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Finite-Horizon Optimal Spatio-Temporal Pattern Control under Spatio-Temporal Logic Specifications

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SUMMARY In spatially distributed systems such as smart buildings and intelligent transportation systems, control of spatio-temporal patterns is an important issue. In this paper, we consider a finite-horizon optimal spatio-temporal pattern control problem where the pattern is specified by a signal spatio-temporal logic formula over finite traces, which will be called an SSTL$_f$ formula. We give the syntax and Boolean semantics of SSTL$_f$. Then, we show linear encodings of the temporal and spatial operators used in SSTL$_f$ and we convert the problem into a mixed integer programming problem. We illustrate the effectiveness of this proposed approach through an example of a heat system in a room.

key words: *optimal control, spatio-temporal logic, spatially distributed system*

1. Introduction

Spatio-temporal logics can describe both spatial and temporal properties formally [1] and have been studied for spatially distributed systems in chemistry, biology, and physics [2], [3]. With the recent IoT technologies and ICT, the control of the spatially distributed systems has been paid much attention to and spatio-temporal logics are used to specify the properties of spatially distributed IoT and cyber-physical systems [4]–[6].

Recently, many kinds of logics that specify spatio-temporal properties have been proposed. Haghighi et al. [2] introduced SpaTeL that is composed of the spatial component based on *Tree Spatial Superposition Logic* [7] and the temporal component based on *Signal Temporal Logic* (STL) [8]. Nenzi et al. [9] proposed *Signal Spatio-Temporal Logic* (SSTL) that integrates STL with the two spatial operators, called the *somewhere* operator and the *bounded surrounded* operator. An SSTL formula is interpreted only on a static graph. Bartocci et al. [6] proposed *Spatio-Temporal Reach and Escape Logic* (STREL) that is an extension of the STL with two spatial operators called the *reach* operator and the *escape* operator. Li et al. [5] presented a *Spatio-Temporal Specification Language* (STSL) by combining STL with the spatial modal logic $S4_u$ and considered a falsification problem. Ma et al. [10] introduced *Spatial aggregation Signal Temporal Logic* (SaSTL) which is suitable to represent properties of aggregated data.

Formal approaches using temporal logics have been used not only for verification and monitoring problems but also for control problems. Wolff et al. [11] proposed a control method of a nonlinear system with specifications described by *Linear Temporal Logic* (LTL) formulas which are encoded as mixed-integer linear constraints. Raman et al. [12] presented a method to encode STL formulas into a set of linear inequalities and convert a model predictive control problem with specifications described by STL formulas into a mixed integer programming (MIP) problem. For the controller synthesis of a multi-agent system, Sahin et al. [13] introduced counting LTL to specify both individual and collective behaviors of agents. Liu et al. [14] formulated a motion planning problem of a multi-robot system with the optimization of the quality of service for communication among the robots where the motion and communication constraints are described by STL and STREL formulas, respectively. Haghighi et al. [2] considered a parameter synthesis problem using SpaTeL and Bartocci et al. [15] considered both a parameter synthesis problem and a pattern classification problem. Penedo et al. [16] formulated a control problem for systems modeled by partial differential equations under control specifications described by STL formulas that are constructed from predicates over spatio-temporal signals.

On the other hand, temporal logic specifications for finite traces are often used in several engineering problems such as path planning [17]. Recently, Giacomo and Vardi [18] proposed *Linear Temporal Logic over finite traces* (LTL$_f$). He et al. [19] considered a reactive planning problem where tasks are described by LTL$_f$ formulas. LTL$_f$ based synthesis is extended to probabilistic systems and timed discrete event systems [20], [21].

In this paper, we consider a finite-horizon optimal spatio-temporal pattern control problem of spatially distributed discrete-time systems where the specifications of the patterns are described by *Signal Spatio-Temporal Logic over finite traces* (SSTL$_f$). We convert the problem into an MIP problem using linear encodings of the spatial and the temporal operators. The rest of this paper is organized as follows. In Sect. 2, we describe model of spatially distributed discrete-time systems. In Sect. 3, we give the syntax and the semantics of SSTL$_f$ and formulate a finite horizon optimal spatio-temporal pattern control problem under SSTL$_f$ specifications. In Sect. 4, we introduce linear encodings of SSTL$_f$ formulas and convert the problem into an MIP problem. In Sect. 5, as an example, we consider a temperature control problem of a room under a spatio-temporal constraints. In Sect. 6, we conclude the paper.

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2. Spatially Distributed Controlled System

Notation: For a set A, denoted by $|A|$ is its cardinality.

We consider an N-dimensional Euclidean space divided into a grid represented by an undirected graph $G = (L, E)$, where L is a set of locations, $E \subseteq L \times L$ is a set of edges between adjacent locations with no direction.

For $\ell, \ell' \in L$, a sequence $\sigma = \ell_0 \ell_1 \ldots \ell_{H_\sigma - 1} \ell_{H_\sigma}$ of locations is called a path from ℓ to ℓ', where H_σ is a non-negative integer, if $\ell_0 = \ell$, $\ell_{H_\sigma} = \ell'$, $(\ell_j, \ell_{j+1}) \in E$ for $j \in \{0, 1, \ldots, H_\sigma - 1\}$. H_σ is called the *distance of the path* σ. We denote a set of all paths from ℓ to ℓ' by $\Sigma(\ell, \ell')$.

Definition 1 (Shortest distance): For any $\ell \in L$, $d(\ell, \ell) = 0$ and, for any ℓ and $\ell' \in L$ with $\ell \neq \ell'$, the shortest distance $d(\ell, \ell')$ is defined as follows.

$$d(\ell, \ell') := \min \{H_\sigma | \, \sigma \in \Sigma(\ell, \ell')\}.$$

Note that $\min \emptyset = \infty$. □

We denote the set of locations ℓ' whose distances from ℓ are between d_1 and d_2 by $L^{\ell}_{[d_1, d_2]} = \{\ell' | \, d_1 \leq d(\ell, \ell') \leq d_2\}$.

Definition 2 (External boundary of A): Given a subset of locations $A \subseteq L$, the external boundary of A is defined by

$$B^+(A) := \{\ell' \in L | \, \ell' \notin A \wedge \exists \ell \in A. \, (\ell', \ell) \in E\}. \tag{1}$$

□

Note that $B^+(\{\ell\}) = \{\ell' | \, \ell \neq \ell' \wedge (\ell', \ell) \in E\}$ represents a set of adjacent locations of $\ell \in L$. We denote the state of the location $\ell \in L$ at the time $t \in \mathbb{T}$ by $\mathbf{x}(t, \ell) = [x_1(t, \ell), x_2(t, \ell), \ldots, x_n(t, \ell)]^\mathsf{T}$ where $\mathbb{T} = \{0, \ 1, \ldots, H\}$ with a finite horizon $H \in \mathbb{N} \cup \{0\}$ is a set of time indices, and $x_i(t, \ell) \in \mathbb{D}_i \subseteq \mathbb{R}$ for each $i \in \{1, 2, \ldots, n\}$ is the i-th state. Let $\mathbb{D} = \mathbb{D}_1 \times \mathbb{D}_2 \times \ldots \times \mathbb{D}_n$. Let $L' \subseteq L$ be a set of locations without control inputs. Then, we consider the following spatially distributed controlled discrete-time system over the graph G.

$$\begin{cases} \mathbf{x}(t+1, \ell) = g_\ell \left(\mathbf{x}(t, \ell), \mathbf{x}(t, \ell_1), \ldots, \mathbf{x}(t, \ell_{|B^+(\{\ell\})|}), \mathbf{u}(t, \ell) \right), \\ \mathbf{x}(t+1, \ell') = g_{\ell'} \left(\mathbf{x}(t, \ell'), \mathbf{x}(t, \ell'_1), \ldots, \mathbf{x}(t, \ell'_{|B^+(\{\ell'\})|}) \right), \end{cases} \tag{2}$$

where $\ell \in L \setminus L'$, $\ell' \in L'$, $\{\ell_1, \ldots, \ell_{|B^+(\{\ell\})|}\} = B^+(\{\ell\})$, $\left\{\ell'_1, \ldots, \ell'_{|B^+(\{\ell'\})|}\right\} = B^+(\{\ell'\})$, $\mathbf{u} : \mathbb{T} \times L \setminus L' \to \mathbb{U}$ is an input function with a compact set $\mathbb{U} \subseteq \mathbb{R}^{n_u}$, and $g_\ell : \mathbb{D}^{|B^+(\{\ell\})|+1} \times \mathbb{U} \to \mathbb{D}$ and $g_{\ell'} : \mathbb{D}^{|B^+(\{\ell'\})|+1} \to \mathbb{D}$ are real-valued continuos functions. Let $S_{\mathbb{U}}$ be the set of all input functions.

The function $\mathbf{x} : \mathbb{T} \times L \to \mathbb{D}$ is called a *spatio-temporal trace* or simply a *trace* over the system (2) if it satisfies (2) for some input function $\mathbf{u} : \mathbb{T} \times L \setminus L' \to \mathbb{U}$.

3. Finite-Horizon Optimal Control

3.1 Spatio-Temporal Logic Specificaions

We consider optimal control under a spatio-temporal pattern specification described by a spatio-tempral logic formula. Since we deal with finite traces with real-valued signals, we leverage the same syntax of siganl spatio-temporal logic (SSTL) defined in [9] and modifies its semantics and call the modified logic SSTL$_f$. Its syntax is defined over a set of m atomic predicates $\mathcal{M} = \{\mu_j(x_1, \ldots, x_n) | \, j \in \{1, \ldots, m\}, \ \mu_j(x_1, \ldots, x_n) \equiv (f_j(x_1, \ldots, x_n) \geq 0)\}$ where $f_j : \mathbb{D} \to \mathbb{R}$ is a real-valued function. We introduce the syntax and the Boolean semantics of SSTL$_f$.

Definition 3 (Syntax of SSTL$_f$): An SSTL$_f$ formula is recursively defined by the following grammar.

$$\begin{aligned} \varphi ::= & \, True \mid \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \mathcal{U}_{[t_1, t_2]} \varphi_2 \mid \mathcal{G}_{[t_1, t_2]} \varphi \\ & \mid \diamond_{[d_1, d_2]} \varphi \mid \varphi_1 \mathcal{S}_{[d_1, d_2]} \varphi_2, \end{aligned}$$

where φ, φ_1, and φ_2 are SSTL$_f$ formulas, $\mu \in \mathcal{M}$, $t_1, t_2 \in \mathbb{T}$ with $t_1 \leq t_2$, and $d_1, d_2 \in \mathbb{R}_{\geq 0}$ with $d_1 \leq d_2$. □

$\mathcal{U}_{[t_1, t_2]}$ and $\mathcal{G}_{[t_1, t_2]}$ are the temporal operators called the *until* and the *globally* operator, respectively. Additionally, the *eventually* operator, denoted by $\mathcal{F}_{[t_1, t_2]}$, is given by $\mathcal{F}_{[t_1, t_2]}\varphi = True \, \mathcal{U}_{[t_1, t_2]} \varphi$. Spatial operators in SSTL$_f$ are the *somewhere* operator, denoted by $\diamond_{[d_1, d_2]}$, and the *bounded surrounded* operator, denoted by $\mathcal{S}_{[d_1, d_2]}$. The *everywhere* operator, denoted by $\boxdot_{[d_1, d_2]}$, is given by $\boxdot_{[d_1, d_2]} \varphi = \neg(\diamond_{[d_1, d_2]} \neg\varphi)$.

Intuitively, $\diamond_{[d_1, d_2]} \varphi$ is satisfied if and only if at least one location whose shortest distance from ℓ is between d_1 and d_2 satisfies φ at the time t, and $\varphi_1 \mathcal{S}_{[d_1, d_2]} \varphi_2$ is satisfied if and only if there is a set A ($\subseteq L^{\ell}_{[0, d_2]}$) including ℓ such that all locations in A satisfy φ_1 and all locations in $B^+(A)$ ($\subseteq L^{\ell}_{[d_1, d_2]}$) satisfy φ_2. Then, the semantics of SSTL$_f$ is defined as follows.

Definition 4 (Boolean semantics): Given a finite spatio-temporal trace $\mathbf{x}$, the satisfaction of an SSTL$_f$ formula ϕ at the time $t \in \mathbb{T}$ and the location ℓ, denoted by $(\mathbf{x}, t, \ell) \models \phi$, is defined recursively as follows.

$$\begin{aligned}
&(\mathbf{x}, t, \ell) \models \mu_j && \Leftrightarrow \left(f_j(x_1, \ldots, x_n) \geq 0 \right) \\
&(\mathbf{x}, t, \ell) \models \neg\varphi && \Leftrightarrow (\mathbf{x}, t, \ell) \not\models \varphi \\
&(\mathbf{x}, t, \ell) \models \varphi_1 \wedge \varphi_2 && \Leftrightarrow (\mathbf{x}, t, \ell) \models \varphi_1 \wedge (\mathbf{x}, t, \ell) \models \varphi_2 \\
&(\mathbf{x}, t, \ell) \models \varphi_1 \mathcal{U}_{[t_1, t_2]} \varphi_2 && \\
&\quad \Leftrightarrow t + t_1 \leq H && \\
&\qquad \wedge (\exists t' \in \{t + t_1, \ldots, \min\{t + t_2, H\}\}. \, (\mathbf{x}, t', \ell) \models \varphi_2) && \\
&\qquad \wedge (\forall t'' \in \{t, \ldots, t' - 1\}. \, (\mathbf{x}, t'', \ell) \models \varphi_1) && \\
&(\mathbf{x}, t, \ell) \models \mathcal{G}_{[t_1, t_2]} \varphi && \\
&\quad \Leftrightarrow t + t_2 \leq H && \\
&\qquad \wedge (\forall t' \in \{t + t_1, \ldots, t + t_2\}. \, (\mathbf{x}, t', \ell) \models \varphi) && \\
&(\mathbf{x}, t, \ell) \models \diamond_{[d_1, d_2]} \varphi &&
\end{aligned}$$

$$\Leftrightarrow \exists \ell' \in L.\ (d_1 \leq d(\ell,\ell') \leq d_2) \wedge (\mathbf{x}, t, \ell') \models \varphi$$

$$(\mathbf{x}, t, \ell) \models \varphi_1 \mathcal{S}_{[d_1,d_2]} \varphi_2$$

$$\Leftrightarrow \exists A \subseteq L^{\ell}_{[0,d_2]}.\ \ell \in A \wedge (\forall \ell' \in A.\ (\mathbf{x}, t, \ell') \models \varphi_1)$$

$$\wedge\ B^{+}(A) \subseteq L^{\ell}_{[d_1,d_2]} \wedge (\forall \ell'' \in B^{+}(A).\ (\mathbf{x}, t, \ell'') \models \varphi_2).$$

□

By the above definition, it is noted that $(\mathbf{x}, H, \ell) \models \varphi_1 \mathcal{U}_{[0,t_2]} \varphi_2$ if and only if $(\mathbf{x}, H, \ell) \models \varphi_2$. A spatio-temporal trace $\mathbf{x}$ satisfies φ at the location ℓ, denoted by $(\mathbf{x}, \ell) \models \varphi$, if and only if $(\mathbf{x}, 0, \ell) \models \varphi$.

3.2 Problem Formulation

In this paper, we consider the following finite horizon optimal spatio-temporal pattern control problem.

Problem 1: Given an undirected graph $G = (V, L)$ with a set L' of locations without control inputs, a finite horizon H, a spatially distributed controlled discrete-time system (2) over G with the initial state $x_\ell \in \mathbb{D}$ for each $\ell \in L$, an SSTL$_f$ formula ϕ_ℓ for each $\ell \in L$, and immediate cost functions $J : \mathbb{D} \times \mathbb{U} \to \mathbb{R}$ and $J' : \mathbb{D} \to \mathbb{R}$, find a control input $\mathbf{u} \in S_{\mathbb{U}}$ that minimizes the following accumulated cost function subject to $(\mathbf{x}, \ell) \models \phi_\ell$ and $\mathbf{x}(0, \ell) = x_\ell$ for each $\ell \in L$, and (2).

$$\sum_{t=0}^{H-1} \left(\sum_{\ell \in L \setminus L'} J(\mathbf{x}(t+1, \ell), \mathbf{u}(t, \ell)) + \sum_{\ell \in L'} J'(\mathbf{x}(t+1, \ell')) \right). \tag{3}$$

We assume that the functions J and J' are continuous.

In the next section, we will introduce linear encodings of the temporal and spatial operators and convert Problem 1 into a mixed integer programming (MIP) problem.

4. Conversion Into an MIP Problem

We consider an undirected graph $G = (L, E)$ where $L = \{\ell_1, \ldots, \ell_{|L|}\}$. We introduce $|L|$ binary vectors $v_{\ell_i} \in \{0,1\}^{|L|}$ for $\ell_i \in L$, where $v_{\ell_i,i} = 1$ (the i-th component of v_{ℓ_i} is 1) and $v_{\ell_i,j} = 0$ for $i \neq j$. Denoted by $D \in \mathbb{R}^{|L| \times |L|}$ is the *distance matrix* whose (i, j)-th element $D_{i,j}$ is given by

$$D_{i,j} = \begin{cases} d(\ell_i, \ell_j) & \text{if } \Sigma(\ell_i, \ell_j) \neq \emptyset, \\ M^d & \text{otherwise,} \end{cases}$$

where M^d is a sufficiently large positive number that satisfies $M^d > \max\{d(\ell_i, \ell_j) |\ \ell_i, \ell_j \in L, \Sigma(\ell_i, \ell_j) \neq \emptyset\}$. To encode an SSTL$_f$ formula, we introduce the following binary variable $z_\varphi(t, \ell)$ for each $t \in \mathbb{T}$ and each $\ell \in L$, where φ is an SSTL$_f$ formula. For a given spatio-temporal trace $\mathbf{x}$,

$$z_\varphi(t, \ell) = \begin{cases} 1 & \text{if} (\mathbf{x}, t, \ell) \models \varphi, \\ 0 & \text{otherwise.} \end{cases}$$

We describe a method of linear encodings of atomic predicates, the Boolean operators, and the temporal operators, which is a slight modification of the method proposed in [12], [22]. Then, we introduce linear encodings of the three spatial operators based on their semantics given by Definition 4.

4.1 Boolean and Temporal Operators

Atomic predicate: Let $\varphi = \mu_j(x_1, \ldots, x_n)$. Then, the satisfaction of φ is encoded as

$$f_j(\mathbf{x}(t, \ell)) \leq M^{\mu_j} z_\varphi(t, \ell) - \epsilon,$$
$$-f_j(\mathbf{x}(t, \ell)) \leq M^{\mu_j}(1 - z_\varphi(t, \ell)),$$

where M^{μ_j} is a sufficiently large positive number compared with the maximum value of f_j for $j \in \{1, \ldots, m\}$ and ϵ is a sufficiently small number that represents the tolerance of satisfaction of atomic predicates.

Negation: Let $\varphi = \neg\psi$. Then,

$$z_\varphi(t, \ell) = 1 - z_\psi(t, \ell).$$

Conjunction: Let $\varphi = \bigwedge_{k=1}^{K} \psi_k$. Then,

$$z_\varphi(t, \ell) \leq z_{\psi_k}(t, \ell),\ \forall k \in [1,\ K],$$
$$z_\varphi(t, \ell) \geq 1 - K + \sum_{k=1}^{K} z_{\psi_k}(t, \ell).$$

Disjunction: Let $\varphi = \bigvee_{k=1}^{K} \psi_k$. Then,

$$z_\varphi(t, \ell) \geq z_{\psi_k}(t, \ell),\ \forall k \in [1,\ K],$$
$$z_\varphi(t, \ell) \leq \sum_{k=1}^{K} z_{\psi_k}(t, \ell).$$

With a slight abuse of notation, Boolean operators are used for binary variables. For example, when we consider $\varphi = \bigwedge_{k=1}^{K} \psi_k$, we write $z_\varphi = \bigwedge_{k=1}^{K} z_{\psi_k}$. Then, we describe linear encodings of the temporal operators for finite traces over $\mathbb{T}$.

Globally: Let $\varphi = \mathcal{G}_{[t_1,t_2]}\psi$ with $t_1 \leq t_2$. Then,

$$z_\varphi(t, \ell) = \begin{cases} \displaystyle\bigwedge_{k=t+t_1}^{t+t_2} z_\psi(k, \ell), & \text{if } t \in \{0, \ldots, H - t_2\}, \\ 0 & \text{otherwise.} \end{cases}$$

Eventually: Let $\varphi = \mathcal{F}_{[t_1,t_2]}\psi$ with $t_1 \leq t_2$. Then,

$$z_\varphi(t, \ell) = \begin{cases} \displaystyle\bigvee_{k=t+t_1}^{b_t^H} z_\psi(k, \ell), & \text{if } t \in \{0, \ldots, H - t_1\}, \\ 0 & \text{otherwise.} \end{cases}$$

where $b_t^H = \min(t + t_2, H)$.

Until: Let $\varphi = \psi_1 \mathcal{U}_{[t_1,t_2]} \psi_2$ with $t_1 \leq t_2$. Then, for each $t \in \{0, \ldots, H - 1\}$,

$$z_\varphi(t, \ell)$$

$$= \begin{cases} z_{\psi_1}(t,\ell) \wedge z_{\psi_1 \mathcal{U}_{[t_1-1,t_2-1]}\psi_2}(t+1,\ell) & \text{if } t_1 \geq 1, \\ (z_{\psi_1}(t,\ell) \wedge z_{\psi_1 \mathcal{U}_{[0,t_2-1]}\psi_2}(t+1,\ell)) & \\ \quad \vee z_{\psi_2}(t,\ell) & \text{if } t_1 = 0 \wedge t_2 \geq 1, \\ z_{\psi_2}(t,\ell) & \text{if } t_1 = t_2 = 0, \end{cases}$$

and

$$z_\varphi(H,\ell) = \begin{cases} z_{\psi_2}(H,\ell) & \text{if } t_1 = 0, \\ 0 & \text{otherwise.} \end{cases}$$

4.2 Spatial Operators

We introduce linear encodings of the somewhere operator, the everywhere operator, and the bounded surrounded operator.

Somewhere operator: Let $\varphi = \diamonddiamond_{[d_1,d_2]}\psi$. Then, the satisfaction of φ is encoded as

$$z_\varphi(t,\ell) = \bigvee_{\ell' :\, d_1 \leq v_\ell^\top D v_{\ell'} \leq d_2} z_\psi(t,\ell'). \tag{4}$$

Everywhere operator: Let $\varphi = \boxbox_{[d_1,d_2]}\psi$. Then,

$$z_\varphi(t,\ell) = \bigwedge_{\ell' :\, d_1 \leq v_\ell^\top D v_{\ell'} \leq d_2} z_\psi(t,\ell'). \tag{5}$$

Before we introduce the linear encoding of the bounded surrounded operator, we define a binary variable $\underline{z}_{d_1}(\ell,\ell') \in \{0,\ 1\}$ for $\ell,\ell' \in L$ such that $\underline{z}_{d_1}(\ell,\ell') = 1$ if and only if $d_1 \leq d(\ell,\ell')$. Then, the variable $\underline{z}_{d_1}(\ell,\ell')$ satisfies the following equation.

$$d_1 - M^d \leq v_\ell^\top D v_{\ell'} - M^d \underline{z}_{d_1}(\ell,\ell') \leq d_1 - \epsilon^d,$$

where ϵ^d is a sufficiently small positive number that represents the tolerance of satisfaction of this predicate. This encoding is inspired by the method proposed in [13]. Then, we have the following linear encoding of the bounded surrounded operator with the variables $\underline{z}_{d_1}(\ell,\ell')$.

Bounded surrounded operator: Let $\varphi = \psi_1 \mathcal{S}_{[d_1,d_2]}\psi_2$. Then, the satisfaction of φ is encoded as

$$z_\varphi(t,\ell) = \left(\bigwedge_{\ell' \in \left(B^+(\{\ell\}) \cap L^\ell_{[0,d_2]}\right) \setminus \{\ell\}} \tilde{z}_\varphi(t,\ell,\ell') \right) \wedge z_{\psi_1}(t,\ell), \tag{6}$$

where, for each $\ell' \in L^\ell_{[0,d_2]} \setminus \{\ell\}$,

$$\tilde{z}_\varphi(t,\ell,\ell') = \left(z_{\psi_2}(t,\ell') \wedge \underline{z}_{d_1}(\ell,\ell')\right) \vee \left\{ \left(\bigwedge_{\ell'' \in \left(B^+(\{\ell'\}) \cap L^\ell_{[0,d_2]}\right) \setminus \{\ell\}} \tilde{z}_\varphi(t,\ell,\ell'') \right) \wedge z_{\psi_1}(t,\ell') \right\}, \tag{7}$$

and for each $\ell^\dagger \in L \setminus L^\ell_{[0,d_2]}$,

$$\tilde{z}_\varphi(t,\ell,\ell^\dagger) = 0. \tag{8}$$

In (7), ℓ' can be in $B^+(A)$ if the first term $z_{\psi_2}(t,\ell') \wedge \underline{z}_{d_1}(\ell,\ell')$ is true, and ℓ' can be in A if the second term is true.

4.3 Overall MIP Problem

Let $LE(\phi_\ell)$ be the linear encoding of an SSTL$_f$ formula ϕ_ℓ using the method described above. Then, we convert Problem 1 into the following MIP problem.

Problem 2: Find an input function $\mathbf{u} \in S_{\mathbb{U}}$ that minimizes (3) subject to $LE(\phi_\ell)$, $z_{\phi_\ell}(0,\ell) = 1$, and $\mathbf{x}(0,\ell) = x_\ell$ for each $\ell \in L$, and (2).

5. Simulation

In this section, we demonstrate the effectiveness of the proposed approach by simulation. We consider a room with six heaters and two windows on a 2-dimensional Euclidean space and control the temperature distribution of the room with specifications described by SSTL$_f$ formulas. The room is divided into a 10×10 grid and is modeled by an undirected graph $G = (L,E)$ where $L = \{\ell_{i,j} | \ i,j \in \{0,\ldots,9\}\}$ and $(\ell_{i,j},\ell_{i',j'}) \in E$ if and only if $\ell_{i,j}$ and $\ell_{i',j'}$ are adjacent locations horizontally or vertically. Let $L_h = \{\ell_{0,0},\ell_{0,9},\ell_{4,6},\ell_{6,4},\ell_{9,0},\ell_{9,9}\}$ be a set of locations where heaters are placed. The windows are placed at the left side and the front side of the room except the corners of the room, that is, the set of locations with a window is denoted by $L_w = \{\ell_{0,1},\ell_{0,2},\ldots,\ell_{0,8},\ell_{1,0},\ell_{2,0},\ldots,\ell_{8,0}\}$. Note that $L \backslash L_h$ is a set of locations without control inputs and $L_w \subset L \backslash L_h$. Moreover, let $\bar{L} = \{\ell_{3,6},\ell_{4,7},\ell_{5,5},\ell_{5,6},\ell_{5,7},\ell_{6,3},\ell_{6,5},\ell_{7,4}\}$ be a set of locations where there are persons. Shown in Fig. 1 is the settings of the room in this simulation.

At the time $t \in \mathbb{T}$ and the location $\ell \in L$, let $T(t,\ell) \in \mathbb{R}$ and $u(t,\ell) \in \mathbb{R}$ be a temperature and a control input, respectively. Then, the distribution of the temperature is modeled by the following equations for $\ell_h \in L_h$, $\ell \in L \setminus L_h$, and $t \in \mathbb{T}$.

$$T(t+1,\ell_h) = (1 - V \cdot A^{\ell_h})T(t,\ell_h) + Vu(t,\ell_h)$$

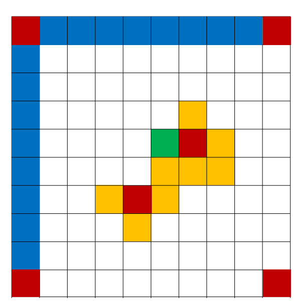


Fig. 1 Grid model of the room. The left-top location is $\ell_{0,0}$, the right-top location is $\ell_{0,9}$, the left-bottom location is $\ell_{9,0}$ and the right-bottom location is $\ell_{9,9}$. Red, blue, yellow locations are in L_h, L_w, and $\bar{L}$, respectively. The green location is $\ell_{4,5}$.

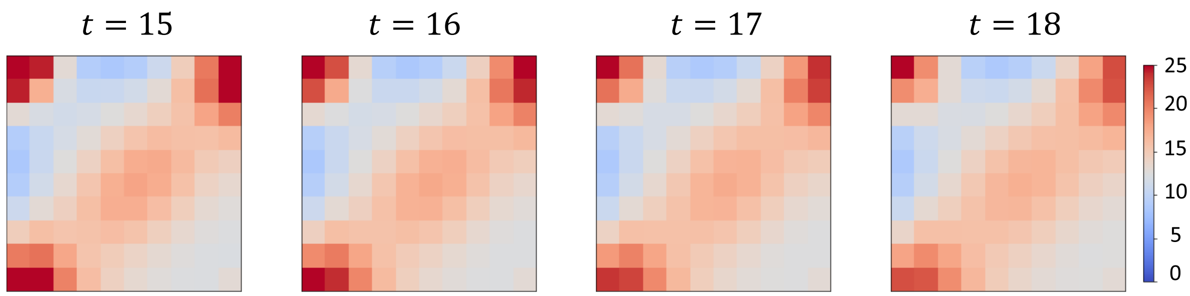
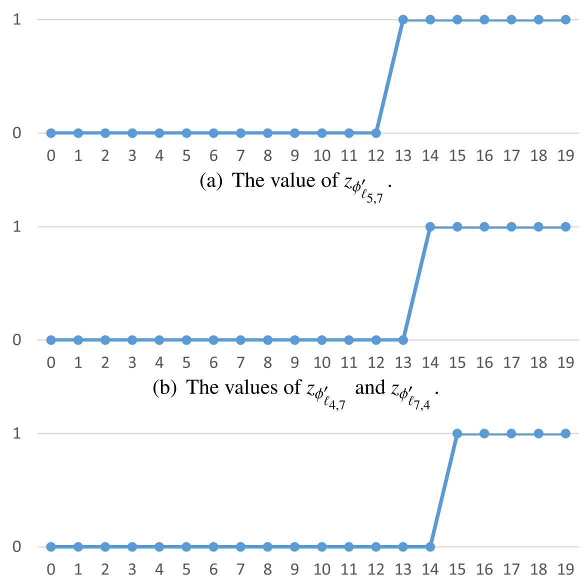


Fig. 2 The values of T at $t = 15, \ldots, 18$.



(a) The value of $z_{\phi'_{\ell_{5,7}}}$.

(b) The values of $z_{\phi'_{\ell_{4,7}}}$ and $z_{\phi'_{\ell_{7,4}}}$.

(c) The values of $z_{\phi'_{\ell}}$ for $\ell \in \bar{L} \setminus \{\ell_{4,7}, \ell_{5,7}, \ell_{7,4}\}$.

Fig. 3 The values of $z_{\phi'_{\bar{\ell}}}$ for $\bar{\ell} \in \bar{L}$. The horizon axes represent time t.

$$+ W \left\{ \frac{1}{|B^+(\{\ell_h\})|} \sum_{\ell'_h \in B^+(\{\ell_h\})} T(t, \ell'_h) - T(t, \ell_h) \right\}, \tag{9}$$

$$\begin{aligned} T(t+1,\ell) &= (1 - V \cdot A^{\ell}) T(t, \ell) \\ &+ W \left\{ \frac{1}{|B^+(\{\ell\})|} \sum_{\ell' \in B^+(\{\ell\})} T(t, \ell') - T(t, \ell) \right\}, \end{aligned} \tag{10}$$

where A^{ℓ}, V, and W are constant. These equations are derived from the the discretization of the heat conduction equation. For details, please see Appendix. The initial value of T for each $\ell \in L$ is given by

$$T(0, \ell) = 11.5 + \mathit{uniform}(0, 1),$$

where $\mathit{uniform}(a, b)$ is a uniformly random number over the interval from a to b ($a, b \in \mathbb{R}$).

Let $f_1(x) = x - 13$ and $f_2(x) = x - 18$ for each $x \in \mathbb{R}$, and atomic predicates $\mu_i(x)$ ($i = 1, 2$) are given by $\mu_1(x) \equiv (f_1(x) \geq 0)$ and $\mu_2(x) \equiv (f_2(x) \geq 0)$. In the time interval

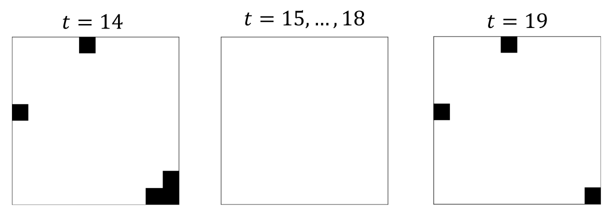


Fig. 4 The values of $z_{\phi'_{\ell_{4,5}}}$. $\phi'_{\ell_{4,5}}$ is satisfied (*resp.* is not satisfied) at the locations colored by white (*resp.* black).

[15, 18], we give two control specifications. The first is that the temperature around a person at each $\bar{\ell} \in \bar{L}$ is equal to or more than 13 and the temperature of the locations close to a person is less than 18. The second is that every location ℓ in L has at least one location in $L^{\ell}_{[0,2]}$ where the temperature is equal to or more than 13. These specifications are described by the following SSTL$_f$ formulas.

$$\begin{aligned} (T, \bar{\ell}) &\models \phi_{\bar{\ell}} = \mathcal{G}_{[15,18]} \left((\mu_1 \wedge \neg \mu_2) \mathcal{S}_{[2,3]} \mu_1 \right), \ \bar{\ell} \in \bar{L} \\ (T, \ell_{4,5}) &\models \phi_{\ell_{4,5}} = \boxdot_{[0,10]} \mathcal{G}_{[15,18]} \left(\diamondsuit_{[0,2]} \mu_1 \right), \\ (T, \ell') &\models \mathit{True}, \ \ell' \in L \setminus (\bar{L} \cup \{\ell_{4,5}\}). \end{aligned}$$

Note that $L^{\ell_{4,5}}_{[0,10]} = L$, and $\phi_{\ell_{4,5}}$ represents the second specification. The immediate cost functions are given by $J(T(t+1, \ell), u(t, \ell)) = |u(t, \ell)|$, and $J'(T(t+1, \ell)) = 0$ with $L' = L \setminus L_h$.

Let $H = 20$, $V = 1$, $W = 0.4$, and

$$A^{\ell} = \begin{cases} 0.05 & \ell \in L_w, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that all control inputs lie in the interval [0, 25]. The simulation was run by a machine with AMD Ryzen9 5950X and 128GB memory, and the solver Gurobi[†] was used to compute an optimal solution of the MIP problem. It takes 4,403 seconds to encode all SSTL$_f$ formulas and 1,644 seconds to compute an optimal solution. Shown in Fig. 2 is the spatio-temporal response of the temperature T. Let $\phi'_{\bar{\ell}} = (\mu_1 \wedge \neg \mu_2) \mathcal{S}_{[2,3]} \mu_1$ for $\bar{\ell} \in \bar{L}$ and $\phi'_{\ell_{4,5}} = \diamondsuit_{[0,2]} \mu_1$. Figures 3 and 4 show that every $\bar{\ell} \in \bar{L}$ satisfies $\phi'_{\bar{\ell}}$ and that all locations in L satisfy $\phi'_{\ell_{4,5}}$ in the time interval [15, 18], respectively. Thus, the obtained optimal inputs satisfy the control specifications.

[†]https://www.gurobi.com/

6. Conclusion

In this paper, we considered a finite horizon optimal spatio-temporal pattern control problem of spatially distributed discrete-time systems where the specifications of the pattern are described by SSTL_f formulas. We introduce linear encodings of the spatial operators of SSTL_f based on their Boolean semantics and convert the control problem into an MIP problem.

SSTL_f is useful for describing spatio-temporal pattern specifications and it is future work to apply the proposed approach to real problems such as management of smart buildings.

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Appendix: Derivations of (9) and (10)

We consider the heat conduction equation of the air in a 2-dimensional Euclidean space with the x-axis (horizontal direction) and the y-axis (vertical direction). Let $T_c(\tau, x, y)$ and $\dot{q}_c(\tau, x, y)$ be the temperature and the heating value per unit area at the time $\tau \in \mathbb{R}_{\geq 0}$ and the coordination (x, y). We denote the heat conductivity of the air by α in the x-axis and y-axis directions. Then, the two-dimensional heat conduction equation of the air is given by the following partial differential equation.

$$\rho c \frac{\partial T_c(\tau, x, y)}{\partial \tau} = \alpha \frac{\partial^2 T_c(\tau, x, y)}{\partial x^2} + \alpha \frac{\partial^2 T_c(\tau, x, y)}{\partial y^2} + \dot{q}_c(\tau, x, y), \tag{A· 1}$$

where ρ is the specific heat of the air and c is the density of the air. Consider a location ℓ that is not adjacent to the wall of the room and it's coordinate denotes (x, y). Then, discretizing (A· 1) for time with a time-step size $\Delta t \in \mathbb{R}_{>0}$ and a time index $t \in \mathbb{Z}_{\geq 0}$, and for the x-axis and y-axis with a space step-size $h \in \mathbb{R}_{>0}$, we have

$$\begin{aligned}
&\rho c \frac{T_c((t+1)\Delta t, x, y) - T_c(t\Delta t, x, y)}{\Delta t} \\
&= \alpha \frac{T_c(t\Delta t, x+h, y) + T_c(t\Delta t, x-h, y) - 2T_c(t\Delta t, x, y)}{h^2} \\
&\quad + \alpha \frac{T_c(t\Delta t, x, y+h) + T_c(t\Delta t, x, y-h) - 2T_c(t\Delta t, x, y)}{h^2} \\
&\quad + \dot{q}_c(t\Delta t, x, y).
\end{aligned}$$

Let $V = \frac{\Delta t}{\rho c}$ and $W = \frac{4\Delta t\alpha}{h^2 \rho c}$, then

$$\begin{aligned}
T_c((t+1)\Delta t, x, y) &= T_c(t\Delta t, x, y) + V\dot{q}_c(t\Delta t, x, y) \\
&\quad + W\Bigg\{\frac{1}{4}\Big(T_c(t\Delta t, x+h, y) + T_c(t\Delta t, x-h, y) \\
&\qquad + T_c(t\Delta t, x, y+h) + T_c(t\Delta t, x, y-h)\Big) \\
&\qquad - T_c(t\Delta t, x, y)\Bigg\}. \qquad (\mathrm{A}\cdot 2)
\end{aligned}$$

Let h be the distance between adjacent locations. Let $T(t, \ell) = T_c(t\Delta t, x, y)$ and $\dot{q}(t, \ell) = \dot{q}_c(t\Delta t, x, y)$. If the heater is located at ℓ, (A·2) becomes (9) with $\dot{q}(t, \ell) = u(t, \ell) - A^{\ell} T(t, \ell)$, otherwise it becomes (10) with $\dot{q}(t, \ell) = -A^{\ell} T(t, \ell)$. Similarly, it is shown that, for any location adjacent to the wall, (9) and (10) hold.

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