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LETTER

Moments Added Statistical Shape Model for Boundary Extraction

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SUMMARY In this paper, we propose a method for extracting an object boundary from a low-quality image such as an infrared one. To take full advantage of a training set, the overall shape is modeled by incorporating statistical characteristics of moments into the point distribution model (PDM). Furthermore, a differential equation for the moment of overall shape is derived for shape refinement, which leads to accurate and rapid deformation of a boundary template toward real object boundary. The simulation results show that the proposed method has better performance than conventional boundary extraction methods.

key words: *boundary extraction, statistical shape model, moment*

1. Introduction

Object boundary is an important feature in various areas including medical electronics, computer vision and object-based video coding. However, internal edges of an object, background edges, and occlusions by other objects make it quite difficult to extract a target object boundary from a low-quality image that has heavy noise and low contrast. That is why most researches that aim to find the boundary have focused on the shape model-based approach, where the boundary is extracted with the aid of shape information derived from a training set [1]. Among these methods, the PDM [2], [3] is a powerful method for describing the shape of an object using statistical characteristics on locations of labeled boundary points (so-called landmarks). It effectively represents a rigid shape as linear combinations of the principal components of a covariance matrix for mean shape. However, point-based approaches including PDM have a correspondence problem between associated landmarks since boundary should be sampled into a constant number of labeled points. This problem degrades the performance of boundary extraction.

As a solution to this correspondence problem, region-based feature, i.e., moments for overall shape, is adopted in this paper. The moment of shape has useful properties of rotation, scale, and translation-invariance and can be computed without sampling boundary. In [4], characteristics of moments on boundary regions were used to detect all of boundaries in a given image. However, it had difficulty in discerning the target object boundary from the detected boundary regions, since it did not employ any prior information regarding the particular target object. The moment has been rarely used in the prior knowledge based boundary extraction methods, because reconstruction of shape is not possible when a finite number of moments are given. To employ the moments in boundary extraction, the proposed boundary extraction process consists of the following two steps. Points that are expected to be located on boundary are found in the first step where prior knowledge derived from statistical shape model (SSM) is used, and then the extracted boundary is refined by means of statistical moment model (SMM) for solving the correspondence problem of SSM.

2. Shape Modeling

The SSM represents each object boundary in a training set as labeled points, for which critical points with high curvature are firstly extracted and then equally spaced points are interpolated between the critical points. Each of M aligned training shapes is described as a position vector $\boldsymbol{L}_i = [x_i(1), y_i(1), x_i(2), y_i(2), \ldots, x_i(N), y_i(N)]^T (i = 1, \ldots, M)$, where N is the number of total labeled points. According to the principal component analysis, any shape $\boldsymbol{L}$ in the training set can be approximated using the mean shape $\overline{\boldsymbol{L}}$ and the first t eigenvectors $\boldsymbol{Q} = (\boldsymbol{q}_1|\boldsymbol{q}_2|\ldots|\boldsymbol{q}_t)$ of the covariance matrix about the mean: $\boldsymbol{L} = \overline{\boldsymbol{L}} + \boldsymbol{Qb}$, where $\boldsymbol{b} = (b_1, b_2, \ldots, b_t)^T$ is a weighting vector that indicates the amount of variation with respect to each of the eigenvectors.

The SMM represents overall shape of an object more precisely without information loss caused by sampling and the correspondence problem. The moment with the order p and q is defined as

$$m_{p,q} = \iint_{\text{object region}} x^p y^q \, dxdy \tag{1}$$

Let the sum of the maximum p and the maximum q be equal to N', and the moment vector of the i-th training shape be $\boldsymbol{M}_i = [m_{0,0}(i), m_{1,0}(i), m_{0,1}(i), \ldots, m_{N',0}(i), \ldots, m_{0,N'}(i)]^T$, then the mean and the covariance of moments are defined as

$$\overline{m}_{p,q} = \frac{1}{M}\sum_{i=1}^{M} m_{p,q}(i), \; \boldsymbol{C}_m = \frac{1}{M}\sum_{i=1}^{M}(\boldsymbol{M}_i - \overline{\boldsymbol{M}})(\boldsymbol{M}_i - \overline{\boldsymbol{M}})^T \tag{2}$$

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In the same way as in SSM, the moments of an individual shape are modeled as linear combinations of the principal components $\boldsymbol{Q}_m$ of covariance matrix $\boldsymbol{C}_m$: $\boldsymbol{M} = \overline{\boldsymbol{M}} + \boldsymbol{Q}_m\boldsymbol{a}$, where $\boldsymbol{a} = (a_1, a_2, \dots, a_t)^T$ is a weighting vector like $\boldsymbol{b}$.

3. Boundary Extraction Process

Bayesian objective function is a cost function used for deforming a boundary template toward the real object boundary. Let $\boldsymbol{p} = (s, \theta, T_x, T_y, b_1, b_2, \dots, b_t)$ denote a deformation parameter, where s, θ and (T_x, T_y) are scale, rotation and translation parameters, respectively. Then, the n-th point on boundary $(n = 1, 2, \dots, N)$ is represented by

$$x(\boldsymbol{p}, n) = s\cos\theta\left(\overline{x}_n + \sum_{k=1}^{t} Q_{2n-1,k}b_k\right) - s\sin\theta\left(\overline{y}_n + \sum_{k=1}^{t} Q_{2n,k}b_k\right) + T_x,$$
$$y(\boldsymbol{p}, n) = s\sin\theta\left(\overline{x}_n + \sum_{k=1}^{t} Q_{2n-1,k}b_k\right) + s\cos\theta\left(\overline{y}_n + \sum_{k=1}^{t} Q_{2n,k}b_k\right) + T_y \tag{3}$$

where $Q_{2n,k}(Q_{2n+1,k})$ is the $2n$-th ($2n$ + 1-th) row and the k-th column element of Q. Given the edge map $E(x, y)$ of an image, the objective function for a given deformation parameter $\boldsymbol{p}$ is formulated as *Bayesian rule* [1].

$$M(\boldsymbol{p}) = \sum_{j=1}^{t+4}\left(-\frac{(p_j - m_j)^2}{2\sigma_j^2}\right) + \frac{1}{\sigma^2}\sum_{n=1}^{N} E(x(\boldsymbol{p}, n), y(\boldsymbol{p}, n)), \tag{4}$$

where m_j and σ_j^2 is the mean and the variance of the j-th element of $\boldsymbol{p}$, respectively. These values are calculated from the training set alignment. Also σ^2 is the variance of the white zero mean Gaussian noise associated with the image noise model [1]. In (4), the first term is *a priori* modeled by SSM and the second term is *likelihood*.

The Boundary extracted using the SSM is depicted as a straightforwardly connected line along points represented by an optimal solution $\boldsymbol{p}'$ of the objective function. It should be noted that the obtained boundary inherits the correspondence problem, so the *shape refinement* is needed to address this problem. The moments of the boundary are first calculated and they are projected to the SMM. Then an approximated moments reflecting prior knowledge on moments of training set are obtained. Here, the difference between original moments and projected moments implies how much the boundary differs from the training set. If the SSM has a serious correspondence problem, the difference appears to be large. The shape refinement will then be needed to deform a boundary iteratively in a way of minimizing projection error. Here, the shape refinement is turned into the minimization problem of normalized mean square error between the moments of a boundary and its projected moments. If the instantaneous rate of the change in the moments caused by moving of a point $(x(\boldsymbol{p}, n), y(\boldsymbol{p}, n))$ is estimated, this can help in finding a solution of the minimization problem. We derived a moment differential equation as follows. In 2D image plane, the boundary is simplified with vertices and straight lines connecting these vertices. Let *sub-moment* α for a line C_n connecting (x_n, y_n) to the neighbor vertex (x_{n+1}, y_{n+1}) be defined as

$$\alpha_n^{p,q} = \int_{C_n} x^p y^q dx, \tag{5}$$

where C_n can be represented as $y = a_n x + b_n$ and the slope and the constant term are specified with (x_n, y_n) and (x_{n+1}, y_{n+1}). So *sub-moment* α can be represented as

$$\alpha_n^{p,q} = \int_{x_n}^{x_{n+1}} x^p (a_n x + b_n)^q dx = \sum_{k=0}^{q} \frac{{}_qC_k a_n^k b_n^{q-k}\left(x_{n+1}^{k+p+1} - x_n^{k+p+1}\right)}{k+p+1}, \tag{6}$$

According to Green's theorem, region integral can be represented as contour integral. Thus (1) can be rewritten as

$$m_{p,q} = -\frac{1}{q+1}\oint_{\text{boundary}} x^p y^{q+1} dx = -\frac{1}{q+1}\sum_{n=1}^{N}\alpha_n^{p,q+1}. \tag{7}$$

The differential equation of moment $m_{p,q}$ about x_n has a relation with *sub-moment* α like

$$\frac{\partial}{\partial x_n}(m_{p,q}) = -\frac{1}{q+1}\left(\frac{\partial\alpha_{n-1}^{p,q+1}}{\partial x_n} + \frac{\partial\alpha_n^{p,q+1}}{\partial x_n}\right). \tag{8}$$

Let X_n^p is $(x_n^p - x_{n-1}^p)/(x_n - x_{n-1})$, then $\partial\alpha_{n-1}^{p,q+1}/\partial x_n$ and $\partial\alpha_n^{p,q+1}/\partial x_n$ are expanded using (6) like

$$\frac{\partial}{\partial x_n}\alpha_{n-1}^{p,q+1} = \sum_{k=0}^{q+1}\frac{{}_{q+1}C_k}{k+p+1}\left\{-ka_{n-1}^k b_{n-1}^{q-k+1} X_n^{k+p+1} + (q-k+1)a_{n-1}^{k+1} b_{n-1}^{q-k} x_{n-1} X_n^{k+p+1} + (k+p+1)a_{n-1}^k b_{n-1}^{q-k+1} x_n^{k+p}\right\}, \tag{9}$$

$$\frac{\partial}{\partial x_n}\alpha_n^{p,q+1} = \sum_{k=0}^{q+1}\frac{{}_{q+1}C_k}{k+p+1}\left\{ka_n^k b_n^{q-k+1} X_{n+1}^{k+p+1} + (k-q-1)a_n^{k+1} b_n^{q-k} x_{n+1} X_{n+1}^{k+p+1} - (k+p+1)a_n^k b_n^{q-k+1} x_{n+1}^{k+p}\right\}. \tag{10}$$

If x_{n-1} is equal to x_n, (9) is equal to zero. Also, if x_n is equal to x_{n+1}, (10) is equal to zero. Using (9) and (10), the differential values of moment $m_{p,q}$ can be calculated.

4. Experimental Results

The proposed method is demonstrated using the steepest decent methods for finding the optimum solutions of the maximum *a posteriori* $M(\boldsymbol{p})$ and the minimum moment projection error. Firstly, to analyze especially how well a shape is refined, an original shape like Fig. 1 (a) is coarsely sampled into equally spaced points like Fig. 1 (b). If critical points with high curvature are selected as landmarks, the boundary can be more nicely described. However, since the number of high curvature points can vary according to the viewpoint and/or pose of an object, e.g., Fig. 2 (b), we simply described a boundary with constant number of points to get the covariance matrix. Figure 1 (c) shows the boundary represented by SSM. As shown in Fig. 1 (b) and (c), the shape around corner regions is smoothed due to sampling and the correspondence problem. The SSM uses this kind of sampled points as a training set, so that it may lose a detail of the shape, which will directly affect its boundary extraction performance as below experiments. On the other hand, the training set of SMM is not the sampled shape but the original shape. Figure 1 (d) and (e) show the results in which the sampled shape is evolved iteratively toward minimizing differences of moments from the original shape. This evolution is guided by Eq. (9). This experiment proves that a shape represented by SSM can be refined well by means of moments. In Fig. 1 (e), the error means distance between a local template point resulted from shape refinement and the closest point on original boundary.

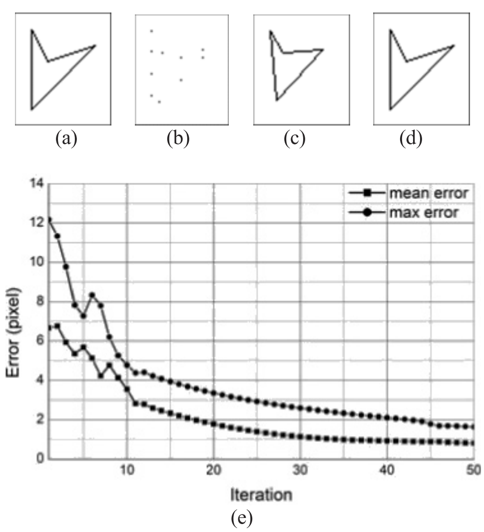

Fig. 1 Shape refinement using moments, (a) original shape, (b) sampled points, (c) shape represented by using SSM, (d) result of the refinement, and (e) shape refinement behavior.

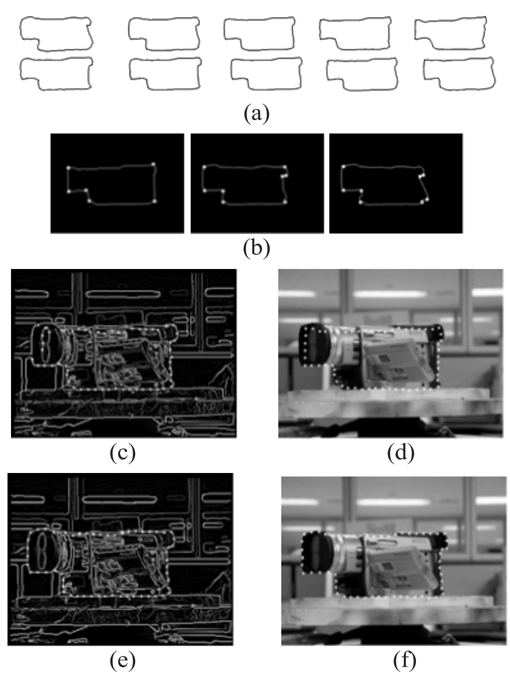

Fig. 2 Shape modeling and boundary extraction results, (a) the effect of varying each of the first two shape parameters, (b) critical points, (c) and (d) PDM results, (e) and (f) the proposed method results.

For low-contrast real images, the experimental results are shown in Fig. 2. Figure 2 (a) shows the effect of varying weighting values related to the first two principle components of the shape model, which indicates how large deformation of the boundary can be tolerated in boundary extraction. Figure 2 (b) shows that the numbers of the critical points generated automatically with the aid of curvature, as well as their positions, differ from one another, which brings the correspondence problem to the shape modeling. The correspondence problem may allow local template points to be deformed inordinately, which leads to sensitivity on neighboring edges and noise as shown in Fig. 2 (c) and (d). In Fig. 2 (c) to (f), it can be observed that the proposed method works well, whereas the boundary extracted by PDM is easily attached to the neighboring edges. This can be attributed to the additional shape refinement that is achieved by using region-based feature.

5. Conclusion

An enhanced boundary extraction scheme that can be applied to low-quality images such as infrared ones was proposed in this paper. When the training set is given, the proposed method can describe shape more accurately with statistical characteristic for moments as well as position of sampled boundary points. Moreover, it can moderate the affects of sampling and the correspondence problem resulting from the conventional methods based on point-based feature.

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References

[1] H. Choi and S. Kim, "Boudnary extraction using statistical shape descriptor," Electron. Lett., vol.38, pp.1330–1332, Oct. 2002.
[2] Y. Wang and L.H. Staib, "Boundary finding with prior shape and smoothness methods," IEEE Trans. Pattern Anal. Mach. Intell., vol.22, no.7, pp.738–743, July 2000.
[3] S. Ordas, E. Oubel, and R. Leta, "A statistical shape model of the heart and its application to model-based segmentation," Proc. SPIE, vol.6511, 65111K, March 2007.
[4] S. Tenginakai, J. Lee, and R. Machiraju, "Salient Iso-Surface detection with model-independent statistical signatures," Proc. Visualization 2001, pp.231–238, Oct. 2001.