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Five Cells and Tilepaint are NP-Complete*

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SUMMARY   Five Cells and Tilepaint are Nikoli's pencil puzzles. We study the computational complexity of Five Cells and Tilepaint puzzles. It is shown that deciding whether a given instance of each puzzle has a solution is NP-complete.

key words: Five Cells, Tilepaint, pencil puzzle, NP-complete

1. Introduction

The Five Cells puzzle is played on a rectangular grid of cells (see Fig. 1 (a)). Initially, some of the cells contain numbers. The purpose of the puzzle is to divide the grid into *blocks* of five cells according to the following rules [1]: (i) A number in a cell shows how many lines of a block there are around it (see Fig. 1 (f)), where the number includes the lines of the outer frame. (ii) Lines must surround a five-cell block and cannot enter the block (see Fig. 1 (e) which is an invalid placement of lines).

Figure 1 (a) is the initial configuration of a Five Cells puzzle. From Figs. 1 (b)–(f), the reader can understand the basic technique for finding a solution. (b) The cell with number 0 and its four neighbors form a five-cell block, and they are surrounded by a wall. (In this paper, a set of connected line segments is called a *wall*.) Since there is a wall (outer frame) around the cell with red number 1, four blue cells belong to a block. However, the green cell with number 2 does not belong to the same block as the four blue cells, since at least one of the cells ③ and ① belongs to the same block as the green cell. (If the green cell and blue cells belong to a single block, then this block has at least six cells.) Thus, there must be a wall between the green cell and blue cells (see (c)). (c) Since the number of blue and red cells is 10 (= 4 + 6), those 10 cells are separated from the remaining cells by a wall (see (d)). (e) is an invalid placement of walls, since a red wall enters the green five-cell block. (f) is one of the multiple solutions.

The Tilepaint puzzle is played on a rectangular grid of cells (see Fig. 2 (a)). Initially, the grid is divided into *tiles* which are the areas enclosed by bold lines (see 16 tiles
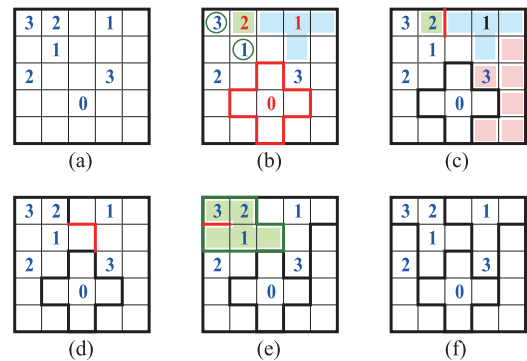
Fig. 1   (a) Initial configuration of a Five Cells puzzle. (b)–(f) are the progress from the initial configuration to a solution. (e) is an invalid placement of lines (walls).
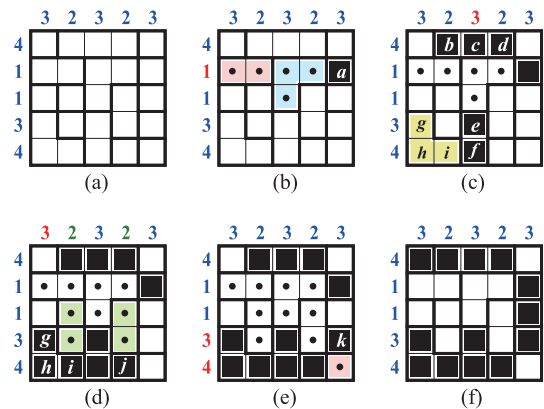
Fig. 2   (a) Initial configuration of a Tilepaint puzzle. (b)–(f) are the progress from the initial configuration to a solution.

of Fig. 2 (a)), and a number is assigned to each row and each column (see numbers 4, 1, 1, 3, 4 and 3, 2, 3, 2, 3 of Fig. 2 (a)). The purpose of the puzzle is to paint cells in black according to the following rules [2]: (i) The number of each row (resp. each column) indicates the number of black cells in the row (resp. each column) (see Fig. 2 (f)). (ii) The cells of each tile must all be colored black or left uncolored.

Figure 2 (a) is the initial configuration of a Tilepaint puzzle. Figures 1 (b)–(f) are the progress from the initial configuration to a solution. (b) Since the number of the second row is 1, cell *a* must be colored black, and cells in the red and blue tiles must not be colored black; such cells are indicated by • in Fig. 2. (c) Since the number of the third column is 3, cells *b*, *c*, *d* and *e*, *f* are colored black. If the
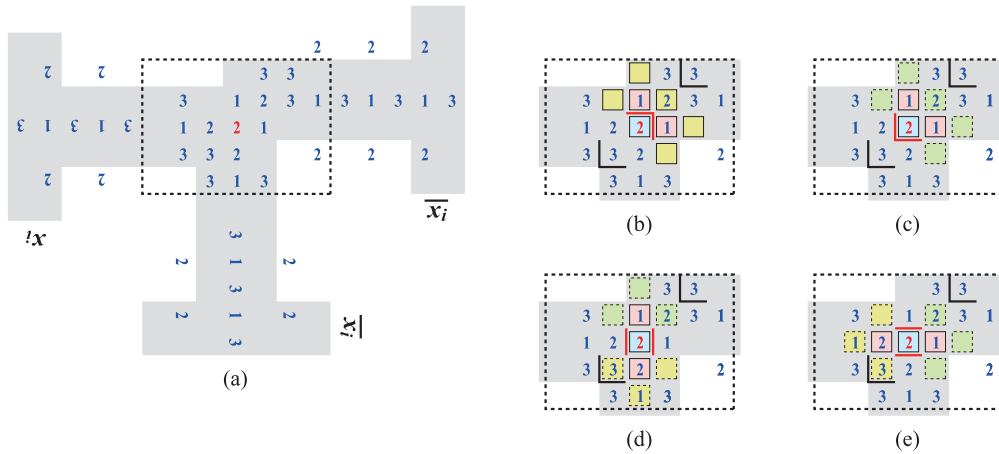
Fig. 3 (a) Variable gadget transformed from x_i. (b)–(e) Invalid placements of walls on the variable gadget.
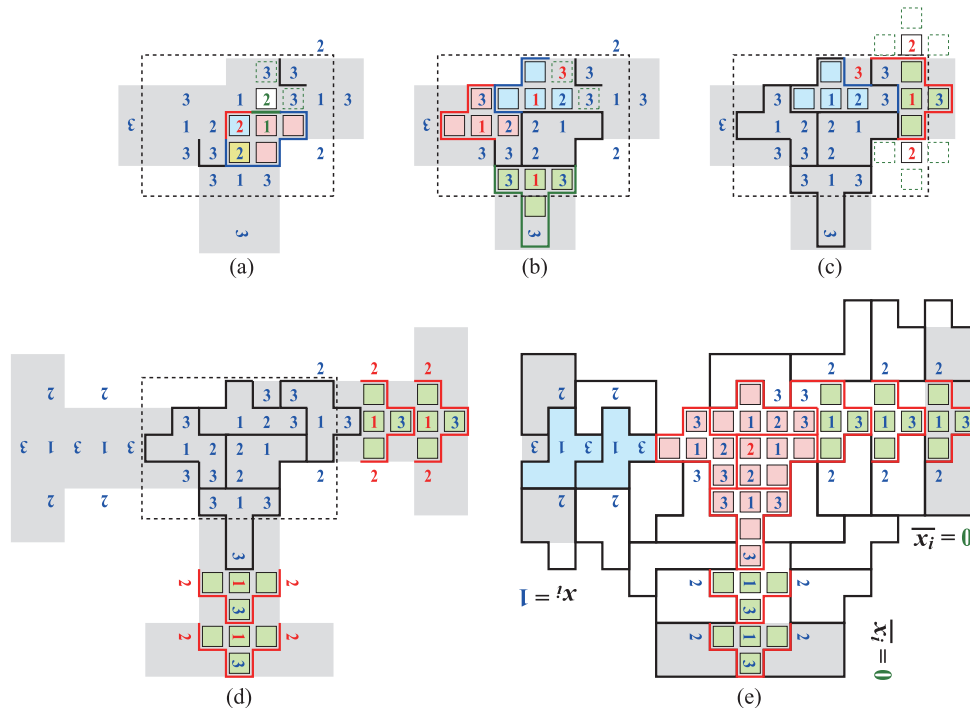
Fig. 4 Variable gadget when $x_i = 1$.

yellow tile consisting of cells g, h, i is not colored black, then the first column cannot contain 3 black cells. Therefore, cells g, h, i must be colored black (see (d)). (d) Four green cells must not be colored black. (e) Since the fourth and fifth rows have numbers 3 and 4 respectively, cell k is colored black and the red cell is not colored black. (f) is one of the multiple solutions.

In this paper, we study the computational complexity of the decision version of the Five Cells and Tilepaint puzzles. The instance of the *Five Cells puzzle problem* is a rectangular grid of cells, where the number of cells is a multiple of five, and some of the cells contain numbers. The instance of the *Tilepaint puzzle problem* is a rectangular grid

of cells which are divided into tiles, and a number is assigned to each row and each column. Each problem is to decide whether there is a solution to the instance.

Theorem 1: The Five Cells and Tilepaint puzzle problems are NP-complete.

It is clear that both problems belong to NP, since the Five Cells puzzle ends when all edges (of cells) become a thick line (wall) or a thin line (no wall), and the Tilepaint puzzle ends when all cells become a black cell or a white cell.

Nikoli is a famous Japanese publisher specializing in pencil puzzles. One of the reasons that people are attracted by Nikoli's pencil puzzles is their difficulty in finding a solu-
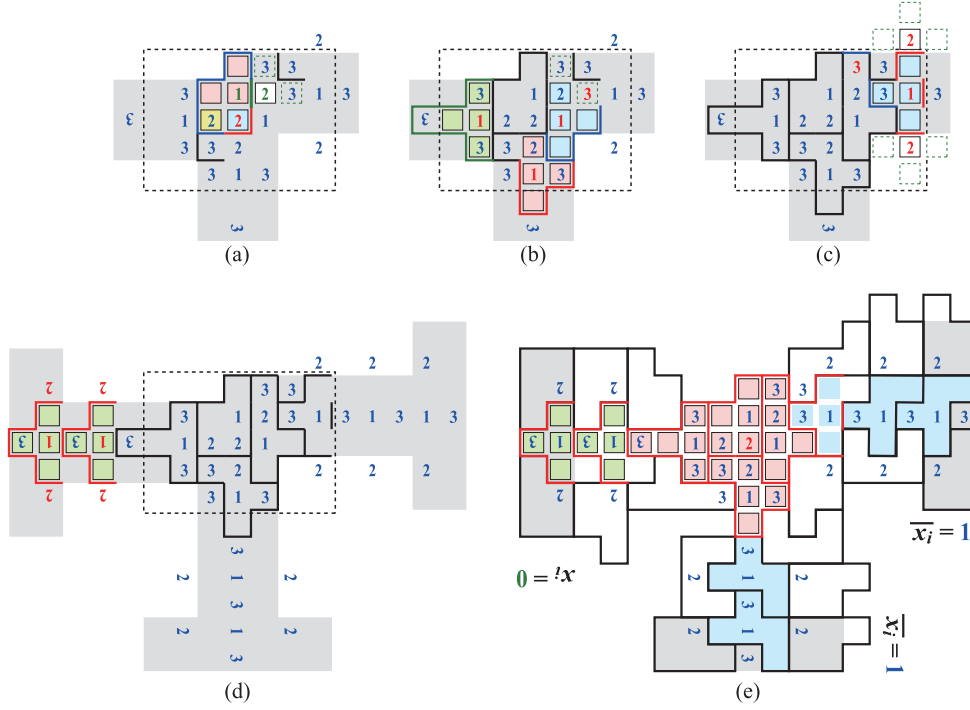
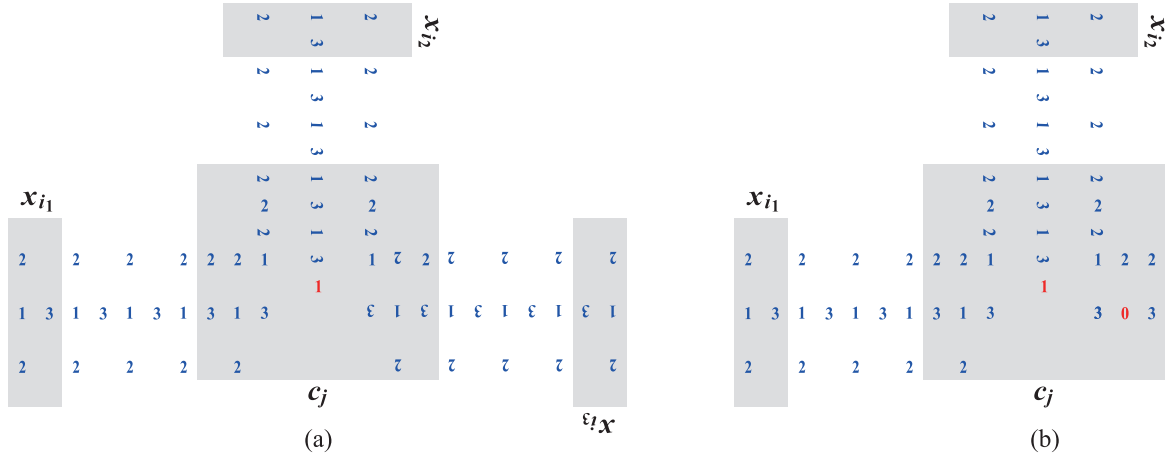**Fig. 5**   Variable gadget when $x_i = 0$.

Fig. 6   (a) Clause gadget transformed from $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. (b) When $c_j = \{x_{i_1}, x_{i_2}\}$, (a) is replaced with (b).

tion. From this perspective, many theoretical computer scientists have studied the computational complexity of pencil puzzles.

In 2009, a survey of games, puzzles, and their complexities was reported by Hearn and Demaine [11]. This book provides underlying mathematical reasons why puzzles are challenging, which explain why they are so much fun. It also mentions that puzzles serve as models of computations and offer a new way of thinking about computation. In a chapter of one-player games in this book, the authors mention about the NP-completeness of several Nikoli's puzzles. After the publication of this book, the following Nikoli's pencil puzzles were shown to be NP-complete:

Dosun-Fuwari [17], Fillmat [24], Hashiwokakero [6], Hebi, Satogaeri, and Suraromu [20], Herugolf and Makaro [16], Kurodoko [21], Kurotto and Juosan [18], LITS and Nori-nori [7], Moon-or-Sun, Nagareru, and Nurimeizu [12], Numberlink [3], Nurimisaki and Sashigane [19], Pencils [22], Pipe Link [25], Shakashaka [9], Shikaku and Ripple Effect [23], Sto-Stone [5], Tatamibari [4], Usowan [15], Yajilin and Country Road [13], and Yosenabe [14].

However, three of Nikoli's puzzles, Double Choco, Five Cells, and Tilepaint, have not been proved to be NP-complete, although those puzzles are difficult (and fun) enough to be worth challenging. This paper provides the NP-completeness of Five Cells and Tilepaint puzzles. Prov-
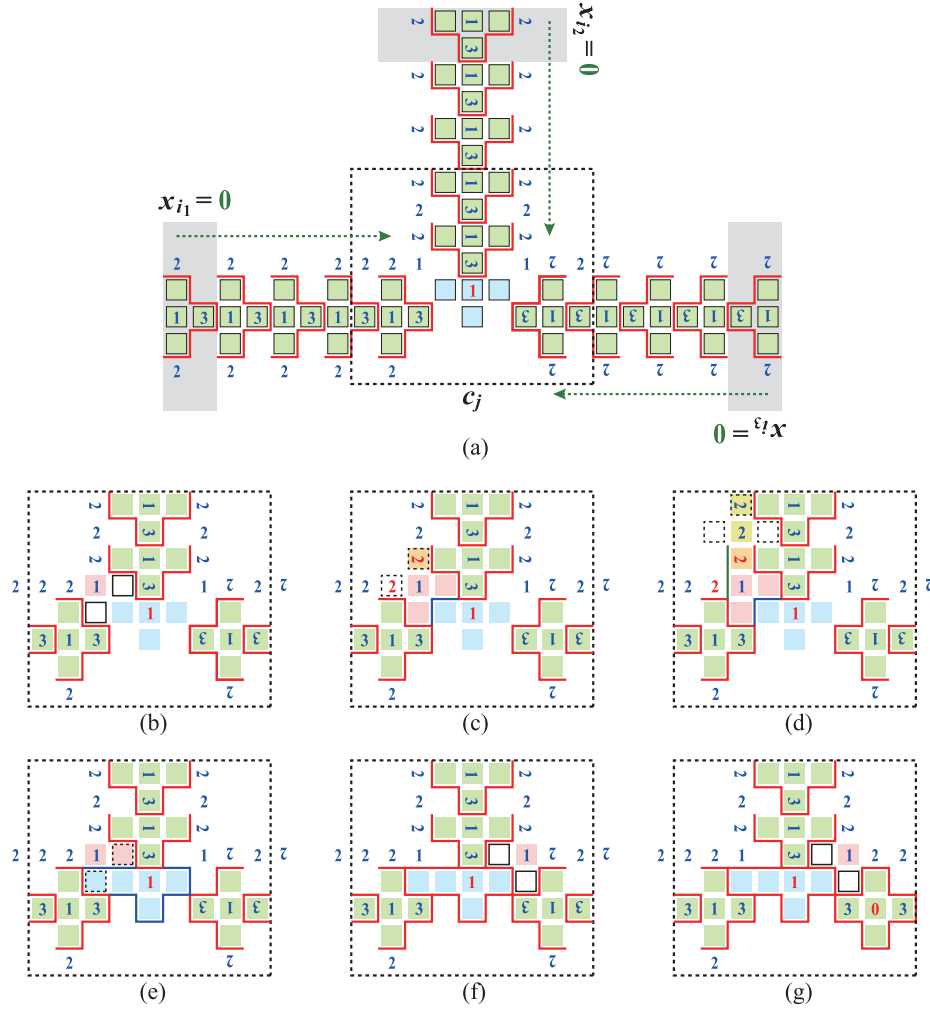
Fig. 7 (a)–(f) If $x_{i_1} = x_{i_2} = x_{i_3} = 0$, there is no solution to red number 1. (g) When $c_j = \{x_{i_1}, x_{i_2}\}$, (f) is replaced with (g).

ing the computational complexity of Double Choco remains an open problem.

2. NP-Completeness of Five Cells

2.1 3SAT Problem

The definition of 3SAT is mostly from [10]. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a set of Boolean *variables*. Boolean variables take on values 0 (false) and 1 (true). If x is a variable in U, then x and $\overline{x}$ are *literals* over U. The value of $\overline{x}$ is 1 (true) if and only if x is 0 (false). A *clause* over U is a set of literals over U, such as $\{\overline{x_1}, x_3, x_4\}$. It represents the disjunction of those literals and is *satisfied* by a truth assignment if and only if at least one of its members is true under that assignment.

An instance of PLANAR 3SAT is a collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses over U such that (i) $|c_j| \leq 3$ for each $c_j \in C$ and (ii) the bipartite graph $G = (V, E)$, where $V = U \cup C$ and E contains exactly those pairs $\{x, c\}$ such that either literal x or $\overline{x}$ belongs to the clause c, is planar.

The PLANAR 3SAT problem asks whether there exists some truth assignment for U that simultaneously satisfies all the clauses in C. This problem is known to be NP-complete. For example, $U = \{x_1, x_2, x_3, x_4\}$, $C = \{c_1, c_2, c_3, c_4\}$, and $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$ provide an instance of PLANAR 3SAT. For this instance, the answer is "yes," since there is a truth assignment $(x_1, x_2, x_3, x_4) = (0, 1, 0, 0)$ satisfying all clauses. It is known that PLANAR 3SAT is NP-complete even if each variable occurs exactly once positively and exactly twice negatively in C [8].

2.2 Transformation from an Instance of 3SAT to a Five Cells Puzzle

We present a polynomial-time transformation from an arbitrary instance C of PLANAR 3SAT to a Five Cells puzzle such that C is satisfiable if and only if the puzzle has a solution.

Each variable $x_i \in \{x_1, x_2, \ldots, x_n\}$ is transformed into the variable gadget as illustrated in Fig. 3 (a). (Note that the
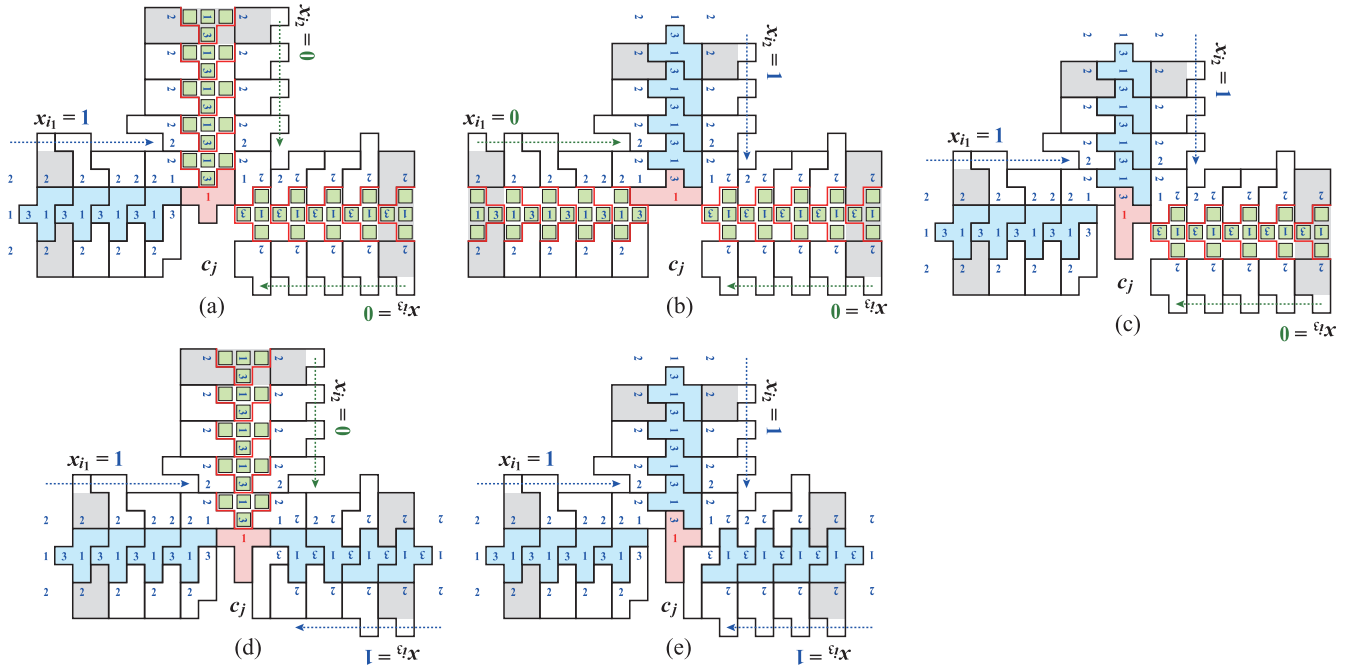
Fig. 8   If at least one of x_{i_1}, x_{i_2}, and x_{i_3} is 1, then there is a solution.
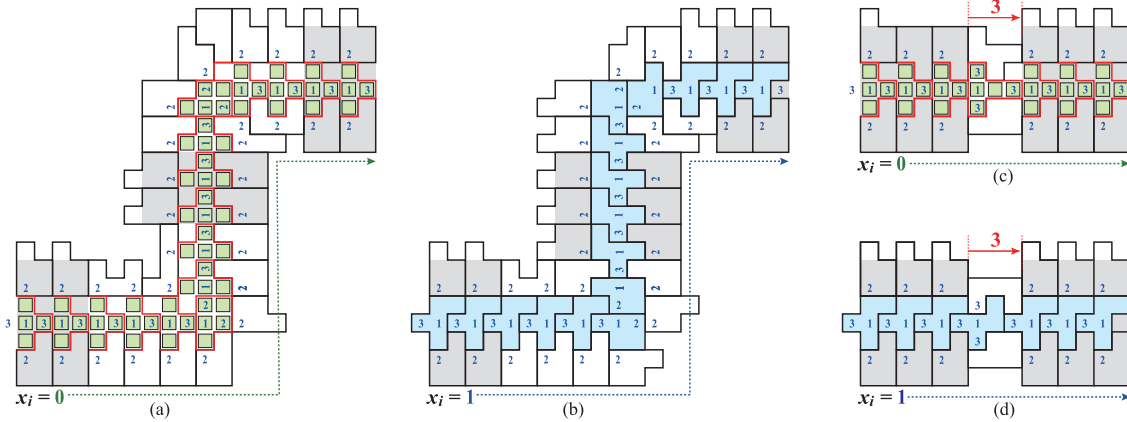
Fig. 9   (a,b) Connection gadget. (c,d) Connection gadget of odd length.

instances of 3SAT considered in this paper have the restriction explained at the end of Sect. 2.1.) Consider the red number 2. There are two types of solutions: The red number 2 is surrounded by a "⌐-shaped wall" (see Figs. 4 (a) and 4 (e)) or a "⌐-shaped wall" (Figs. 5 (a) and 5 (e)). On the other hand, there are no solutions such that the red number 2 is surrounded by ⌐-shaped, ∟-shaped, ∥-shaped, or =-shaped walls (see Figs. 3 (b)–(e)).

Consider the red number 2 in the blue cell of Fig. 3 (b). Two red cells with number 1 must belong to the same block as the five yellow cells. Since those red and yellow cells form a seven-cell block, Fig. 3 (b) is an invalid placement of walls. (In the following, a "cell with number i" is sometimes simply called a "cell i.")

Consider the red walls in Fig. 3 (d). The blue cell with red number 2 must belong to the same block as the two red

cells 1 and 2. The red cell 1 (resp. red cell 2) belongs to the same block as two of the three green cells (resp. one of the three yellow cells). This implies that the blue cell with red number 2 belongs to a block of at least six cells. Thus, Fig. 3 (d) is an invalid placement of walls. By a similar reason, Figs. 3 (c) and 3 (e) are also invalid placements of walls. Note that if two cells with number 3 are adjacent, then there must be a wall between them (see black walls in Figs. 3 (b)–(e)).

Consider the blue cell with red number 2 of Fig. 4 (a). This blue cell belongs to the same block as red cell 1 and yellow cell 2. If the red cell 1 and white cell 2 belong to a single block, then this block has at least six cells. Thus, there is a wall between them (see the green wall in Fig. 4 (a)), and hence red and yellow cells are surrounded by a blue wall.

In Fig. 4 (b), consider blue, red, and green cells with

red number 1. They belong to three four-cell sets of blue, red, and green cells, which are surrounded by blue, red, and green walls, respectively. Since the red number 3 cannot


$x_i = 0$ (a)

$x_i = 1$ (b)
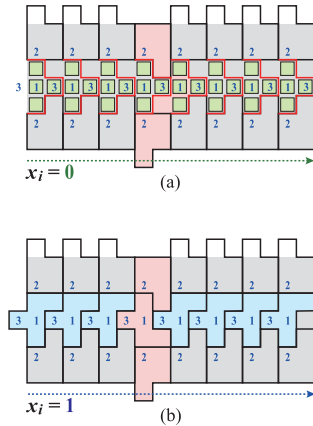
Fig. 10 If we flip three five-cell blocks upside down (see red blocks), then we can increase (resp. decrease) the area above (resp. below) the connection gadget.

belong to the same block as the blue cells, two blue walls are placed in Fig. 4 (c).

In Fig. 4 (c), any white cell with red number 2 cannot belong to the same block as the four green cells. (If the red number 1 and a red number 2 belong to a single block, then this block contains at least six cells.) Thus, three green cells are surrounded by a red wall. For a similar reason, four red walls are placed in Fig. 4 (d). Figure 4 (e) is one of multiple solutions to the variable gadget. In this figure, there are sets of four green cells which form ⊢-shaped blocks surrounded by red walls, and there are blue five-cell blocks. They play important roles in this section; the former and latter correspond to assignments $\overline{x_i} = 0$ and $x_i = 1$, respectively. Explanations for Fig. 5 are omitted, since they are similar to Fig. 4.

Clause $c_j \in \{c_1, c_2, \ldots, c_m\}$ is transformed into the clause gadget as illustrated in Fig. 6 (a). In this figure, each of the three 7×2 grey areas corresponds to a 7×2 grey area of Figs. 4 (e) and 5 (e). Let $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. Suppose $x_{i_1} = x_{i_2} = x_{i_3} = 0$ (see Fig. 7 (a)). There are no solutions such that red walls are placed as in Fig. 7 (a); the reason is
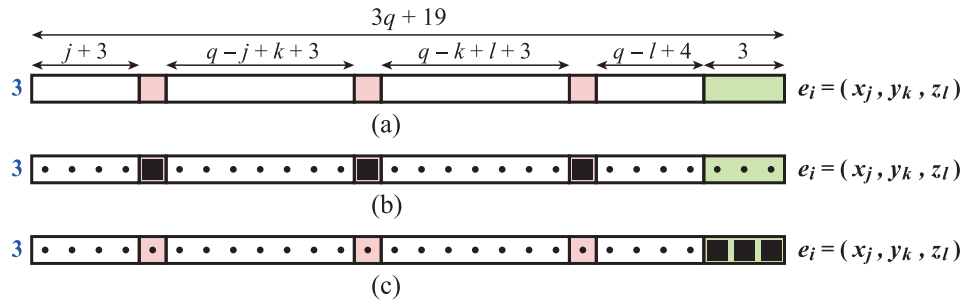
Fig. 11 (a) Gadget transformed from a triple $e_i = (x_j, y_k, z_l) \in M$ when $|X| = |Y| = |Z| = q = 3$ and $(j, k, l) = (1, 2, 3)$. (b) and (c) are two possible solutions to this gadget.
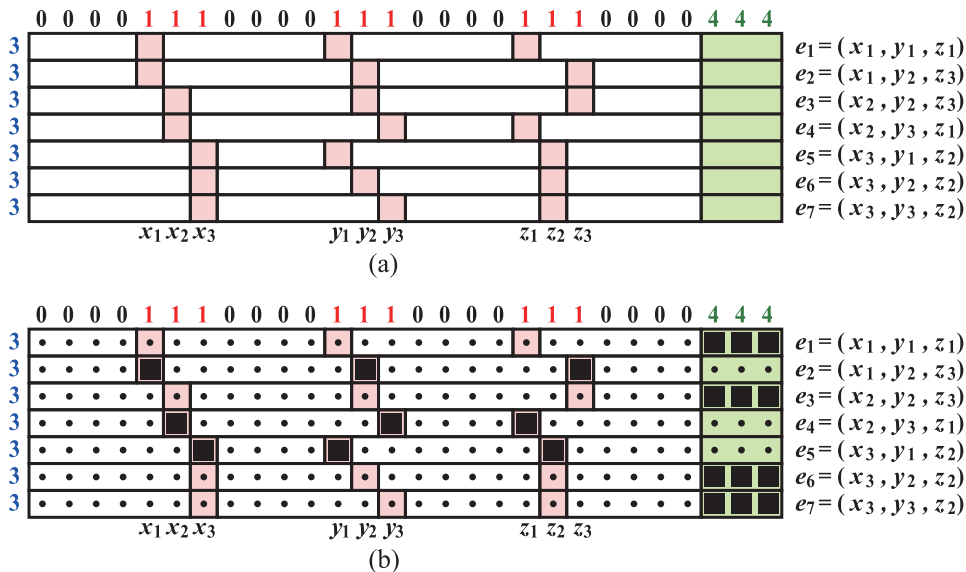
Fig. 12 (a) Tilepaint puzzle transformed from 3DM $M = \{e_1, e_2, \ldots, e_7\}$. Each column of the green area has number 4 ($= m - q = 7 - 3$). (b) From the solution of the Tilepaint puzzle, one can see there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.
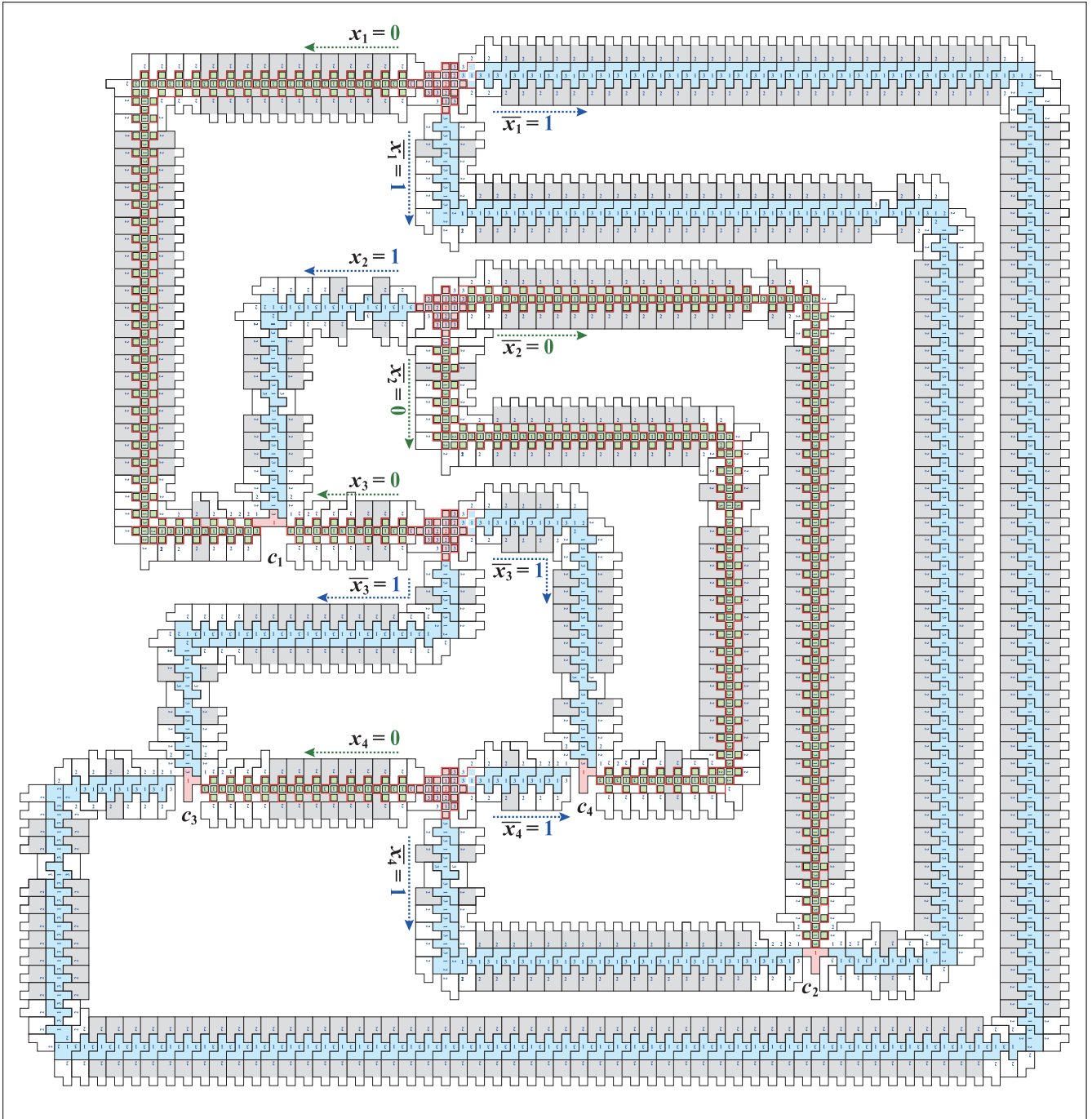
Fig. 13 A Five Cells puzzle transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. From the solution of the puzzle, one can see that the assignment $(x_1, x_2, x_3, x_4) = (0, 1, 0, 0)$ satisfies all clauses of C. (The reader should see this figure on a big-screen PC.)

given in the next paragraph (see Figs. 7 (b)–(f)).

Assume for contradiction that two white-square cells of Fig. 7 (b) belong to the same block as the red cell 1 (see Fig. 7 (c)). Then, one of the two cells 2 in dotted squares of Fig. 7 (c) must belong to the same block as the red cell 1 (see the orange cell). In Fig. 7 (d), red, orange, and yellow cells form a block of at least six cells, a contradiction. Therefore,

the two white-square cells of Fig. 7 (b) must belong to two different blocks (see dotted squares of Fig. 7 (e)), and blue cells form a five-cell block. However, the two white-square cells of Fig. 7 (f) belong to a single block. By a reason similar to Fig. 7 (b), there is no solution such that the two white-square cells belong to a single five-cell block. Namely, the placement of red walls in Fig. 7 (a) is invalid. Hence, there

is no solution to the clause gadget when $x_{i_1} = x_{i_2} = x_{i_3} = 0$. On the other hand, if at least one of the variables x_{i_1}, x_{i_2}, and x_{i_3} is 1 (see Fig. 8), there is a valid placement of walls.

If c_j consists of two literals, then the corresponding clause gadget is Fig. 6 (b) (see also Fig. 7 (g)). The gadget of Fig. 6 (b) is essential, since it is known that 3SAT is polynomial-time solvable if (i) every clause $c_j \in C$ has size $|c_j| = 3$ and (ii) every variable occurs exactly once positively and exactly twice negatively in C [8].

Figure 9 (a,b) is a connection gadget. In Fig. 9 (a), if the bottom left ⊢-shaped block of green cells is surrounded by a red wall, then the upper right ⊢-shaped block must be surrounded by a red wall. On the other hand, in Fig. 9 (b), if the bottom left pair of numbers 3 and 1 belong to the blue five-cell block, then the upper right pair can belong to the blue five-cell block.

In the connection gadget, numbers 3 and 1 appear alternately (see $3131\cdots$ in Fig. 9 (a,b)). If you want the distance between two numbers 3 to be odd, then the gadget of Fig. 9 (c,d) are used. (Fig. 10 is explained later.)

Figure 13 is a Five Cells puzzle transformed from $C = \{c_1, c_2, c_3, c_4\}$, where $c_1 = \{x_1, x_2, x_3\}$, $c_2 = \{\overline{x_1}, \overline{x_2}, \overline{x_4}\}$, $c_3 = \{\overline{x_1}, \overline{x_3}, x_4\}$, and $c_4 = \{\overline{x_2}, \overline{x_3}, \overline{x_4}\}$. In this figure, there are six large white areas separated by connection, variable, and clause gadgets. Those gadgets separating six white areas are embedded on a sufficiently large space so that those white areas can easily be filled up with five-cell blocks. If there are white areas such that the number of cells is not a multiple of five, then you can use the connection gadget given in Fig. 10. Consider two white areas separated by a connection gadget. If we flip three five-cell blocks upside down (see red blocks of Fig. 10), then we can decrease and increase the numbers of cells in the lower and upper areas separated by the connection gadget, respectively. Recall that the puzzle is played on a rectangular grid of cells such that the number of cells is a multiple of five (see Figs. 1 and 13). Since connection, variable, and clause gadgets consist of five-cell blocks, the total number of cells in all the white areas is also a multiple of five. Therefore, by using the gadget of Fig. 10, all the white areas can be filled up with five-cell blocks. From this construction, the instance C of PLANAR 3SAT is satisfiable if and only if the corresponding Five Cells puzzle has a solution.

3. NP-Completeness of Tilepaint

3.1 3-Dimensional Matching Problem

An instance of the *3-Dimensional Matching Problem* (3DM) [10] is a set $M \subseteq X \times Y \times Z$, where X, Y, and Z are disjoint sets having the same number q of elements. The 3DM problem asks whether M contains a matching, i.e., a subset $M' \subseteq M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate. This problem is known to be NP-complete. For example, $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $Z = \{z_1, z_2, z_3\}$, $M = \{e_1, e_2, \ldots, e_7\}$, and

$$e_1 = (x_1, y_1, z_1), \ e_2 = (x_1, y_2, z_3), \ e_3 = (x_2, y_2, z_3),$$

$$e_4 = (x_2, y_3, z_1), \ e_5 = (x_3, y_1, z_2), \ e_6 = (x_3, y_2, z_2),$$
$$e_7 = (x_3, y_3, z_2)$$

provide an instance of 3DM. For this instance, the answer is "yes," since there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.

3.2 Transformation from an Instance of 3DM to a Tilepaint Puzzle

We present a polynomial-time transformation from an arbitrary instance M of 3DM to a Tilepaint puzzle such that M contains a matching if and only if the puzzle has a solution.

Each triple $e_i = (x_j, y_k, z_l) \in M$ is transformed into the gadget as illustrated in Fig. 11 (a). This gadget is of height 1 and width $3q + 19$, and it is composed of eight tiles of length $j + 3$, 1, $q - j + k + 3$, 1, $q - k + l + 3$, 1, $q - l + 4$, and 3, where $|X| = |Y| = |Z| = q$. (In Fig. 11 (a), $q = 3$ and $(j, k, l) = (1, 2, 3)$.) Since the number 3 is assigned to this one-row gadget, there are two possible solutions (see Figs. 11 (b) and 11 (c)).

The instance $M = \{e_1, e_2, \ldots, e_m\}$ is transformed into the $m \times (3q + 19)$ cells (see Fig. 12) which is composed of m gadgets of Fig. 11 (a). For every $d \in \{1, 2, 3\}$, the $((q + 4)d - q + 1)$th through $((q + 4)d)$th columns have numbers 1 (see red numbers 1 of Fig. 12). The $(3q+17)$th through $(3q+19)$th columns have numbers $m - q$ (see green numbers 4 (= $m - q = 7 - 3$)). From this construction, the Tilepaint puzzle of Fig. 12 (a) has a solution if and only if the instance M of 3DM has a matching. From Fig. 12 (b), one can see that there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.

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