Provable-Security Analysis of Authenticated Encryption Based on Lesamnta-LW in the Ideal Cipher Model

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SUMMARY   Hirose, Kuwakado and Yoshida proposed a nonce-based authenticated encryption scheme Lae0 based on Lesamnta-LW in 2019. Lesamnta-LW is a block-cipher-based iterated hash function included in the ISO/IEC 29192-5 lightweight hash-function standard. They also showed that Lae0 satisfies both privacy and authenticity if the underlying block cipher is a pseudorandom permutation. Unfortunately, their result implies only about 64-bit security for instantiation with the dedicated block cipher of Lesamnta-LW. In this paper, we analyze the security of Lae0 in the ideal cipher model. Our result implies about 120-bit security for instantiation with the block cipher of Lesamnta-LW.
key words: *authenticated encryption, hash function, Lesamnta-LW, ideal cipher model*

1. Introduction

1.1 Background

Authenticated encryption (AE) is symmetric cryptography providing both privacy and authenticity. Informally, privacy is confidentiality of plaintexts and authenticity is integrity of ciphertexts. AE schemes often take additional input called associated data which only require authenticity. Such AE schemes are referred to as authenticated encryption with associated data (AEAD).

There are some kinds of approaches for AEAD construction. Among them, one of the most common approaches is to construct it as a mode of operation of a block cipher such as AES [1]. The other is to construct it based on the sponge construction [2]. The sponge construction [3] was invented originally for cryptographic hash functions as well as for MAC functions and stream ciphers. The sponge-based hash function Keccak [4] was selected for the SHA-3 standard [5]. The sponge construction is also popular for lightweight hashing.

The ISO/IEC 29192-5 lightweight hash function standard [6] was released in 2016, which specifies three lightweight cryptographic hash functions: PHOTON [7],

SPONGENT [8], and Lesamnta-LW [9]. PHOTON and SPONGENT follow the sponge construction, and the sponge-based AEAD mode can be applied to them. On the other hand, Lesamnta-LW is a Merkle-Damgård [10], [11] hash function using a dedicated block cipher whose key size is half the block size as a compression function. In addition, Lesamnta-LW is optimized for software implementation, while both PHOTON and SPONGENT are optimized for hardware implementation. In fact, a software result [9] shows that Lesmanta-LW provides 120-bit collision resistance with 54 bytes of RAM, achieving 20% faster short-message performance over SHA-256, while hardware results show that SPONGENT provides 80-bit collision resistance with 1329 GE and PHOTON provides the same security level with 1396 GE.

In 2019, Hirose, Kuwakado and Yoshida [12] proposed a nonce-based AEAD scheme based on Lesamnta-LW, which they called Lae0. It can be implemented with the block cipher of Lesamnta-LW. Thus, it is an efficient option for lightweight AEAD on low-cost 8-bit microcontrollers where RAM requirement is critical for cryptographic functionality.

1.2 Our Contribution

Hirose, Kuwakado and Yoshida [12] also showed that Lae0 is secure in the standard model: Lae0 satisfies both privacy and authenticity if the block cipher of the Lesamnta-LW hashing mode is a pseudorandom permutation (PRP). Unfortunately, their result is not entirely satisfactory in that it implies only about 64-bit security for instantiation of Lae0 with the block cipher of Lesamnta-LW. Their upper bound on the advantage of any adversary **A** against Lae0 has the term $\ell q \, \mathrm{adv}_E^{\mathrm{prp}}$, where $\mathrm{adv}_E^{\mathrm{prp}}$ is the advantage of an adversary constructed from **A** against the underlying block cipher E, ℓ is the maximum length of the queries made by **A**, and q is the number of the queries made by **A**. Due to the simple key-guessing attack on E, $\mathrm{adv}_E^{\mathrm{prp}} = \Omega(t/2^w)$, where t is the run time of **A** and w is the key length of E. Thus, the upper bound is $\Omega(1)$ if both ℓq and t are $\Omega(2^{w/2})$. For the block cipher of Lesamnta-LW, $w = 128$.

In this paper, we analyze the security of Lae0 in the ideal cipher model. In terms of both privacy and authenticity, our result implies about 120-bit security for instantiation of Lae0 with the block cipher of Lesamnta-LW. We discuss the authenticity of Lae0 under two typical misuses: nonce repetition (NR) and releasing unverified plaintexts (RUP).

Though our analysis assumes an ideal cipher, our result is still significant in that it implies security of Lae0 against generic attacks regarding the underlying block cipher just as a black box.

1.3 Related Work

Authenticated encryption received the first formal treatments from Katz and Yung [13] and Bellare and Namprempre [14], which are followed by Jutla [15].

There are many block-cipher modes of operation for AEAD. OCB [16] is one of the earliest but most efficient modes, and it is inspired by IAPM [15]. CCM [17] and GCM [18] are specified by NIST and ISO/IEC 19772 [19].

As far as we know, there is only one proposal for AEAD based on cryptographic hashing except for the sponge-based proposals. It is OMD (Offset Merkle-Damgård) by Cogliani et al. [20], which is a mode of operation of a compression function for the Merkle-Damgård hashing such as SHA-2 [21].

Nonce-based symmetric encryption was introduced with its formalization by Rogaway [22]. The generic composition of nonce-based AEAD was discussed by Namprempre et al. [23].

For misuse resistance of authenticated encryption, security under NR was formalized by Rogaway and Shrimpton [24]. Security under RUP was formalized by Andreeva et al. [25]. Robust authenticated encryption was introduced and formalized by Hoang et al. [26], which is secure under NR and RUP.

Improved and/or new security analyses of the Lesamnta-LW block cipher have recently been conducted by Hirose, Sasaki and Yoshida [27] and by Shiba et al. [28].

1.4 Organization

Notations and definitions used in the remaining parts are given in Sect. 2. Syntax and security are formalized for AEAD in Sect. 3. The nonce-based AEAD scheme Lae0 is described in Sect. 4. Lae0 is shown to satisfy both privacy and authenticity in Sect. 5. A brief concluding remark is given in Sect. 6.

2. Preliminaries

2.1 Notations

Let $\Sigma = \{0, 1\}$. For any integer $l \geq 0$, let Σ^l be identified with the set of all Σ-sequences of length l. $\Sigma^0 = \{\varepsilon\}$, where ε is the empty sequence. $\Sigma^1 = \Sigma$. Let $(\Sigma^l)^* = \bigcup_{i \geq 0} (\Sigma^l)^i$ and $(\Sigma^l)^+ = \bigcup_{i \geq 1} (\Sigma^l)^i$.

For $x \in \Sigma^*$, the length of x is denoted by $|x|$. For $x_1, x_2 \in \Sigma^*$, $x_1 \| x_2$ represents their concatenation. For $x \in \Sigma^*$ and an integer $0 \leq l \leq |x|$, $\mathsf{msb}_l(x)$ represents the most significant l bits of x, and $\mathsf{lsb}_l(x)$ represents the least significant l bits of x.

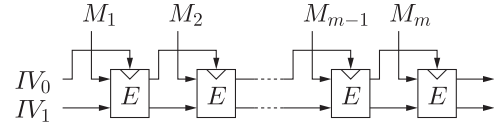For $x, y \in \Sigma^*$ such that $|x| \geq |y|$, let $x \oplus y$ and $y \oplus x$

Fig. 1 The hashing mode of Lesamnta-LW. The input of the block cipher E from the top is its key input.

represent bitwise XOR of x and $y \| 0^{|x|-|y|}$.

Selecting an element s from a set $\mathcal{S}$ uniformly at random is denoted by $s \leftarrow \mathcal{S}$.

The set of all functions from $\mathcal{X}$ to $\mathcal{Y}$ is denoted by $\mathcal{F}(\mathcal{X}, \mathcal{Y})$. The set of all permutations on $\mathcal{X}$ is denoted by $\mathcal{P}(\mathcal{X})$. ι represents an identity permutation. The set of all block ciphers with a key size κ and a block size n is denoted by $\mathcal{B}(\kappa, n)$. A block cipher in $\mathcal{B}(\kappa, n)$ is called a (κ, n) block cipher. For a keyed function $f : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$, $f(K, \cdot)$ is often denoted by $f_K(\cdot)$.

2.2 Hashing Mode of Lesamnta-LW

The hashing mode of Lesamnta-LW [9] is the plain Merkle-Damgård iteration of a block cipher E in $\mathcal{B}(n/2, n)$, where n is a positive even integer. E works as a compression function with domain $\Sigma^{3n/2}$ and range Σ^n. It is depicted in Fig. 1. $IV_0 \| IV_1 \in \Sigma^n$ is an initialization vector, where $|IV_0| = |IV_1| = n/2$. $M_1, M_2, \ldots, M_m$ are message blocks, where $M_i \in \Sigma^{n/2}$ for $i = 1, 2, \ldots, m$.

The dedicated block cipher of Lesamnta-LW is in $\mathcal{B}(128, 256)$.

3. Authenticated Encryption with Associated Data

3.1 Syntax

A scheme of nonce-based authenticated encryption with associated data (AEAD) consists of a pair of functions for encryption and decryption. The encryption function is $\mathsf{Enc} : \mathcal{K} \times \mathcal{N} \times \mathcal{A} \times \mathcal{M} \to C \times \mathcal{T}$ and the decryption function is $\mathsf{Dec} : \mathcal{K} \times \mathcal{N} \times \mathcal{A} \times C \times \mathcal{T} \to \mathcal{M} \cup \{\bot\}$, where $\mathcal{K}$ is a key space, $\mathcal{N}$ is a nonce space, $\mathcal{A}$ is an associated-data space, $\mathcal{M}$ is a message space, C is a ciphertext space, and $\mathcal{T}$ is a tag space. $\mathcal{M} \subset \Sigma^*$, $\bot \notin \mathcal{M}$ and $\mathcal{A} \subset \Sigma^*$. If $M \in \mathcal{M}$, then $\Sigma^{|M|} \subset \mathcal{M}$. For any $K \in \mathcal{K}$, if $(C, T) \leftarrow \mathsf{Enc}_K(N, A, M)$ for some $(N, A, M) \in \mathcal{N} \times \mathcal{A} \times \mathcal{M}$, then $M \leftarrow \mathsf{Dec}_K(N, A, C, T)$. Otherwise, $\bot \leftarrow \mathsf{Dec}_K(N, A, C, T)$, which means that (N, A, C, T) is invalid with respect to $K \in \mathcal{K}$.

3.2 Security

The security requirements for AEAD are privacy and authenticity. Informally, privacy is confidentiality of encrypted messages, and authenticity is integrity of ciphertexts and associated data.

(1) Privacy

Let \$ be a random function taking $(N, A, M) \in \mathcal{N} \times \mathcal{A} \times$

$\mathcal{M}$ as input and returning a binary sequence of length $|\mathsf{Enc}_K(N, A, M)|$, which is chosen uniformly at random. The privacy of an AEAD scheme $(\mathsf{Enc}, \mathsf{Dec})$ is defined by the indistinguishability between Enc_K and $:

$$\text{Adv}^{\text{priv}}_{(\mathsf{Enc},\mathsf{Dec})}(\mathbf{A}) = \left| \Pr[\mathbf{A}^{\mathsf{Enc}_K} = 1] - \Pr[\mathbf{A}^{\$} = 1] \right|,$$

where $K \twoheadleftarrow \mathcal{K}$. $\mathbf{A}$ is assumed to be nonce-respecting. Namely, $\mathbf{A}$ is not allowed to make multiple encryption queries with the same nonce.

(2) Authenticity

The authenticity of an AEAD scheme $(\mathsf{Enc}, \mathsf{Dec})$ is defined by the unforgeability:

$$\text{Adv}^{\text{auth}}_{(\mathsf{Enc},\mathsf{Dec})}(\mathbf{A}) = \Pr[\mathbf{A}^{\mathsf{Enc}_K,\mathsf{Dec}_K} \text{ succeeds in forgery}],$$

where $K \twoheadleftarrow \mathcal{K}$. $\mathbf{A}$ succeeds in forgery if it succeeds in making a decryption query such that its corresponding reply from Dec_K is not $\bot$. $\mathbf{A}$ is not allowed to make a trivial decryption query. Namely, if $\mathbf{A}$ gets (C, T) as an answer to some encryption query (N, A, M), then it is not allowed to ask (N, A, C, T) as a decryption query.

4. AEAD Based on Lesamnta-LW: **Lae0**

Let E be a block cipher in $\mathcal{B}(n/2, n)$, where n is an even integer. Hereafter, let $n/2 = w$ just for simplicity.

The padding function used in the construction is defined as follows: For any $X \in \Sigma^*$,

$$\mathsf{pad}(X) = \begin{cases} X & \text{if } |X| > 0 \text{ and } |X| \equiv 0 \pmod{w} \\ X\|10^t & \text{if } |X| = 0 \text{ or } |X| \not\equiv 0 \pmod{w}, \end{cases}$$

where t is the minimum non-negative integer satisfying $|X| + 1 + t \equiv 0 \pmod{w}$. For any $X \in \Sigma^*$, $|\mathsf{pad}(X)|$ is the minimum positive multiple of w, which is greater than or equal to $|X|$. Notice that pad is not injective. For example, $\mathsf{pad}(\varepsilon) = \mathsf{pad}(10^{w-1}) = 10^{w-1}$.

Let $\mathsf{pad}(X) = (X_1, X_2, \ldots, X_x)$, where $|X_i| = w$ for every i such that $1 \le i \le x$. $x = 1$ if $|X| = 0$, and $x = \lceil |X|/w \rceil$ if $|X| > 0$. X_i is called the i-th block of $\mathsf{pad}(X)$.

For $E \in \mathcal{B}(w, n)$ and $\pi_0, \pi_1 \in \mathcal{P}(\Sigma^w)$, the nonce-based AEAD scheme $\mathsf{Lae0} = (\mathsf{E0}, \mathsf{D0})$ is presented by Algorithm 1. The encryption function $\mathsf{E0}$ is also depicted in Fig. 2. For $\mathsf{Lae0}$, the key space is Σ^w, the nonce space is Σ^n,

and the tag space is Σ^τ, where $0 < \tau \le n$. The associated-data space, the message space and the ciphertext space are Σ^*. If $(C, T) \leftarrow \mathsf{E0}_K(N, A, M)$, then $|C| = |M|$.

Algorithm 1 Encryption E0 and decryption D0 of Lae0

function $\mathsf{E0}_K(N, A, M)$
  $(A_1, A_2, \ldots, A_a) \leftarrow \mathsf{pad}(A)$;
  $(M_1, M_2, \ldots, M_m) \leftarrow \mathsf{pad}(M)$;
  $Y_0 \leftarrow E_K(N)$;
  for $i = 1$ **to** $a - 1$ **do**   ▷ $Y_{i-1,0} = \mathsf{msb}_w(Y_{i-1}), Y_{i-1,1} = \mathsf{lsb}_w(Y_{i-1})$
    $Y_i \leftarrow E_{Y_{i-1,0}}(A_i\|Y_{i-1,1})$;
  if $|A| > 0 \wedge |A| \equiv 0 \pmod{w}$ **then**
    $Y_a \leftarrow E_{Y_{a-1,0}}(A_a\|\pi_0(Y_{a-1,1}))$;
  else
    $Y_a \leftarrow E_{Y_{a-1,0}}(A_a\|\pi_1(Y_{a-1,1}))$;
  for $i = 1$ **to** $m - 1$ **do**
    $C_i \leftarrow M_i \oplus Y_{a+i-1,1}$;
    $Y_{a+i} \leftarrow E_{Y_{a+i-1,0}}(M_i\|Y_{a+i-1,1})$;
  $C_m \leftarrow M_m \oplus Y_{a+m-1,1}$;
  if $|M| > 0 \wedge |M| \equiv 0 \pmod{w}$ **then**
    $T \leftarrow E_{Y_{a+m-1,0}}(M_m\|\pi_0(Y_{a+m-1,1}))$;
  else
    $T \leftarrow E_{Y_{a+m-1,0}}(M_m\|\pi_1(Y_{a+m-1,1}))$;
  $C \leftarrow C_1\|\cdots\|C_{m-1}\|\mathsf{msb}_{|M|-(m-1)w}(C_m)$;
  return C, T;

function $\mathsf{D0}_K(N, A, C, T)$
  $(A_1, A_2, \ldots, A_a) \leftarrow \mathsf{pad}(A)$;
  $(C_1, C_2, \ldots, C_m) \leftarrow \mathsf{pad}(C)$;
  $Y_0 \leftarrow E_K(N)$;
  for $i = 1$ **to** $a - 1$ **do**
    $Y_i \leftarrow E_{Y_{i-1,0}}(A_i\|Y_{i-1,1})$;
  if $|A| > 0 \wedge |A| \equiv 0 \pmod{w}$ **then**
    $Y_a \leftarrow E_{Y_{a-1,0}}(A_a\|\pi_0(Y_{a-1,1}))$;
  else
    $Y_a \leftarrow E_{Y_{a-1,0}}(A_a\|\pi_1(Y_{a-1,1}))$;
  for $i = 1$ **to** $m - 1$ **do**
    $M_i \leftarrow C_i \oplus Y_{a+i-1,1}$;
    $Y_{a+i} \leftarrow E_{Y_{a+i-1,0}}(M_i\|Y_{a+i-1,1})$;
  $M_m \leftarrow C_m \oplus \mathsf{msb}_{|C|-(m-1)w}(Y_{a+m-1,1})$;
  if $|C| > 0 \wedge |C| \equiv 0 \pmod{w}$ **then**
    $T' \leftarrow E_{Y_{a+m-1,0}}(M_m\|\pi_0(Y_{a+m-1,1}))$;
  else
    $T' \leftarrow E_{Y_{a+m-1,0}}(M_m\|\pi_1(Y_{a+m-1,1}))$;
  $M \leftarrow M_1\|\cdots\|M_{m-1}\|\mathsf{msb}_{|C|-(m-1)w}(M_m)$;
  if $T' = T$ **then**
    return M;
  else
    return $\bot$;

Fig. 2 The encryption function E0 of the nonce-based AEAD scheme Lae0. $(C, T) \leftarrow \mathsf{E0}_K(N, A, M)$, where $\mathsf{pad}(A) = (A_1, A_2)$, $\mathsf{pad}(M) = (M_1, M_2, M_3)$, and $C = C_1\|C_2\|C_3$. This figure assumes that $|A| \not\equiv 0 \pmod{w}$, $|M| \equiv 0 \pmod{w}$ and $\tau = n$.

5. Security of Lae0 in the Ideal Cipher Model

The security of $\mathsf{Lae0} = (\mathsf{E0}, \mathsf{D0})$ is analyzed in the ideal cipher model. Thus, adversaries are given oracle access to encryption E and decryption E^{-1} of the block cipher used in Lae0. Without loss of generality, it is assumed that adversaries do not make trivial queries to E and E^{-1}. Namely, once an adversary obtains a triplet (S, U, V) such that $E_S(U) = V$ by a query to E or E^{-1}, it makes no new queries on the triplet.

A combinatorial theorem used in the analysis is first presented:

Lemma 1 (Theorem 3.1 in [29]) Suppose that there are t balls and t bins and that each ball is placed in a bin chosen independently and uniformly at random. Then, with probability at least $1 - 1/t$, no bin has more than $\mathrm{e}\ln t/\ln\ln t$ balls in it.

For Lemma 1, let $t = 2^w$. Then,

$$\mathrm{e}\ln t/\ln\ln t = \mathrm{e}w/(\log_2 w - \log_2\log_2 \mathrm{e}),$$

which is denoted by $\gamma(w)$ in the remaining part.

Example 1 $\gamma(128) \approx 53.77$.

5.1 Privacy

From the theorem given below, for privacy, Lae0 is secure against nonce-respecting adversaries causing at most $O(2^w/\gamma(w))$ evaluations of its underlying block cipher. For $w = 128$, $2^w/\gamma(w) \approx 2^{122.25}$.

The main idea of the proof of the following theorem is simple: The outputs of E0 look random to an adversary if the set of the triplets (S, U, V) such that $E_S(U) = V$ used by the process of E0 and the set of them obtained by the queries to E and E^{-1} made by the adversary are disjoint.

Theorem 1 Let **A** be any adversary against Lae0 for privacy. Suppose that **A** makes at most q_{e} and q_{d} queries to E and E^{-1}, respectively. Suppose that σ is the total number of the queries to E induced by the queries to E0 and D0 made by **A**. Let $q = q_{\mathrm{e}} + q_{\mathrm{d}}$ and suppose that $q + \sigma \leq 2^w$. Then,

$$\mathrm{Adv}^{\mathrm{priv}}_{\mathsf{Lae0}}(\mathbf{A}) \leq \frac{\gamma(w)q_{\mathrm{e}} + q_{\mathrm{d}} + q + \sigma + 1}{2^w} + \frac{(q+\sigma)^2}{2^{n-1}}$$

in the ideal cipher model.

Proof This proof uses the game transformation technique. In the game PGr1 given in Fig. 3, BCenc and BCdec implement E and E^{-1} using lazy evaluation, respectively, and AEenc implements E0. Thus,

$$\mathrm{Pr}[\mathbf{A}^{\mathsf{E0}_K} = 1] = \mathrm{Pr}[\mathbf{A}^{\mathrm{PGr1}} = 1].$$

PGr2 differs from PGr1 only in $\mathscr{E}$ and $\mathscr{D}$, which are

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Initialization:
100: $K \xleftarrow{} \Sigma^w$;
101: $\mathrm{E}[S, U] \leftarrow \perp$ for every (S, U);
102: $\mathrm{D}[S, V] \leftarrow \perp$ for every (S, V);
103: $P_S \leftarrow \{\}$ for every S; $C_S \leftarrow \{\}$ for every S;
104: $bad \leftarrow \mathtt{false}$;

Function $\mathscr{E}(S, U)$:	**Function $\mathscr{D}(S, V)$:**
200: **if** $\mathrm{E}[S, U] = \perp$ **then**	300: **if** $\mathrm{D}[S, V] = \perp$ **then**
201: $\quad V \xleftarrow{} \Sigma^n \setminus C_S$	301: $\quad U \xleftarrow{} \Sigma^n \setminus P_S$
202: $\quad P_S \leftarrow P_S \cup \{U\}$	301: $\quad P_S \leftarrow P_S \cup \{U\}$
203: $\quad C_S \leftarrow C_S \cup \{V\}$	303: $\quad C_S \leftarrow C_S \cup \{V\}$
204: $\quad \mathrm{E}[S, U] \leftarrow V$	304: $\quad \mathrm{E}[S, U] \leftarrow V$
205: $\quad \mathrm{D}[S, V] \leftarrow U$	305: $\quad \mathrm{D}[S, V] \leftarrow U$
206: **return** $\mathrm{E}[S, U]$	306: **return** $\mathrm{D}[S, V]$

Oracle AEenc(N, A, M):
500: $(A_1, A_2, \ldots, A_a) \leftarrow \mathsf{pad}(A)$;
501: $(M_1, M_2, \ldots, M_m) \leftarrow \mathsf{pad}(M)$;
502: $Y_0 \leftarrow \mathscr{E}(K, N)$;
503: **for** $i = 1$ **to** $a - 1$ **do**
504: $\quad Y_i \leftarrow \mathscr{E}(Y_{i-1,0}, A_i \| Y_{i-1,1})$;
505: **if** $|A| > 0 \wedge |A| \equiv 0 \pmod{w}$ **then**
506: $\quad Y_a \leftarrow \mathscr{E}(Y_{a-1,0}, A_a \| \pi_0(Y_{a-1,1}))$;
507: **else**
508: $\quad Y_a \leftarrow \mathscr{E}(Y_{a-1,0}, A_a \| \pi_1(Y_{a-1,1}))$;
509: **for** $i = 1$ **to** $m - 1$ **do**
510: $\quad C_i \leftarrow M_i \oplus Y_{a+i-1,1}$;
511: $\quad Y_{a+i} \leftarrow \mathscr{E}(Y_{a+i-1,0}, M_i \| Y_{a+i-1,1})$;
512: $C_m \leftarrow M_m \oplus Y_{a+m-1,1}$;
513: **if** $|M| > 0 \wedge |M| \equiv 0 \pmod{w}$ **then**
514: $\quad T \leftarrow \mathscr{E}(Y_{a+m-1,0}, M_m \| \pi_0(Y_{a+m-1,1}))$;
515: **else**
516: $\quad T \leftarrow \mathscr{E}(Y_{a+m-1,0}, M_m \| \pi_1(Y_{a+m-1,1}))$;
517: $C \leftarrow C_1 \| \cdots \| C_{m-1} \| \mathsf{msb}_{|M|-(m-1)w}(C_m)$;
518: **return** C, T;

Oracle BCenc(S, U):	**Oracle BCdec(S, V):**
600: **return** $\mathscr{E}(S, U)$;	600: **return** $\mathscr{D}(S, V)$;

Fig. 3 Game PGr1

Function $\mathscr{E}(S, U)$:	**Function $\mathscr{D}(S, V)$:**
200: **if** $\mathrm{E}[S, U] = \perp$ **then**	300: **if** $\mathrm{D}[S, V] = \perp$ **then**
201: $\quad V \xleftarrow{} \Sigma^n$	301: $\quad U \xleftarrow{} \Sigma^n$
202: $\quad$ **if** $V \in C_S$ **then**	302: $\quad$ **if** $U \in P_S$ **then**
203: $\qquad bad \leftarrow \mathtt{true}$	303: $\qquad bad \leftarrow \mathtt{true}$
204: $\quad P_S \leftarrow P_S \cup \{U\}$	304: $\quad P_S \leftarrow P_S \cup \{U\}$
205: $\quad C_S \leftarrow C_S \cup \{V\}$	305: $\quad C_S \leftarrow C_S \cup \{V\}$
206: $\quad \mathrm{E}[S, U] \leftarrow V$	306: $\quad \mathrm{E}[S, U] \leftarrow V$
207: $\quad \mathrm{D}[S, V] \leftarrow U$	307: $\quad \mathrm{D}[S, V] \leftarrow U$
208: **return** $\mathrm{E}[S, U]$	308: **return** $\mathrm{D}[S, V]$

Fig. 4 Game PGr2

given in Fig. 4. PGr2 is equivalent to PGr1 until bad gets true in $\mathscr{E}$ or $\mathscr{D}$. Thus,

$$\left| \mathrm{Pr}[\mathbf{A}^{\mathrm{PGr1}} = 1] - \mathrm{Pr}[\mathbf{A}^{\mathrm{PGr2}} = 1] \right| \leq (q+\sigma)^2/2^{n+1}.$$

PGr3 differs from PGr2 only in **Initialization**, $\mathscr{E}$ and $\mathscr{D}$, which are given in Fig. 5. The differences are minor, and

$$\mathrm{Pr}[\mathbf{A}^{\mathrm{PGr3}} = 1] = \mathrm{Pr}[\mathbf{A}^{\mathrm{PGr2}} = 1].$$

PGr4 differs from PGr3 only in $\mathscr{E}$ and $\mathscr{D}$, which are

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Initialization:
100: K ⫫ Σ^w;
101: E[S, U] ← ⊥ for every (S, U);
102: D[S, V] ← ⊥ for every (S, V);
103: bad ← false;
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Function $\mathscr{E}(S, U)$:	**Function** $\mathscr{D}(S, V)$:
200: **if** $E[S, U] = \bot$ **then**	300: **if** $D[S, V] = \bot$ **then**
201: $\quad V \twoheadleftarrow \Sigma^n$	301: $\quad U \twoheadleftarrow \Sigma^n$
202: $\quad E[S, U] \leftarrow V$	302: $\quad E[S, U] \leftarrow V$
203: $\quad D[S, V] \leftarrow U$	303: $\quad D[S, V] \leftarrow U$
204: **return** $E[S, U]$	304: **return** $D[S, V]$

Fig. 5   Game PGr3

Function $\mathscr{E}(S, U)$:	**Function** $\mathscr{D}(S, V)$:
200: **if** $E[S, U] \neq \bot$ **then**	300: **if** $D[S, V] \neq \bot$ **then**
201: $\quad bad \leftarrow \text{true}$	301: $\quad bad \leftarrow \text{true}$
202: **else**	302: **else**
203: $\quad V \twoheadleftarrow \Sigma^n$	303: $\quad U \twoheadleftarrow \Sigma^n$
204: $\quad E[S, U] \leftarrow V$	304: $\quad E[S, U] \leftarrow V$
205: $\quad D[S, V] \leftarrow U$	305: $\quad D[S, V] \leftarrow U$
206: **return** $E[S, U]$	306: **return** $D[S, V]$

Fig. 6   Game PGr4

given in Fig. 6. The differences are also minor, and

$$\Pr[\mathbf{A}^{\text{PGr4}} = 1] = \Pr[\mathbf{A}^{\text{PGr3}} = 1].$$

In the game PGi1 given in Fig. 7, BCenc and BCdec implement E and E^{-1} using lazy evaluation, respectively, and AEenc implements \$. Thus,

$$\Pr[\mathbf{A}^\$ = 1] = \Pr[\mathbf{A}^{\text{PGi1}} = 1].$$

PGi2 differs from PGi1 only in $\mathscr{E}$ and $\mathscr{D}$, which are given in Fig. 8. Similar to the transformation from PGr1 to PGr3,

$$\left| \Pr[\mathbf{A}^{\text{PGi1}} = 1] - \Pr[\mathbf{A}^{\text{PGi2}} = 1] \right| \leq q^2 / 2^{n+1}.$$

Notice that $\mathscr{E}$ and $\mathscr{D}$ are called only by BCenc and BCdec, respectively.

PGr4 is equivalent to PGi2 until bad gets true in PGr4. Let Bad be the event that $\mathbf{A}^{\text{PGr4}}$ sets bad true. Then,

$$\left| \Pr[\mathbf{A}^{\text{PGr4}} = 1] - \Pr[\mathbf{A}^{\text{PGi2}} = 1] \right| \leq \Pr[\text{Bad}].$$

For PGr4, let Hit be the event that $\mathscr{E}$ receives a query (K, U) for some U except for the cases that (K, N) is the first query made by AEenc to respond to a query (N, A, M) made by $\mathbf{A}$, or $\mathscr{D}$ receives a query (K, V) for some V. Then,

$$\Pr[\text{Bad}] \leq \Pr[\text{Hit}] + \Pr[\text{Bad} \,|\, \overline{\text{Hit}}]$$

and

$$\Pr[\text{Hit}] \leq (q + \sigma)/2^w.$$

For Bad, let Bad_{AE} be the event that a query from AEenc to $\mathscr{E}$ sets bad true for the first time and Bad_{BC} be the event that a query from BCenc or BCdec sets bad true for the

```
Initialization:
100: E[S, U] ← ⊥ for every (S, U);
101: D[S, V] ← ⊥ for every (S, V);
102: P_S ← {} for every S; C_S ← {} for every S;
```

Function $\mathscr{E}(S, U)$:	**Function** $\mathscr{D}(S, V)$:
200: **if** $E[S, U] = \bot$ **then**	300: **if** $D[S, V] = \bot$ **then**
201: $\quad V \twoheadleftarrow \Sigma^n \setminus C_S$	301: $\quad U \twoheadleftarrow \Sigma^n \setminus P_S$
202: $\quad P_S \leftarrow P_S \cup \{U\}$	302: $\quad P_S \leftarrow P_S \cup \{U\}$
203: $\quad C_S \leftarrow C_S \cup \{V\}$	303: $\quad C_S \leftarrow C_S \cup \{V\}$
204: $\quad E[S, U] \leftarrow V$	304: $\quad E[S, U] \leftarrow V$
205: $\quad D[S, V] \leftarrow U$	305: $\quad D[S, V] \leftarrow U$
206: **return** $E[S, U]$	306: **return** $D[S, V]$

```
Oracle AEenc(N, A, M):
500: C ⫫ Σ^|M|; T ⫫ Σ^n;
501: return C, T;
```

Oracle BCenc(S, U):	**Oracle** BCdec(S, V):
600: **return** $\mathscr{E}(S, U)$;	600: **return** $\mathscr{D}(S, V)$;

Fig. 7   Game PGi1

Function $\mathscr{E}(S, U)$:	**Function** $\mathscr{D}(S, V)$:
200: $V \twoheadleftarrow \Sigma^n$	300: $U \twoheadleftarrow \Sigma^n$
201: $E[S, U] \leftarrow V$	301: $E[S, U] \leftarrow V$
202: $D[S, V] \leftarrow U$	302: $D[S, V] \leftarrow U$
203: **return** $E[S, U]$	303: **return** $D[S, V]$

Fig. 8   Game PGi2

first time. Then,

$$\Pr[\text{Bad} \,|\, \overline{\text{Hit}}] \leq \Pr[\text{Bad}_{\text{AE}} \,|\, \overline{\text{Hit}}] + \Pr[\text{Bad}_{\text{BC}} \,|\, \overline{\text{Hit}}].$$

$$\Pr[\text{Bad}_{\text{AE}} \,|\, \overline{\text{Hit}}] \leq \sigma(q + \sigma)/2^n.$$

Further, for Bad_{BC}, let Bad_{BCe} and Bad_{BCd} be the events that a query from BCenc and BCdec sets bad true for the first time, respectively. Then,

$$\Pr[\text{Bad}_{\text{BC}} \,|\, \overline{\text{Hit}}] \leq \Pr[\text{Bad}_{\text{BCe}} \,|\, \overline{\text{Hit}}] + \Pr[\text{Bad}_{\text{BCd}} \,|\, \overline{\text{Hit}}].$$

For Bad_{BCe}, from Lemma 1,

$$\Pr[\text{Bad}_{\text{BCe}} \,|\, \overline{\text{Hit}}] \leq \gamma(w)q_{\text{e}}/2^w + 1/2^w.$$

For Bad_{BCd}, considering the probability of collision among the replies from $\mathscr{E}$, we obtain

$$\Pr[\text{Bad}_{\text{BCd}} \,|\, \overline{\text{Hit}}] \leq q_{\text{d}}/2^w + (q + \sigma)^2/2^{n+1}.$$

Thus,

$$\left| \Pr[\mathbf{A}^{\text{PGr4}} = 1] - \Pr[\mathbf{A}^{\text{PGi2}} = 1] \right|$$
$$\leq \frac{\gamma(w)q_{\text{e}} + q_{\text{d}} + q + \sigma + 1}{2^w} + \frac{\sigma(q + \sigma)}{2^n} + \frac{(q + \sigma)^2}{2^{n+1}}.$$

Consequently,

$$\text{Adv}_{\text{Lae0}}^{\text{priv}}(\mathbf{A}) \leq \left| \Pr[\mathbf{A}^{\text{PGr4}} = 1] - \Pr[\mathbf{A}^{\text{PGi2}} = 1] \right|$$
$$+ \left| \Pr[\mathbf{A}^{\text{PGr1}} = 1] - \Pr[\mathbf{A}^{\text{PGr2}} = 1] \right|$$
$$+ \left| \Pr[\mathbf{A}^{\text{PGi1}} = 1] - \Pr[\mathbf{A}^{\text{PGi2}} = 1] \right|$$

$$\leq \frac{\gamma(w)q_e + q_d + q + \sigma + 1}{2^w}$$
$$+ \frac{\sigma(q+\sigma)}{2^n} + \frac{(q+\sigma)^2}{2^n} + \frac{q^2}{2^{n+1}},$$

where

$$\frac{\sigma(q+\sigma)}{2^n} + \frac{(q+\sigma)^2}{2^n} + \frac{q^2}{2^{n+1}} \leq \frac{(q+\sigma)^2}{2^{n-1}}.$$

□

5.2 Authenticity

We discuss the authenticity of Lae0 under misuses. Namely, we assume NR and RUP in the following analysis.

Definition 1 Let $\Pi \subset \mathcal{P}(\mathcal{X})$. We say that Π is pairwise everywhere distinct if, for every $\pi, \pi' \in \Pi$ such that $\pi \neq \pi'$, $\pi(x) \neq \pi'(x)$ for every $x \in \mathcal{X}$.

From the following theorem, for authenticity, Lae0 is secure against adversaries causing at most $O(2^w/\gamma(w))$ evaluations of its underlying block cipher in the setting allowing both NR and RUP.

Theorem 2 For permutations π_0 and π_1 on Σ^w used in Lae0, suppose that $\{\pi_0, \pi_1, \iota\}$ is pairwise everywhere distinct. Let **A** be any adversary against Lae0 for authenticity. Suppose that **A** makes at most q_e and q_d queries to E and E^{-1}, respectively, and q_D queries to D0. Suppose that σ is the total number of the queries to E induced by the queries to E0 and D0 made by **A**. Let $q = q_e + q_d$ and suppose that $q + \sigma \leq 2^w$. Then,

$$\text{Adv}_{\text{Lae0}}^{\text{auth}}(\mathbf{A}) \leq \frac{3\gamma(w)q + q_d}{2^w - 1} + \frac{q+\sigma+1}{2^w} + \frac{q_D + 7\sigma^2 + 3q\sigma}{2^n - q - \sigma}$$

in the ideal cipher model.

Proof In this proof, we refer to the game AG1 given in Fig. 9. In this game, BCenc and BCdec implement E and E^{-1} using lazy evaluation, respectively. AEenc and AEdec implement E0 and D0, respectively.

It is assumed that, for each query made by **A**, AEenc and AEdec give all $Y_{j,1}$'s to **A** together with the corresponding reply. Then, they are more informative than in the RUP setting.

Let $\mathcal{E}$ be the set of input-output pairs of the underlying block cipher obtained by the queries to $\mathcal{E}$ induced by the queries to AEenc and AEdec.

Let MCol be the event that

$$\max_{v \in \Sigma^w}\left|\{(S, U, V) \in \mathcal{E} \mid \text{lsb}_w(V) = v\}\right| > \gamma(w).$$

Then, since $\sigma \leq 2^w$, from Lemma 1,

$$\Pr[\text{MCol}] \leq 1/2^w.$$

For a query to $\mathcal{E}$ or $\mathcal{D}$, let $\mathcal{W}_{\text{AE}}$ be the set of (S, U, V)'s

---

Initialization:
100: $K \xleftarrow{\$} \Sigma^w$;
101: $E[S, U] \leftarrow \perp$ for every (S, U);
102: $D[S, V] \leftarrow \perp$ for every (S, V);
103: $P_S \leftarrow \{\}; C_S \leftarrow \{\}$;

Function $\mathcal{E}(S, U)$:
200: **if** $E[S, U] = \perp$ **then**
201:     $V \xleftarrow{\$} \Sigma^n \setminus C_S$
202:     $P_S \leftarrow P_S \cup \{U\}$
203:     $C_S \leftarrow C_S \cup \{V\}$
204:     $E[S, U] \leftarrow V$
205:     $D[S, V] \leftarrow U$
206: **return** $E[S, U]$

Function $\mathcal{D}(S, V)$:
300: **if** $D[S, V] = \perp$ **then**
301:     $U \xleftarrow{\$} \Sigma^n \setminus P_S$
302:     $P_S \leftarrow P_S \cup \{U\}$
303:     $C_S \leftarrow C_S \cup \{V\}$
304:     $E[S, U] \leftarrow V$
305:     $D[S, V] \leftarrow U$
306: **return** $D[S, V]$

Oracle AEenc(N, A, M):
500: $(A_1, A_2, \ldots, A_a) \leftarrow \text{pad}(A)$;
501: $(M_1, M_2, \ldots, M_m) \leftarrow \text{pad}(M)$;
502: $Y_0 \leftarrow \mathcal{E}(K, N)$;
503: **for** $i = 1$ **to** $a - 1$ **do**
504:     $Y_i \leftarrow \mathcal{E}(Y_{i-1,0}, A_i \| Y_{i-1,1})$;
505: **if** $|A| > 0 \wedge |A| \equiv 0 \pmod{w}$ **then**
506:     $Y_a \leftarrow \mathcal{E}(Y_{a-1,0}, A_a \| \pi_0(Y_{a-1,1}))$;
507: **else**
508:     $Y_a \leftarrow \mathcal{E}(Y_{a-1,0}, A_a \| \pi_1(Y_{a-1,1}))$;
509: **for** $i = 1$ **to** $m - 1$ **do**
510:     $C_i \leftarrow M_i \oplus Y_{a+i-1,1}$;
511:     $Y_{a+i} \leftarrow \mathcal{E}(Y_{a+i-1,0}, M_i \| Y_{a+i-1,1})$;
512: $C_m \leftarrow M_m \oplus Y_{a+m-1,1}$;
513: **if** $|M| > 0 \wedge |M| \equiv 0 \pmod{w}$ **then**
514:     $T \leftarrow \mathcal{E}(Y_{a+m-1,0}, M_m \| \pi_0(Y_{a+m-1,1}))$;
515: **else**
516:     $T \leftarrow \mathcal{E}(Y_{a+m-1,0}, M_m \| \pi_1(Y_{a+m-1,1}))$;
517: $C \leftarrow C_1 \| \cdots \| C_{m-1} \| \text{msb}_{|M|-(m-1)w}(C_m)$;
518: **return** C, T;

Oracle AEdec(N, A, C, T):
600: $(A_1, A_2, \ldots, A_a) \leftarrow \text{pad}(A)$;
601: $(C_1, C_2, \ldots, C_m) \leftarrow \text{pad}(C)$;
602: $Y_0 \leftarrow \mathcal{E}(K, N)$;
603: **for** $i = 1$ **to** $a - 1$ **do**
604:     $Y_i \leftarrow \mathcal{E}(Y_{i-1,0}, A_i \| Y_{i-1,1})$;
605: **if** $|A| > 0 \wedge |A| \equiv 0 \pmod{w}$ **then**
606:     $Y_a \leftarrow \mathcal{E}(Y_{a-1,0}, A_a \| \pi_0(Y_{a-1,1}))$;
607: **else**
608:     $Y_a \leftarrow \mathcal{E}(Y_{a-1,0}, A_a \| \pi_1(Y_{a-1,1}))$;
609: **for** $i = 1$ **to** $m - 1$ **do**
610:     $M_i \leftarrow C_i \oplus Y_{a+i-1,1}$;
611:     $Y_{a+i} \leftarrow \mathcal{E}(Y_{a+i-1,0}, M_i \| Y_{a+i-1,1})$;
612: $M_m \leftarrow C_m \oplus \text{msb}_{|C|-(m-1)w}(Y_{a+m-1,1})$;
613: **if** $|C| > 0 \wedge |C| \equiv 0 \pmod{w}$ **then**
614:     $T' \leftarrow \mathcal{E}(Y_{a+m-1,0}, M_m \| \pi_0(Y_{a+m-1,1}))$;
615: **else**
616:     $T' \leftarrow \mathcal{E}(Y_{a+m-1,0}, M_m \| \pi_1(Y_{a+m-1,1}))$;
617: $M \leftarrow M_1 \| \cdots \| M_{m-1} \| \text{msb}_{|C|-(m-1)w}(M_m)$;
618: **if** $T' = T$ **then**
619:     **return** M;
620: **else**
621:     **return** $\perp$;

Oracle BCenc(S, U):
600: **return** $\mathcal{E}(S, U)$;

Oracle BCdec(S, V):
600: **return** $\mathcal{D}(S, V)$;

Fig. 9 Game AG1

obtained by all the previous queries to $\mathscr{E}$ induced by queries to AEenc or AEdec made by **A**. For a query to $\mathscr{E}$ or $\mathscr{D}$, let $\mathcal{W}_{\mathsf{BC}}$ be the set of (S, U, V)'s obtained by all the previous queries to BCenc or BCdec made by **A**. A query (S, U) to $\mathscr{E}$ is called fresh if $\mathrm{E}[S, U] = \bot$.

Let $\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}$ be the event that, for a fresh query to $\mathscr{E}$ induced by a query to AEenc or AEdec, $\mathscr{E}$ replies V such that $V_0 = K$ or, for some $(S', U', V') \in \mathcal{W}_{\mathsf{AE}}$, $V_0 = V_0'$ and $\{V_1, \pi_0(V_1), \pi_1(V_1)\} \cap \{V_1', \pi_0(V_1'), \pi_1(V_1')\} \neq \emptyset$, that is,

$$V_1 \in \{V_1', \pi_0(V_1'), \pi_1(V_1'), \pi_0^{-1}(V_1'), \pi_0^{-1}(\pi_1(V_1')),$$
$$\pi_1^{-1}(V_1'), \pi_1^{-1}(\pi_0(V_1'))\},$$

where $V = V_0 \| V_1$, $V' = V_0' \| V_1'$ and $|V_0| = |V_1| = |V_0'| = |V_1'| = w$. Then,

$$\Pr[\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}] \leq \frac{\sigma}{2^w} + \frac{7\sigma^2}{2^n - q - \sigma}.$$

Let $\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{BC}}$ be the event that, for a fresh query to $\mathscr{E}$ induced by a query to AEenc or AEdec, $\mathscr{E}$ replies V such that, for some $(S', U', V') \in \mathcal{W}_{\mathsf{BC}}$, $S' = V_0$ and $\mathsf{lsb}_w(U') \in \{V_1, \pi_0(V_1), \pi_1(V_1)\}$. Then,

$$\Pr[\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{BC}}] \leq 3q\sigma/(2^n - q - \sigma).$$

Let $\mathsf{Bad}_{\mathsf{BCe}}$ be the event that **A** makes at least one query (S, U) to BCenc such that $S = K$ or, for some $(S', U', V') \in \mathcal{W}_{\mathsf{AE}}$, $S = V_0'$ and $U_1 \in \{V_1', \pi_0(V_1'), \pi_1(V_1')\}$. Then, since $q + \sigma \leq 2^w$,

$$\Pr[\mathsf{Bad}_{\mathsf{BCe}} \,|\, \overline{\mathsf{MCol}}] \leq \frac{q_{\mathsf{e}}}{2^w} + \frac{2^w \cdot 3\gamma(w)q_{\mathsf{e}}}{2^n - (q + \sigma)}$$
$$\leq \frac{q_{\mathsf{e}}}{2^w} + \frac{3\gamma(w)q_{\mathsf{e}}}{2^w - 1}.$$

Let $\mathsf{Bad}_{\mathsf{BCd}}$ be the event that **A** makes at least one query (S, V) to BCdec such that $S = K$ or, for some $(S', U', V') \in \mathcal{W}_{\mathsf{AE}}$, $S = S'$ and $V = V'$, or $S = V_0'$ and $\mathsf{lsb}_w(U) \in \{V_1', \pi_0(V_1'), \pi_1(V_1')\}$, where U is the reply to the query (S, V). Then,

$$\Pr[\mathsf{Bad}_{\mathsf{BCd}} \,|\, \overline{\mathsf{MCol}} \cap \overline{\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}}] \leq \frac{q_{\mathsf{d}}}{2^w} + \frac{q_{\mathsf{d}}}{2^w - 1} + \frac{3\gamma(w)q_{\mathsf{d}}}{2^w - 1}.$$

Let Forge be the event that **A** succeeds in forgery. Let $\mathsf{Bad} = \mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}} \cup \mathsf{Bad}_{\mathsf{AE}}^{\mathsf{BC}} \cup \mathsf{Bad}_{\mathsf{BCe}} \cup \mathsf{Bad}_{\mathsf{BCd}}$. If Bad does not occur, then, since $\{\pi_0, \pi_1, \iota\}$ is pairwise everywhere distinct, for each query to AEdec made by **A**, the final query to $\mathscr{E}$ induced by the query is fresh. Thus,

$$\mathrm{Adv}_{\mathsf{Lae0}}^{\mathrm{auth}}(\mathbf{A}) = \Pr[\mathsf{Forge}] \leq \Pr[\mathsf{Bad}] + \Pr[\mathsf{Forge} \,|\, \overline{\mathsf{Bad}}],$$

where

$$\Pr[\mathsf{Forge} \,|\, \overline{\mathsf{Bad}}] \leq q_{\mathrm{D}}/(2^n - q - \sigma).$$

In addition,

$$\Pr[\mathsf{Bad}] = \Pr[\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}} \cup \mathsf{Bad}_{\mathsf{AE}}^{\mathsf{BC}} \cup \mathsf{Bad}_{\mathsf{BCe}} \cup \mathsf{Bad}_{\mathsf{BCd}}]$$

$$\leq \Pr[\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}] + \Pr[\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{BC}}]$$
$$+ \Pr[\mathsf{Bad}_{\mathsf{BCe}} \cup (\mathsf{Bad}_{\mathsf{BCd}} \cap \overline{\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}})],$$

and

$$\Pr[\mathsf{Bad}_{\mathsf{BCe}} \cup (\mathsf{Bad}_{\mathsf{BCd}} \cap \overline{\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}})]$$
$$\leq \Pr[\mathsf{MCol}] + \Pr[\mathsf{Bad}_{\mathsf{BCe}} \cup (\mathsf{Bad}_{\mathsf{BCd}} \cap \overline{\mathsf{Bad}_{\mathsf{AE}}^{\mathsf{AE}}}) \,|\, \overline{\mathsf{MCol}}].$$

Thus,

$$\Pr[\mathsf{Bad}] \leq \frac{3\gamma(w)q + q_{\mathrm{d}}}{2^w - 1} + \frac{q + \sigma + 1}{2^w} + \frac{7\sigma^2 + 3q\sigma}{2^n - q - \sigma}.$$

This completes the proof. $\qquad\square$

6. Conclusion

The privacy and authenticity of Lae0 have been analyzed in the ideal cipher model. The analysis implies that, for both privacy and authenticity, the instantiation of Lae0 with the Lesamnta-LW block cipher has about 120-bit security against generic attacks regarding the block cipher as a black box.

Acknowledgments

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