities in detail. Our contributions are as follows: (1) The tendencies of driver nodes towards various nodal centralities are studied. (2) We find that the driver nodes tend to be low in-degree but high out-degree nodes. (3) We find that the driver nodes tend to be low betweenness but relatively high closeness nodes. (4) We find that both the eigenvector centrality and Katz centrality of most driver nodes are low. (5) We find that the vast majority of driver nodes have low PageRank centrality whereas only few driver nodes have high PageRank centrality.

The rest of the paper is organized as follows. In Sect. 2, we give a brief review of network controllability. In Sect. 3, we investigate the relationship between centrality of driver nodes and controllability of complex networks based on numerical results and related discussions. Finally, Sect. 4 concludes the whole paper.

2. Network Controllability

Consider a network of N nodes governed by the following linear time invariant dynamics:

$$\frac{d\boldsymbol{x}(t)}{dt} = \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T$ captures the states of N nodes at time t, $\mathbf{A} \in \mathbb{R}^{N\times N}$ denotes the coupling matrix, in which a_{ij} represents the weight of a directed link from node j to node i (for undirected networks, $a_{ij} = a_{ji}$). $\boldsymbol{u}(t) = (u_1(t), u_2(t), \ldots, u_M(t))^T$ is the input vector of M external controllers, and $\mathbf{B} \in \mathbb{R}^{N\times M}$ $(M \leq N)$ is the input matrix which identifies nodes (called driver nodes) controlled by external controllers.

According to the classic Kalman rank condition [4], the system described by Eq. (1) is said controllable if it can be driven from any initial state to any desired final state within finite time, which is possible if and only if the $N \times NM$ controllability matrix

$$\mathbf{C} = (\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \ldots, \mathbf{A}^{N-1}\mathbf{B}) \tag{2}$$

has full rank, that is

$$\text{rank}(\mathbf{C}) = N \tag{3}$$

The standard way to address the controllability problem is to find a suitable control matrix $\mathbf{B}$ consisting of the minimal number of driver nodes so as to satisfy the Kalman rank condition [12]. However, the practical difficulty lies in that there are 2^N possible combinations of selecting driver nodes, which is computationally prohibitive for large networks. Even though one can enumerate all the combinations efficiently, the link weights (a_{ij}) are often unknown for real-world networks. Therefore, Liu et al. [3] introduced the so called structurally controllability [27] to complex networks to overcome the inherently incomplete knowledge of link weights. A matrix is called structured matrix if its elements are either constant zeros or independent free parameters. The system $(\mathbf{A}, \mathbf{B})$ is said to be structurally controllable if it is possible to fix non-zeros in $\mathbf{A}$ and $\mathbf{B}$ to certain values so that rank$(\mathbf{C}) = N$ [27]. The structurally controllable system can be shown to be controllable for almost all weight combinations except for some pathological cases [3].

Liu et al. [3] also proved that the structural controllability of a network can be mapped into its maximum matching [10], where the unmatched nodes are exactly the driver nodes we are looking for. Since the maximum matching problem can be solved in $O(\sqrt{N}L)$ time, where L is the number of links in the network, it is efficient to detect driver nodes for any directed networks. Finally, the controllability n_D of a network is characterized by the proportion of the minimal number of driver nodes N_D, i. e.

$$n_D = N_D/N \tag{4}$$

3. Relationship between Centrality of Driver Nodes and Controllability of Complex Networks

The method used in this paper is simple and straightforward, which was first adopted by Liu et al. [3] to show the driver nodes tend to avoid hub nodes. Specifically speaking, we divide all the nodes into three groups of equal size according to some metric of centrality low-centrality group, medium-centrality group and high-centrality group, and then we count the fraction of driver nodes falling in each group to characterize the tendency of driver nodes towards this centrality. The canonical Erdös-Rényi (ER) random [28], Barabási-Albert (BA) scale-free [29], Watts-Strogatz (WS) small-world [30], and Newman-Watts (NW) small-world [31] networks are used as benchmarks, and all the network size is set to $N = 1000$.

3.1 In-Degree and Out-Degree Centralities of Driver Nodes

The in-degree k_{v_in} of a node v is defined as the number of edges coming into v whereas the out-degree k_{v_out} is defined as the number of edges going out of v. The degree k_v of a node v is defined as the number of edges adjacent to v, for directed networks, $k_v = k_{v\_\text{in}} + k_{v\_\text{out}}$. Despite that Liu et al. [3] have showed the driver nodes tend to avoid high degree nodes, we want to further know the specific tendency of driver nodes towards in-degree centrality and out-degree centrality.

In Fig. 1, we show the tendency of driver node to in-degree centrality. The bars show the fractions of driver nodes, f_D, among low, medium and high in-degree nodes for

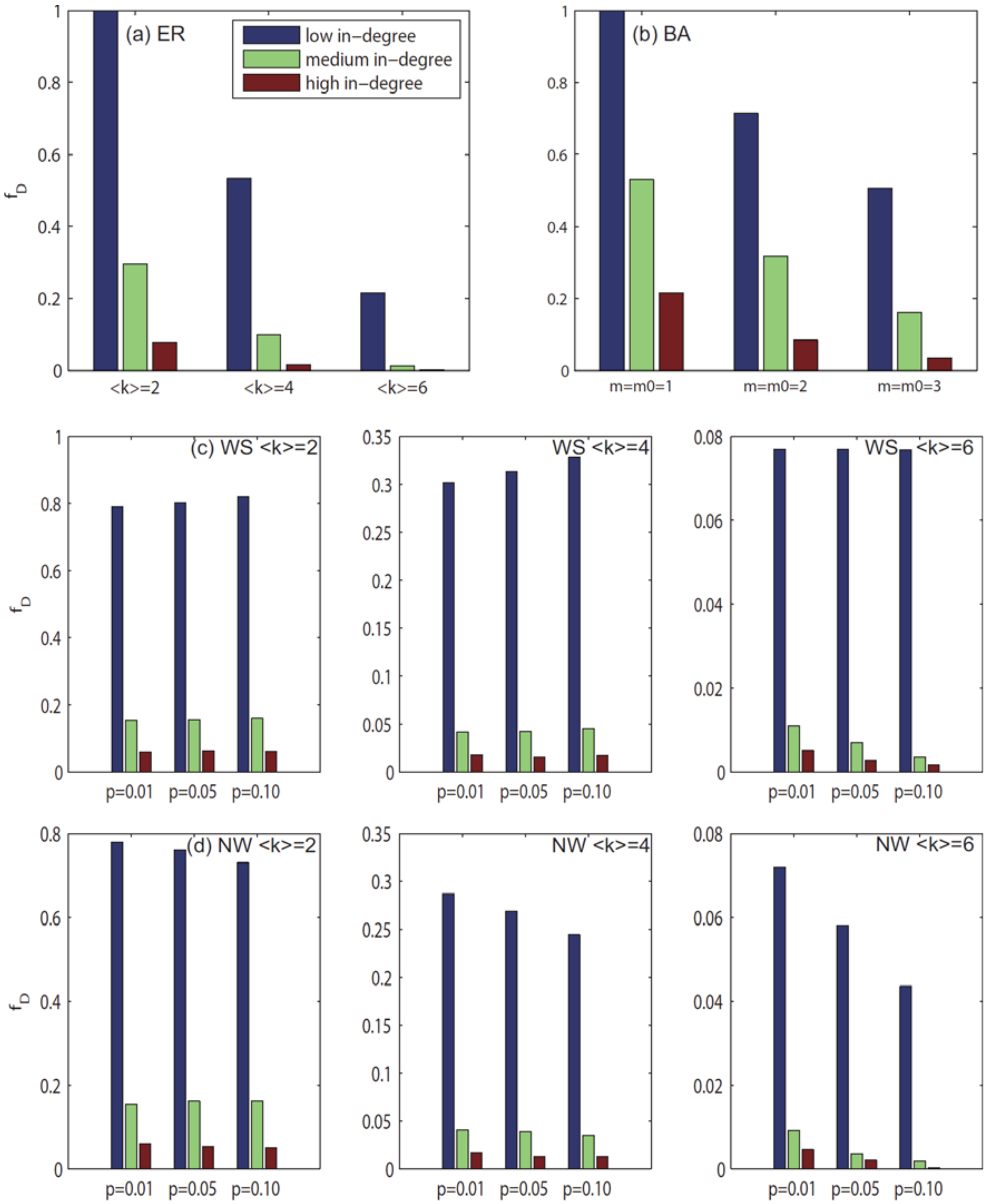

Fig. 1 The tendency of driver node to in-degree centrality.

(a) ER, (b) BA, (c) WS and (d) NW networks with different parameters. The legend is shown in (a), the results are averaged over 50 independent runs. From Fig. 1, we can see that for the ER and BA networks, f_D among low in-degree nodes is significantly higher than both f_D in the medium and high in-degree groups, this trend is even more obvious for the WS and NW small networks shown in Fig. 1 (c) and Fig. 1 (d), which clearly indicates that the driver nodes tend to be low in-degree nodes. In addition, comparing the histograms under different average degree $<k>$, we notice that as $<k>$ increases, f_D in all the groups decreases, which means that the total number of driver nodes N_D is declining, verifying the previous conclusion that denser networks are easier to control [3]. It should be stressed that for WS and NW networks with different $<k>$, the tendencies of driver nodes show similar behaviors as shown in Fig. 1 (c) and Fig. 1 (d), and these behaviors have also been checked in other experiments. Therefore, hereafter for WS and NW networks, we only show the results with $<k> = 2$ as representatives.

Figure 2 shows the tendency of driver nodes towards

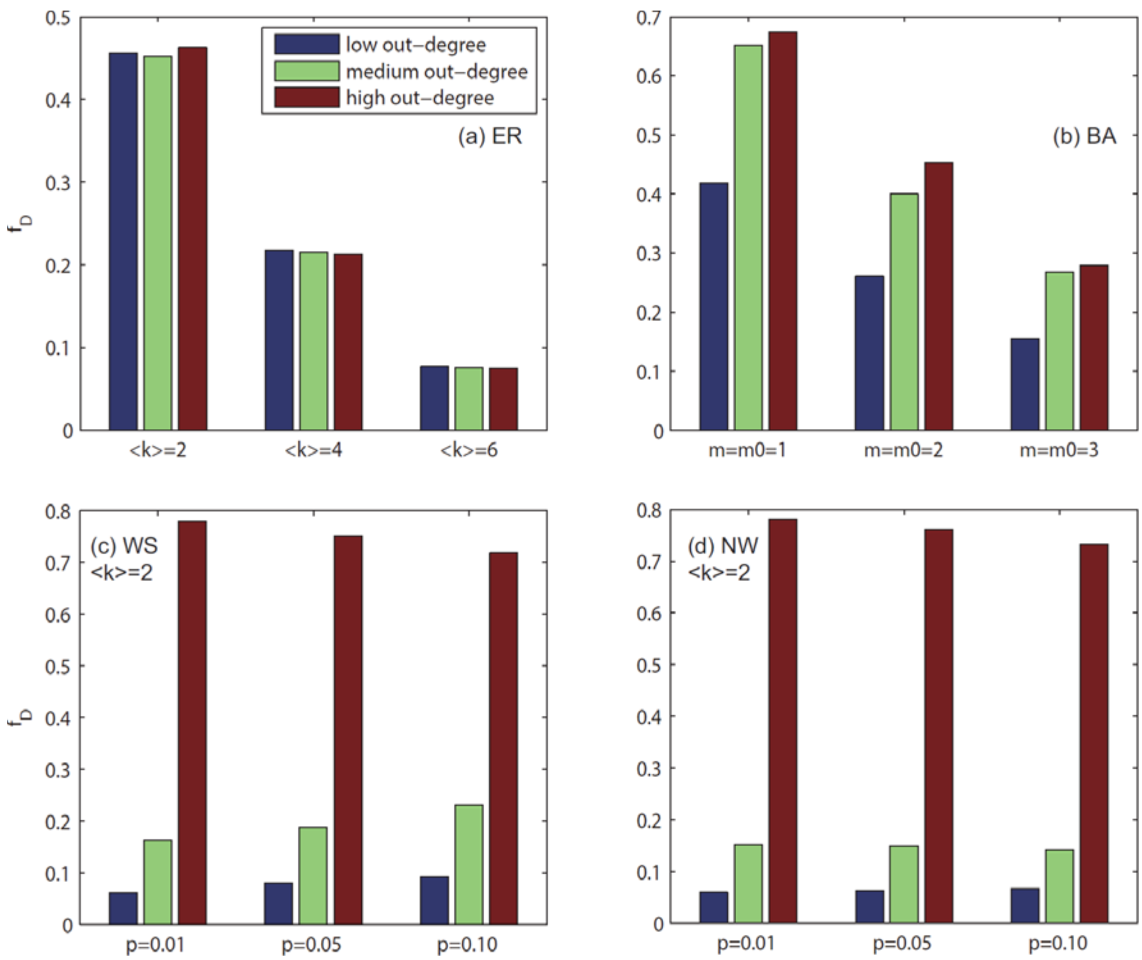

Fig. 2 The tendency of driver nodes towards out-degree centrality.

out-degree centrality. The bars show the fractions of driver nodes, f_D, among the low, medium and high out-degree nodes for (a) ER, (b) BA, (c) WS with $<k> = 2$ and (d) NW with $<k> = 2$ networks under different parameters. The legend is shown in (a), the results are averaged over 50 independent runs. From the results, we can see that for the ER network, f_D in low, medium and high out-degree groups are almost equal, indicating that the driver nodes have no bias towards the out-degree centrality for random networks. However, this unbias behavior starts to change for BA scale-free network, where f_D in the high out-degree group is higher than that in the medium group, and so is f_D in the medium out-degree group than that in the low group, suggesting that the driver nodes, to some extent, prefer to be relatively high out-degree nodes. This preference finally reaches its peak in the WS and NW networks, where the fraction of driver nodes in the high out-degree group is far higher than all the other groups, clearly showing that the driver nodes tend to be high out-degree nodes for small-world networks.

Put these results together, we find that the driver nodes tend to low in-degree but relatively high out-degree nodes, which can be seen as a refinement of previous conclusion that the driver nodes tend to avoid high degree nodes [3]. One exception is the ER random network where the driver nodes show no bias towards the out-degree centrality.

3.2 Betweenness Centrality of Driver Nodes

The nodal betweenness centrality measures the extent to which a node is needed by others when connecting along the shortest paths. Mathematically, the betweenness B_v of a node v is defined as

$$B_v = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{5}$$

where σ_{st} is the total number of shortest paths existing from node s to node t, $\sigma_{st}(v)$ is the number of shortest st-paths that pass through v. In Fig. 3, we show the tendency of driver nodes towards betweenness centrality. The bars show the fractions of driver nodes, f_D, among the low, medium and high betweenness nodes for (a) ER, (b) BA, (c) WS with $<k> = 2$ and (d) NW with $<k> = 2$ networks under different parameters. The legend is shown in (a), the results are averaged over 50 independent runs. From Fig. 3, we can see that the driver nodes in the high betweenness group account for a much smaller proportion than the other two groups for the ER random and BA scale-free networks, the disparity is even more pronounced for the WS and NW small-world net-

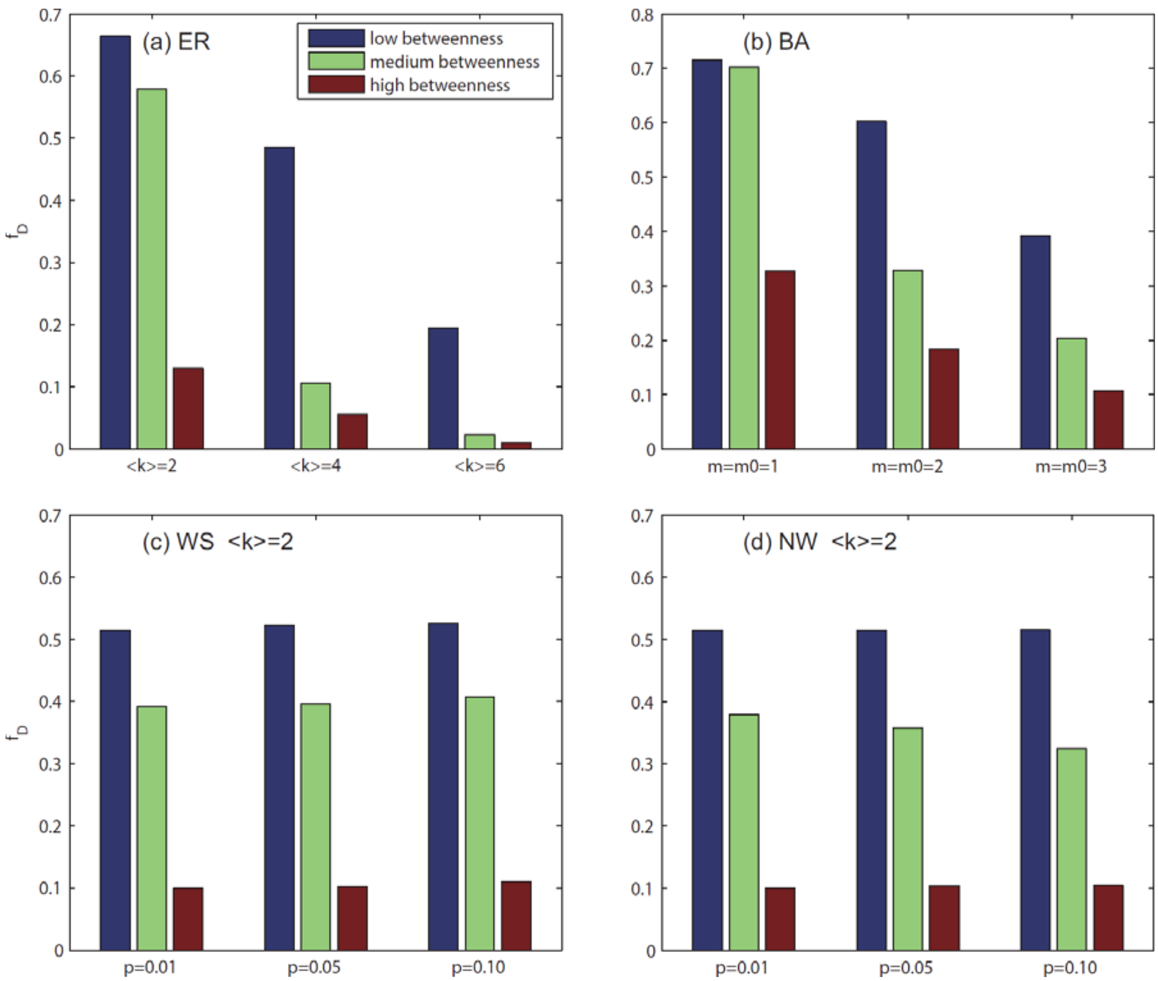

Fig. 3 The tendency of driver nodes towards betweenness centrality.

works where the former is only about 1/4 (or even 1/5) of the latter, indicating that for all the networks, the driver nodes tend to avoid high betweenness nodes. This is an interesting finding as the maximal nodal betweenness centrality has been thought as another key indicator of network dynamics synchronizability [24].

3.3 Closeness Centrality of Driver Nodes

The nodal closeness centrality measures how close a node is to other nodes. Formally, the closeness C_v of a node v is defined as the reciprocal of the sum of its distances $d(v,u)$ to all the other nodes u, i.e., $C_v = 1/\Sigma_{u \neq v} d(v,u)$. In Fig. 4, we show the tendency of driver nodes to closeness centrality. The bars show the fractions of driver nodes, f_D, among the low, medium and high closeness nodes for (a) ER, (b) BA, (c) WS with $<k> = 2$ and (d) NW with $<k> = 2$ networks under different parameters. The legend is shown in (a), the results are averaged over 50 independent runs. From Fig. 4, we can see that for the ER network, the fraction of driver nodes in the low, medium, high closeness groups are almost equal, suggesting that the driver nodes have no inclination to closeness centrality for random networks. However, for BA network, the proportions of driver nodes in the medium and high closeness groups are higher than that in the low group, the gap is even more evident for the WS and NW small-world networks, where the majority of driver nodes fall in the high and medium groups with an overwhelming advantage, which clearly indicates that the driver nodes tend to avoid low closeness nodes. Note that this opposite behavior of driver nodes towards closeness centrality compared with betweenness centrality are within our expectations, which can be more or less guessed from their reciprocal definitions.

3.4 Eigenvector Centrality and Katz Centrality of Driver Nodes

The nodal eigenvector centrality measures the importance of a node via the number of neighbor nodes and the importance of these neighbor nodes. Mathematically, the eigenvector centrality E_v of a node v is defined as

$$E_v = \frac{1}{\lambda} \sum_{u=1}^{N} a_{vu} E_u \tag{6}$$

where λ is a constant, and $a_{vu} = 1$ if there is a link from

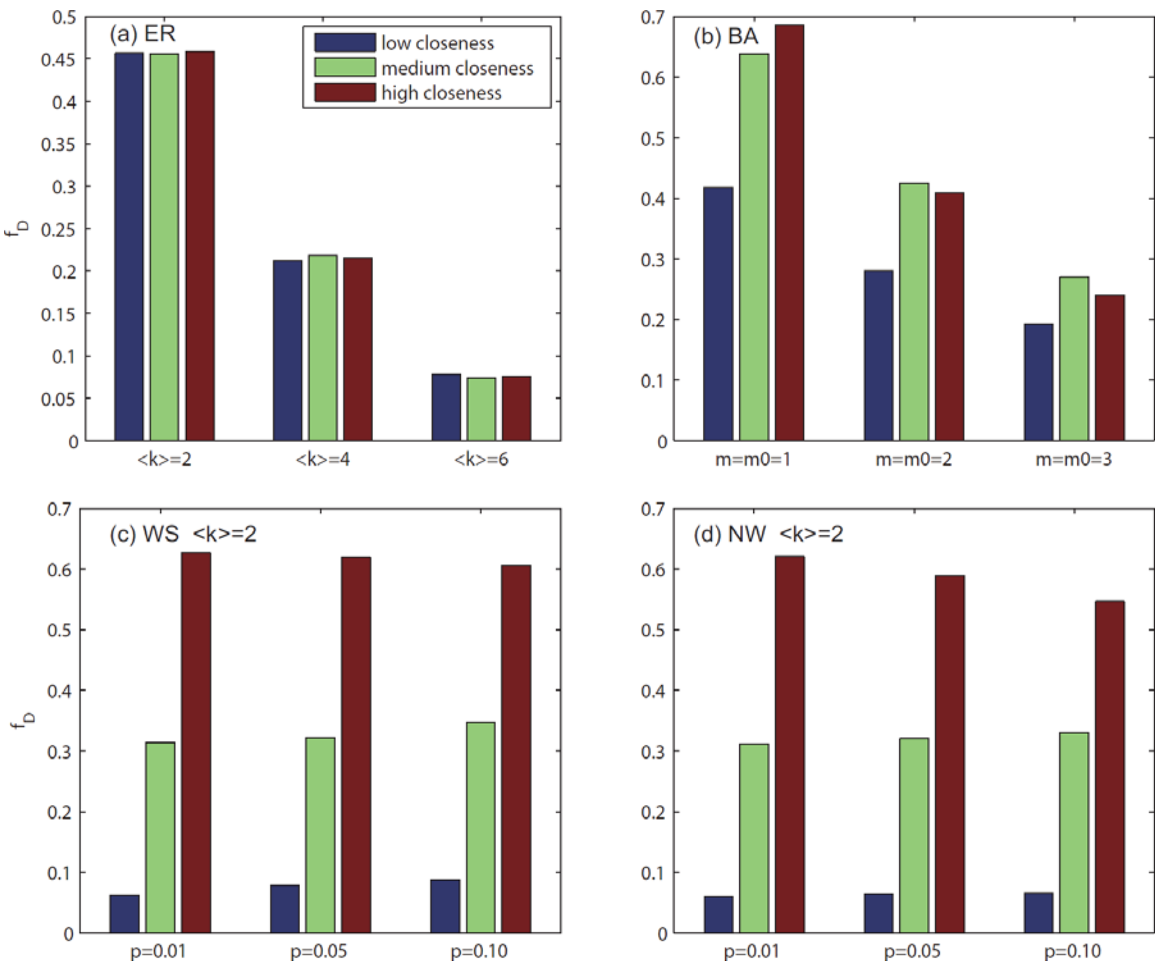

Fig. 4 The tendency of driver nodes to closeness centrality.

node v to node u, otherwise $a_{vu} = 0$.

The nodal Katz centrality [32] is a variant of the eigenvector centrality. Formally, the Katz centrality K_v of a node v is defined as

$$K_v = \alpha \sum_{u=1}^{N} a_{vu}(K_u + 1) \tag{7}$$

where $\alpha \in (0, 1)$ is an attenuation factor, and a_{vu} is the same as Eq. (6).

In Table 1 and Table 2, we show the tendencies of driver nodes towards eigenvector and Katz centralities. The mean degree for WS and NW networks is $<k> = 4$, the results are averaged over 50 independent runs. Network Parameters f_D for eigenvector centrality f_D for Katz centrality. It can be seen that for the ER, WS and NW networks, the fraction of driver nodes f_D in the low eigenvector group is 10 times larger than the other two groups, indicating that the driver nodes are much inclined to be low eigenvector centrality nodes. For the BA scale free network, although the gap is not that significant, the former is still 2 times larger than the latter. The tendency of driver nodes towards Katz centrality shows much similar behaviors, the vast majority of driver nodes fall in the low Katz centrality groups with overwhelming advantages, suggesting that the driver nodes also tend to be low Katz centrality nodes.

Table 1 The tendencies of driver nodes towards eigenvector centrality.

Network	Parameters	$f_D^{\text{low}-E_v}$	$f_D^{\text{mid}-E_v}$	$f_D^{\text{high}-E_v}$
ER	$<k> = 4$	0.5028	0.0919	0.0517
	$<k> = 6$	0.2001	0.0205	0.0080
	$<k> = 8$	0.0690	0.0026	0.0011
BA	$m=m_0=2$	0.5970	0.2504	0.2672
	$m=m_0=4$	0.2071	0.1035	0.0884
	$m=m_0=6$	0.0265	0.0049	0.0029
WS	$p=0.01$	0.2704	0.0544	0.0400
	$p=0.05$	0.2759	0.0537	0.0399
	$p=0.10$	0.2954	0.0533	0.0431
NW	$p=0.01$	0.2532	0.0514	0.0373
	$p=0.05$	0.2437	0.0412	0.0355
	$p=0.10$	0.2190	0.0440	0.0317

3.5 PageRank Centrality of Driver Nodes

The PageRank centrality [33] was proposed to measure the importance of web pages, its main idea is that the more important pages (nodes) are likely to receive more links from other pages (nodes). Mathematically, the PageRank centrality PR_v of a node v is defined as

$$PR_v = \frac{1-d}{N} + d \sum_{u=1}^{N} a_{uv} \frac{PR_u}{L(u)} \tag{8}$$

where PR_u is the PageRank centrality of node u, $L(u)$ is the out-degree of node u, $a_{uv} = 1$ if there exists a link from node-u to node-v and otherwise $a_{uv} = 0$, $d \in (0, 1)$ is a constant damping factor which is usually set around 0.85.

In Fig. 5, we show the tendency of driver nodes towards PageRank centrality. The bars show the fractions of driver nodes, f_D, among the low, medium and high PageRank centrality nodes for (a) ER, (b) BA, (c) WS with $<k> = 2$ and (d) NW with $<k> = 2$ networks under different parameters. The legend is shown in (a), the results are averaged over 50 independent runs. From Fig. 5, we can see that the driver nodes demonstrate a polarized distribution to the PageRank centrality. On the one hand, the overwhelming majority of nodes in the low PageRank groups are driver nodes, for example, for WS and NW networks, about 80% of the nodes in the low PageRank group are driver nodes, this ratio is even up to 100% for ER network with $<k> = 2$ and BA network with $m = m_0 = 1$. This extreme phenomenon has not been observed in other experiments, which clearly show that the driver nodes tend to be low PageRank nodes. On the other hand, the driver nodes in the high PageRank groups only account for less than 20%, this ratio is even down to less than 1% for the WS and NW networks, indicating that the driver nodes tend to avoid being high PageRank nodes.

Based on above results, we can find that: (1) the driver nodes tend to be low in-degree but high out-degree nodes. (2) the driver nodes tend to be low betweenness but relatively high closeness nodes. (3) both the eigenvector central-

Table 2 The tendencies of driver nodes towards Katz centrality.

Network	Parameters	$f_D^{low-E_v}$	$f_D^{mid-E_v}$	$f_D^{high-E_v}$
ER	$<k> = 4$	0.5141	0.1092	0.0232
	$<k> = 6$	0.2114	0.0144	0.0029
	$<k> = 8$	0.0703	0.0020	0.0005
BA	$m=m_0=2$	0.6649	0.2839	0.1661
	$m=m_0=4$	0.2437	0.1028	0.0527
	$m=m_0=6$	0.0282	0.0042	0.0018
WS	p=0.01	0.2794	0.0585	0.0232
	p=0.05	0.2879	0.0605	0.0224
	p=0.10	0.2999	0.0662	0.0238
NW	p=0.01	0.2662	0.0552	0.0238
	p=0.05	0.2498	0.0521	0.0186
	p=0.10	0.2299	0.0443	0.0175

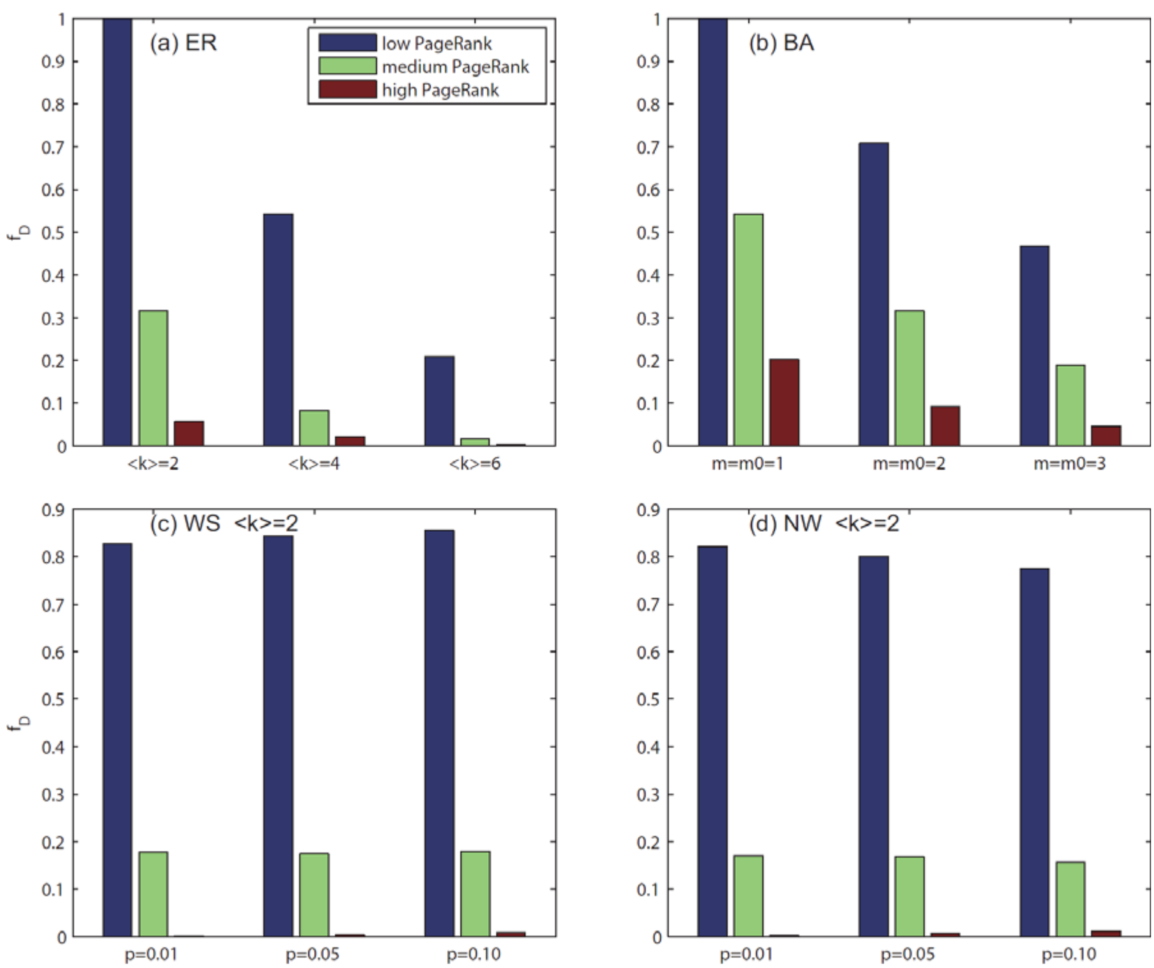

Fig. 5 The tendency of driver nodes towards PageRank centrality.

ity and Katz centrality of most driver nodes are low. (4) the vast majority of driver nodes have low PageRank centrality whereas only few driver nodes have high PageRank centrality.

Compared with previous work [3] that found that the driver nodes tend to avoid high degree nodes, our work further found that the driver nodes tend to low in-degree but relatively high out-degree nodes, which is a refinement of previous work [3]. We also found many properties that previous works have not found. Thus, the properties that our work found can serve as references for other works to design networks. For example, for previous work [22], our results can be used as a reference to design a more efficient centrality measure. For previous work [23], our findings can serve as a partial guidance to design an optimal placement strategy.

4. Conclusion

In this paper, we have studied the tendencies of driver nodes towards various nodal centralities. Extensive numerical results on the canonical model networks show that the driver nodes tend to be low in-degree but high out-degree nodes, which is a refinement of the previous conclusion that driver nodes tend to avoid high degree nodes. We also find that most of the driver nodes tend to have low betweenness centrality but relatively high closeness centrality with one exception, i.e., ER random networks, in which the driver nodes have no inclination towards the closeness centrality. Moreover, the driver nodes towards eigenvector centrality and Katz centrality show much similar behaviors, avoiding being high eigenvector and Katz nodes.

Finally, it is found that the driver nodes towards PageRank centrality demonstrate a polarized distribution, and the vast majority of driver nodes tend to be low PageRank nodes whereas only few driver nodes tend to be high PageRank nodes. Our results may help to understand the structural properties of driver nodes, and the future work will focus on studying the relations between average distance, standard deviation of degree distribution, maximal betweenness and network controllability.

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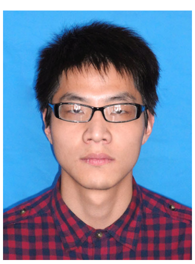

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