LETTER Special Section on Educational Technologies for Sustainable and Expansive Learning

Design and Development of a Card Game for Learning on the Structure of Arithmetic Story by Concatenated Sentence Integration

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SUMMARY This study focuses on creating arithmetical stories as a sub-task of problem posing and proposes a game named "Tri-prop scrabble" as a learning environment based on a fusion method of learning and game. The problem-posing ability has a positive relationship with mathematics achievement and understanding the mathematical structure of problems. In the proposed game, learners are expected to experience creating and concatenating various arithmetical stories by integrating simple sentences. The result of a preliminary feasibility study shows that the participants were able to pose and concatenate a variety of types of arithmetic stories and accept this game is helpful for learning arithmetic word problems.

key words: *arithmetical word problem, problem posing, learning game, a fusion of learning and game*

1. Introduction

A learning method by posing problems rather than solving them is called "learning by problem-posing." Some researchers have reported a positive relationship between problem-posing ability and mathematics achievement [1], [2] and the understanding of the mathematical structure of problems [3]. However, the implementation of learning by problem posing in practical use is facing the issue of inefficiency in the time needed for assessment and giving feedback to students' posed problems.

Hirashima et al. have developed Monsakun as a learning environment for posing arithmetical word problems by sentence integration. In the learning environment, learners can learn the structure of arithmetical word problems [4]. This system provides learners with some sentence cards (e.g., there are three apples) representing a proposition with a quantity, and learners select and assemble these cards to make an arithmetical word problem. Through this activity, Monsakun aims to encourage them to learn the structure of arithmetical stories. The use of Monsakun in elementary schools demonstrated its effectiveness [5].

This paper proposes a learning game named "Tri-prop scrabble" designed based on a fusion method of learning and game [6]. Tri-prop scrabble is the fusion of Monsakun as learning with Scrabble [7] as a game. As mentioned above, Monsakun is the learning environment that requests learners to assemble an arithmetical word problem with sentence cards (one sentence card expresses one proposition). Scrabble is a game that asks players to concatenate words by gathering letter cards. Tri-prop scrabble enables learners to experience creating and comparing various arithmetical stories according to the situation. Because the activities in Tri-prop Scrabble are like the game activities in Scrabble in producing things by assembling and concatenating cards for posing arithmetical stories, it is expected that learners are also able to enjoy the learning activities in the same way with the original Scrabble.

In addition to the fusion with games, the characteristic of this study is augmented reality (AR) technology. Learners can pose arithmetic stories not on a tablet but in the real world. The reason is that, in the real world, learners can have much more space and options of simple sentences for posing arithmetic stories than Monsakun on a tablet. One of the goals of Tri-prop scrabble is to enable learners to consider many possibilities of concatenating arithmetic stories. This needs ample space to pose and link many arithmetic stories. However, it is difficult to judge the correctness of posed arithmetic stories in the real world. We use AR technology to capture and validate stories posed by learners to solve this problem.

There are a few previous studies on a fusion of Monsakun with a game [8] and integration of AR technology [9]. Shintake et al. [8] proposed a collaborative learning environment where learners competitively pose arithmetical word problems. The competitive element is a kind of gamification [10] of Monsakun by adding game elements. This study aims not to add game elements to Monsakun but to fuse Monsakun and a game for proposing new activity of posing arithmetical stories. Yamamoto presents the integration of Monsakun with AR technology [9]. The target task is to pose a unit arithmetical story that can be represented with an equation including only one arithmetic operator. On the other hand, the target task in this study is to pose concatenated arithmetical stories.

2. Triplet Structure Model and Monsakun

A unit arithmetical word problem is a problem that can be solved with a single arithmetical operation. Arithmeti-

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cal word problems can be formulated as two existent sentences that express the existence of a quantity and a relational sentence that describes the relationship between the two quantities [11]. There are four types of arithmetic stories: combination, increase, decrease, comparison (more and less) [12], [13]. These story types can be distinguished by the relational sentence and the roles of existent sentences in an arithmetic story. Furthermore, any problems can be defined within a problem space where unknown facts can be derived from given facts with relations in a context [14]. Therefore, a way to pose an arithmetical word problem is to make a valid arithmetical story with two existent sentences and a relational sentence and then to make them incomplete.

Whereas Monsakun asks learners to conduct both tasks of making a valid arithmetical story and making them incomplete at once, in this study, we focus on the former story-making task and consider concatenating arithmetic stories. In each question on Monsakun, as shown in Fig. 1, learners are required to pose the arithmetical word problem uniquely determined with the combination of the simple sentences given to the learner by the requirements on the formula and the type of story.

It can also be possible to create multiple stories from a simple sentence. For example, with the simple sentence, "there are seven apples," we can make all types of stories: combination, increase, decrease, comparisons. This study aims to realize an exercise to learn the mathematical structure of arithmetical stories by having learners examine the possibility of creating as many arithmetical stories as possible from a simple sentence. Figure 2 shows the possibility of combinations of simple sentences. Here, there are four arithmetic stories of combination, comparison, increase and decrease. These stories share the simple sentence "there are seven apples" in the middle of Fig. 2. This means that one simple sentence can be used for posing many arithmetic stories and have different roles. For example, the simple sentence "there are seven apples" in Fig. 2 has the resultant amount role in the increase story and the larger amount role in the comparison story.

On the other hand, in Monsakun, each problem-posing exercise has only one correct answer, and each provided simple sentence can have only one role in the correct answer. For example, in Fig. 1, "there are seven apples" can have only one role of the initial amount because the correct answer is a decrease story. To recognize the multiple roles of simple sentences in various stories is the learning goal of Tri-prop scrabble.

3. Tri-Prop Scrabble

Tri-Prop Scrabble is a multi-player card game in which players connect simple-sentence cards to create arithmetic stories. As shown in Fig. 3, like the word game Scrabble, players make stories by linking sentence cards that the players are dealt. The player who is first to shed their cards wins. At the first step of the design Tri-prop scrabble borrows only the basic rule of Scrabble in which players place letters to form words. That is because posing arithmetic stories is the essential activity the authors would have learners do to learn arithmetic word problems. In Tri-prop scrabble we replace letters with sentences and words with arithmetic word problems. At the beginning the game, each player draws six sen-

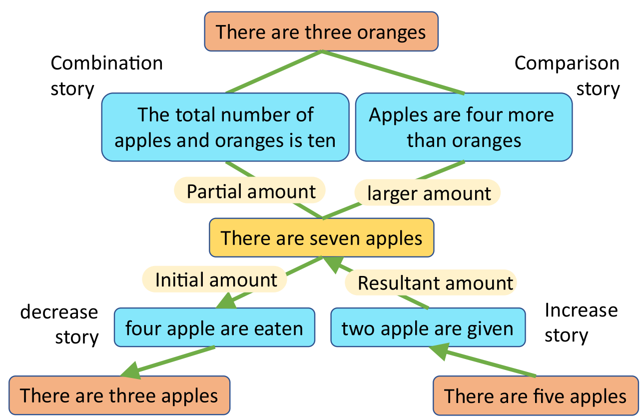

Fig. 2 Possibility of combination of simple-sentences.

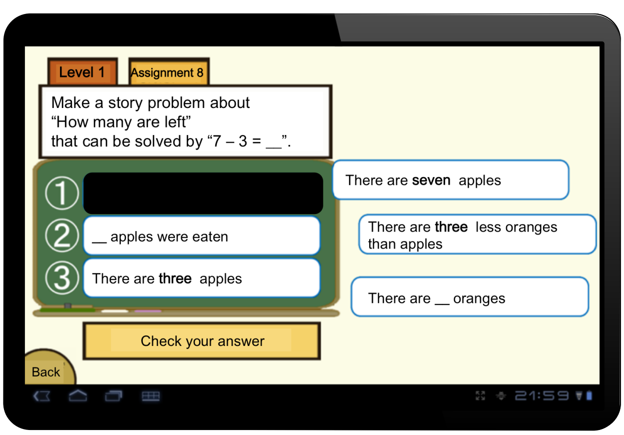

Fig. 1 Monsakun.

Fig. 3 Tri-prop scrabble.

tence cards from the deck. On each turn, the player tries to make an arithmetic word story and check the validity with Story AR checker. When the player cannot make an arithmetic word problem with his/her own cards, she or he can exchange one or two cards from the deck or forfeit the turn. The game ends when all the players cannot make any arithmetic word stories. The player who makes the greatest number of stories is the winner. Although the original scrabble has more complex rules for the scoring, the current Tri-prop scrabble has very simple rules for scoring and the extension of the scoring rule is a future issue.

The learning goal of Tri-Prop Scrabble is that learners consider many possibilities of combinations of simple sentences to pose arithmetic stories. Each card represents a simple sentence. Cards of existent sentence has gray frame and ones of relational sentence has colored frame. In this version cards must be arranged in the order of an existent sentence (gray colored frame), relational sentence (other colored frame), and another existent sentence (gray colored frame). The distinction between colors of cards is just whether gray of existent sentence and other color of relational sentence. In addition, each card has a picture to characterize the card for the identification by Story AR checker. The pictures do not mean the content of simple sentence on the cards. This is just a technique to improve the recognition accuracy of Story AR checker. The ends of cards arranged on the table allow to connect other simple-sentence cards, and players find a place where they can create an arithmetical story and make a story by taking out two of their cards. This activity aims to learn the mathematical structure of arithmetical stories required to understand arithmetical word problems.

There are two ways to pose a new story: forward and reverse thinking. Forward-thinking is the situation in which the equation implied by the story is the same as one to decide sentences to be added. On the other hand, reverse thinking is the situation in which the equation implied by the story is different from one to choose sentences to be added. Figure 4 shows an example of forward- and reverse-thinking. There are two stories of decrease and increase derived from the sentence "there are seven apples" in the middle of Fig. 4. The arrows represent the flow of sentences in each story. The increase story shown at the bottom of Fig. 4 is an example of forward-thinking. If a learner tries to use "three apples are given" to pose a story beginning from "there are seven apples", the learner needs to consider the addition "$7+3 = 10$" for deciding to use "there are ten apples" as the last sentence. In this case the calculation to decide the last sentence is the same as the one implied from the posed story. On the other hand, the decrease story shown at the top of Fig. 4 is an example of reverse thinking. If a learner tries to use "two apples are eaten" to pose a story ending with "there are seven apples", the learner needs to consider the addition "$7 + 2 = 9$" for deciding to use "there are nine apples" as the first sentence. In this case the calculation "$7 + 2 = 9$" to decide the number in the first sentence is different form the one "$9 - 2 = 7$" implied from the posed story.

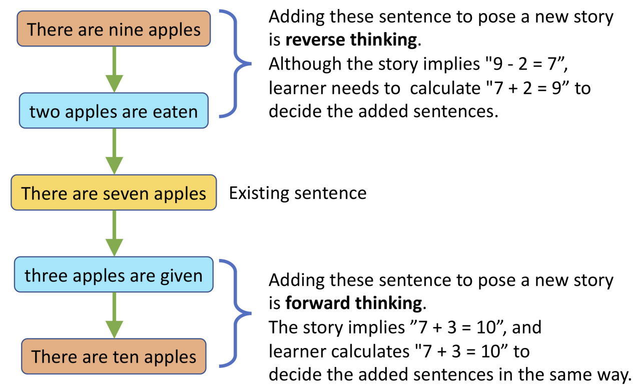

Fig. 4 Forward and reverse thinking.

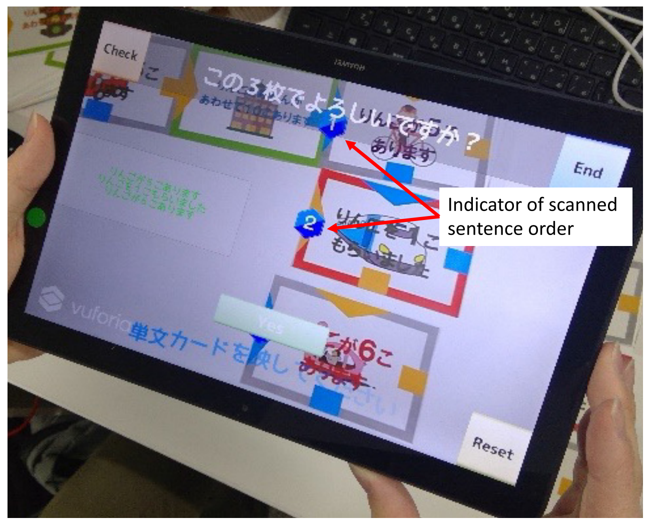

Fig. 5 Story AR checker.

Learners can check the validity of posed stories with Story AR checker shown in Fig. 5. Learners concatenate arithmetical stories with the simple sentence cards in their hands and compete in the number of stories they make. They can judge the correctness of their stories by scanning cards with Story AR checker. This system recognizes a story created as a sequence of cards in the real space by reading the simple sentence cards with markers on the tablet's camera. While playing this game, players take an image of a sequence of cards they make on the table with camera on tablet and then Story AR checker scan the card and identify the story made by the cards. If the system identifies a sequence of cards composing a story, it displays the indicator to represent numbers of sentence order on the image of scanned cards on the tablet as shown in Fig. 5. The system is developed using the Augmented Reality SDK of Vuforia [15] and Unity [16] and C# as the development environment.

4. Preliminary Feasibility Study

We conducted a preliminary feasibility study of playing Tri-prop Scrabble with Story AR checker. 18 female high school students participated in the study. Although the primary target of Tri-prop scrabble is elementary school students, in this experiment, the subjects are high school students because they can make sure to solve arithmetical word problems, that is, they understand the structure of arithmetical stories. If they cannot accept the tasks required in the game, it is almost impossible for elementary school students to play the game.

They are the participants of the lecture for high school students organized by the university the authors belong to. In the lecture, before playing Tri-prop scrabble, the second author explained Triplet structure model and Monsakun. And then, they did exercises in problem posing with Monsakun and took a test confirming the understanding of the structure of arithmetic word stories. The test asks them to make wrong arithmetic stories with provided sentences and to explain the reasons of the wrongness.

For playing Tri-prop scrabble the subjects were divided into groups of three. Each group played the game for 15 minutes. Before playing the game the first author explained the rule of Tri-prop scrabble with a document and how to use Story AR checker. They were also able to check the rule by reading the document or asking authors while they were playing the game.

The research questions of this preliminary feasibility study are the followings:

RQ1: Can the participants pose a variety of types of arithmetic stories?
RQ2: Can the participants accept this game is helpful for learning arithmetic word problems?

Table 1 shows the number and proportion of posed arithmetic stories by type, total numbers of the stories, and the score of the comprehension test. The purpose of this data is to show that they were able to make a certain number of stories and that types of stories they have made is balanced. For the latter item, we calculated the possible rate of story types with sentence cards provided for the participants. This rate was calculated with all the possible combination of cards. Therefore, this rate is different from real rate when they are playing the game because the combination is limited by the card each player has in their hand and posed stories on the table. Since it is too difficult to calculate the exact possible rate of stories in a game playing, we use the possible rate to as a criterion to consider whether there is a bias in posed stories by the players.

They made 9.3 stories per group. The "mean" row shows that they mainly made decrease and increase stories, almost 60% of stories they made. However, compared with the possible rate of each story type by prepared simple sentences in the game shown in the "possible" row, only the number of *less* story is significantly below the possible value by permutation test. In addition to that, excepting one group, each group has at least four types of stories of five types in total. The coefficient of correlation between the test scores and total numbers of posed stories is $r = 0.69$ ($p = 0.16$) though it is not significant and is from a small amount of data.

Table 1 The number and proportion of posed stories by type, total numbers of stories, and test score.

group	combine	decrease	increase	less	more	total	test
1	1 (0.143)	2 (0.286)	1 (0.143)	1 (0.143)	2 (0.286)	7	3.3
2	1 (0.071)	4 (0.286)	6 (0.429)	1 (0.071)	2 (0.143)	14	4.3
3	1 (0.125)	2 (0.250)	2 (0.250)	2 (0.250)	1 (0.125)	8	4.0
4	1 (0.091)	6 (0.545)	3 (0.273)	1 (0.091)	0 (0.000)	11	4.0
5	1 (0.143)	3 (0.429)	2 (0.286)	0 (0.000)	1 (0.143)	7	4.0
6	1 (0.111)	5 (0.556)	3 (0.333)	0 (0.000)	0 (0.000)	9	3.6
mean	1.0 (0.114)	3.7 (0.392)	2.8 (0.286)	0.8 (0.093)	1.0 (0.116)	9.3	3.9
possible	0.108	0.280	0.331	0.191	0.089		
p-value	0.634, n.s.	0.182, n.s.	0.288, n.s.	0.048 *	0.799, n.s.		

Table 2 shows the comparison between the posed stories of forward-thinking and reverse-thinking. This data shows the proportions of forward-thinking and reverse-thinking in each group excepting *combination* stories because, in Tri-prop scrabble, only *combination* stories must be made by reverse thinking. The possibility of them is even in possible posing patterns for other story types. We also use permutation test to test whether the proportion is different or not by comparing the ratios in each group. The result is not significant. This implies that the participants made no less reverse-thinking stories than forward-thinking.

From the results shown in Tables 1 and 2, participants were able to pose a certain number of stories in the activity with Tri-prop scrabble, and there is no bias in types of posed stories. By these results, we can answer yes to RQ1.

Furthermore, the resulted structure implies a further potential of Tri-prop scrabble to facilitate learners to think for linking arithmetic stories. Figure 6 shows the structured arithmetic stories made by group 4 that is second in their number of posed stories and their test score. The characteristic of this structure is that there is a circular ring of arithmetic stories in the middle of it. This structure requires

Table 2 Forward or reverse thinking.

group	1	2	3	4	5	6	mean	p-value
forward	0.333	0.462	0.714	0.600	0.667	0.625	0.567	0.1775
reverse	0.667	0.538	0.286	0.400	0.333	0.375	0.433	

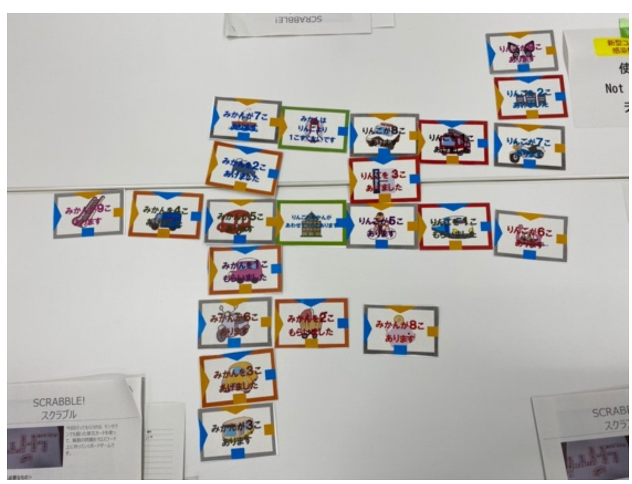

Fig. 6 An example of structured arithmetic stories.

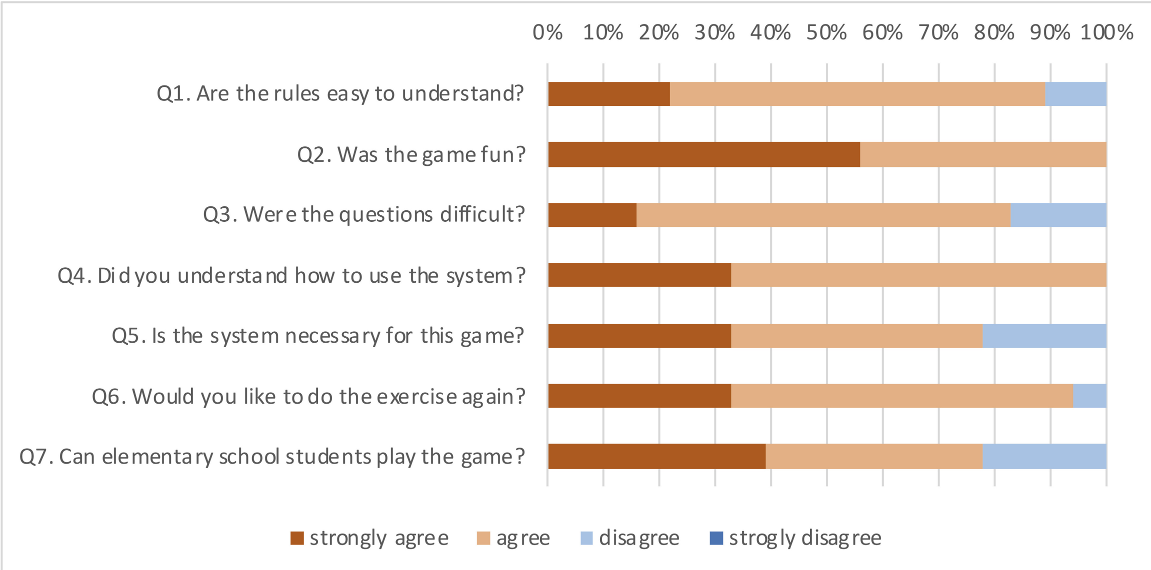

Fig. 7 Questionnaire result.

more complex thinking than just linking stories. Depending on the rules, for example, offering an incentive for players to make circular rings, Tri-prop scrabble has the potential to offer some different perspectives to learners.

The questionnaire result in Fig. 7 showed that they could easily understand the rules, enjoyed the exercises, and felt the proposed activities could be carried out by elementary school students as long as the method and usage of the practices are appropriately explained. Therefore, by this result, we can answer yes to RQ2.

In addition to that, the questionnaire includes the questions Q4 and Q5 about Story AR checker. The answer to these questions indicates that the participants easily understand how to use Story AR checker (from Q4) and that they recognize the need to use the system (from Q5). On the other hand, Story AR checker remains to be improved because, in the free-text field, some participants answer that they have difficulty using it.

5. Conclusion

This study proposed the game promoting exploratory posing arithmetical stories as sentence-integration and developed the system "Story AR checker" supporting the game with AR. The game requires the players to explore possible arithmetical stories with simple sentences on the table and in their hands. Story AR checker recognizes the card with the camera and judges the validity of the stories the players make. The results of the preliminary use of the game with the system show the potential of the use by the elementary school students as the proper user. The future is to verify the proposed game's learning effect further and extend the game and the system to multiplication and division operations.

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