PAPER Regularization Super-Resolution with Inaccurate Image Registration**

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SUMMARY Considering the inaccuracy of image registration, we propose a new regularization restoration algorithm to solve the ill-posed superresolution (SR) problem. Registration error is used to obtain cross-channel error information caused by inaccurate image registration. The registration error is considered as the noise mean added into the within-channel observation noise which is known as Additive White Gaussian Noise (AWGN). Based on this consideration, two constraints are regulated pixel by pixel within the framework of Miller's regularization. Regularization parameters connect the two constraints to construct a cost function. The regularization parameters are estimated adaptively in each pixel in terms of the registration error and in each observation channel in terms of the AWGN. In the iterative implementation of the proposed algorithm, sub-sampling operation and sampling aliasing in the detector model are dealt with respectively to make the restored HR image approach the original one further. The transpose of the sub-sampling operation is implemented by nearest interpolation. Simulations show that the proposed regularization algorithm can restore HR images with much sharper edges and greater SNR improvement. key words: super-resolution, image restoration, regularization, registration error, AWGN

1. Introduction

Super-resolution (SR) image restoration can overcome the inherent resolution limitation of low-resolution (LR) imaging systems and improve the performance of most digital image processing applications, such as medical imaging, video frame stills, remote imaging, and video surveillance. It has been one of the most spotlighted research areas in recent years.

Tsai and Huang [1] carried out earlier research on SR image restoration. Based on the spatial aliasing effect, they proposed a frequency domain approach and demonstrated its ability to reconstruct an improved resolution image from several down-sampled noise-free versions. Kim et al. [2] extended this technique for noisy data and derived a weighted least square algorithm. Tekalp et al. [3] further improved

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the technique by taking into account a linear shift invariant blur Point Spread Function (PSF) and using a least square approach to solve the system of equations. However, the frequency domain approaches are only applicable for translation motion and shift invariant blur.

More and more resolution enhancement techniques are operated in the spatial domain because they can be applied in more cases like other more complex motion models and shift-variant blur. In 1989, Stark and Oskoui [4] proposed a Projection onto Convex Sets (POCS) approach to compute an estimate of desired high-resolution (HR) image from observations obtained by scanning or rotating an image with respect to the CCD image acquisition system. In their method, the desired HR image belongs to an intersection set of convex constraint sets representing desirable image characteristics such as positivity, bounded energy, fidelity to data, smoothness. The POCS approach has been extended to time-varying motion blur in [5], [6]. Nevertheless, the dependence on a set of initial HR images and the possibility of no unique solution limited the use of POCS. In 1991, Irani and Peleg [7] proposed an iterative algorithm using a back projection filter. However, the appropriate back projection filter is very difficult to compute.

The regularization SR algorithms based on determinate conditions or stochastic techniques, such as Constrained Least Squares (CLS) [8], Maximum Likelihood (ML) [9], Maximum a Posteriori (MAP) [10]–[13], are an effective class of resolution enhancement methods to avoid the ill-posed problem in SR image restoration. A MAP/POCS hybrid method was proposed in [14] as well. Bose et al. [15] and [16] pointed out the choice of the regularization parameter influences the quality of the solution significantly, and used the L-curve method to generate the optimum value of the regularization parameter. Nguyen [17] also calculated the regularization parameter by Generalized-Cross-Validation (GCV).

The accuracy of registrations between LR images will influence the quality of the reconstructed HR image. However, image registration, especially on the subpixel level, is very difficult. The restored HR image is distorted since the subpixel motion is estimated inaccurately. We need to consider the inaccuracy of image registration as part of the SR problem. In [18] only the pixels from the LR images are determined to be usable by the three different criterions: maximum distance from the median - MDM, maximum distance from initial image - MDIM, and maximum distance from the SR estimate - MDSRE. But the performance of the re-

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construction algorithm in [18] depends on the selected hard threshold and regularization parameter. In [19], Noha A. El-Yamany etc. proposed an adaptive M-estimation framework to suppress the outliers due to model violations. The framework used a robust error norm in the data fidelity term of the objective function and adapted the estimation process to each of the LR frames and each of the color components, which resulted in color SR images of crisp details and no artifacts. However, the performance of the framework still depended on the selected regularization parameter. In [20], Lee and Kang proposed a regularized adaptive HR reconstruction considering inaccurate subpixel registration. They called the distortion the registration error noise. Two methods for automatically estimating the regularization parameter for each LR channel were advanced, based on the approximation that the registration error noise was modeled as Gaussian with the standard deviation proportional to the degree of registration error. In [21], besides registration error noise and additive white Gaussian noise (AWGN), He and Kondi took blur noise into account. These three types of noise are the sources of the residual noise that was assumed approximately Gaussian. Moreover, they affected the residual norm in the cost function for each LR channel (image). An iterative process was proposed with a regularization parameter to control the within-channel balance between received data and prior information, as well as a channel weight coefficient to control the channel fidelity.

Due to inaccurate image registration, this paper considers the displaced frame differential (DFD) between the warped HR image and forward motion compensation of the original HR image. The DFD creates the cross-channel error information across the estimated observed channel. The error information is called the registration error here. Different from [20] and [21], we consider the registration error as the noise mean added into the within-channel observation noise which is known as AWGN. Based on the idea of Miller's regularization [22], [23], two constraints are regulated pixel by pixel. Regularization parameters connect the two constraints to construct a cost function. The regularization parameters are adaptively estimated based on the within-channel AWGN and the cross-channel registration error. For making the restored HR image approach the original one further, sub-sampling operation and sampling aliasing are dealt with independently in the iterative implementation of the proposed algorithm. The transpose of the sub-sampling operation is implemented by nearest interpolation.

The remainder of the paper is organized as follows. In Sect. 2, the observation model of the LR images is formulated. In Sect. 3, we first analyze the change of the noise due to the DFD. Then we propose a regularization SR restoration approach based on the idea of Miller's regularization. Finally, the method of estimating the regularization parameter is given. Simulations to demonstrate the effectiveness of the proposed algorithm are presented in Sect. 4. Conclusions are given in Sect. 5.

2. Problem Formulations with the Mathematical Model

An HR image of size $L_1N_1 \times L_2N_2$ corresponding to a constant scene during the acquisition of the multiple LR images is written in lexicographical notation as $L_1L_2N \times 1$ vector f, where $N = N_1 \times N_2$, L_1 and L_2 represent the down-sampling factors in the horizontal and vertical directions of the HR grid, respectively. By warping f, we get f_i .

$$\boldsymbol{f}_i = \boldsymbol{M}_i \boldsymbol{f} (1 \le i \le p) \tag{1}$$

where *p* is the channel number for observing LR images. M_i is the $L_1L_2N \times L_1L_2N$ forward motion matrix, and $M_1 = I$, where *I* is the identity matrix. For sufficiently high frame rates, most motion models can be (at least locally) approximated by the translational model, which fairly well approximates the motion contained in image sequences where the scene is stationary and only the imaging device moves [24], [25]. We hereby consider the motion as a translational model.

The observed LR images result from blurring and down-sampling operations performed on f_i . Assuming that each LR image is corrupted by additive noise, the observation model can be represented as a matrix-vector equation [14], [26].

$$\boldsymbol{g}_i = \boldsymbol{D}\boldsymbol{B}_i\boldsymbol{f}_i + \boldsymbol{n}_i = \boldsymbol{D}\boldsymbol{B}_i\boldsymbol{M}_i\boldsymbol{f} + \boldsymbol{n}_i \tag{2}$$

where g_i is the $N \times 1$ lexicographically ordered vector of the *i*-th LR image. n_i is the $N \times 1$ lexicographically ordered vector of the noise, which is generally modeled as i.i.d. AWGN with variance σ_i^2 . B_i is the $L_1L_2N \times L_1L_2N$ blurring matrix, which denotes the image degradation process. In this paper, all the blur operators are assumed the same, i.e. $\forall i, B_i = B$. D is the $N \times L_1L_2N$ down-sampling matrix. A simple detector model with some aliasing, i.e. sub-sampling after $L_1 \times L_2$ pixel averaging, can be used to realize the down-sampling operation from HR grid to LR grid. Let A represents the pixel averaging, and S represents the sub-sampling, then

$$\boldsymbol{g}_i = \boldsymbol{S} \boldsymbol{A} \boldsymbol{B}_i \boldsymbol{M}_i \boldsymbol{f} + \boldsymbol{n}_i \tag{3}$$

The above observation process is shown in Fig. 1.

Let $H_i = DBM_i$ which presents the observation channel. Then the *p* matrix-vector equations from different LR images can be stacked into



Fig. 1 The observation process of *i*-th LR image.

The mathematic model is the same as the classical image restoration problem. Thus, some ideas of solving the classic restoration problem can be used to solve the SR problem.

3. Super-Resolution Image Restoration

The objective of SR image restoration is to obtain a desired HR image from multiple observed LR images while simultaneously reducing the influence of system degradation and additive noise. The process of obtaining the desired HR image can be regarded as the inverse process of image observation. The implementation of SR image restoration generally includes image registration, image interpolation, and image restoration. The accuracy of the image registration is one of the most important factors for influencing the quality of the SR restoration algorithm. However, accurate image registration is a challenging problem. Due to the inaccuracy of image registration, there must exist displaced frame differential (DFD) d_i between the warped image f_i and forward motion compensation of f.

$$\boldsymbol{d}_i = \boldsymbol{f}_i - \hat{\boldsymbol{M}}_i \boldsymbol{f} \tag{5}$$

where \hat{M}_i denotes estimated forward motion matrix.

 f_i can be expressed by f and d_i as

$$\boldsymbol{f}_i = \boldsymbol{M}_i \boldsymbol{f} + \boldsymbol{d}_i \tag{6}$$

Let $\hat{H}_i = DB\hat{M}_i$, which represents the estimated observation channel. The observation model considering the DFD is

$$\boldsymbol{g}_i = \boldsymbol{D}\boldsymbol{B}(\boldsymbol{\hat{M}}_i\boldsymbol{f} + \boldsymbol{d}_i) + \boldsymbol{n}_i = \boldsymbol{\hat{H}}_i\boldsymbol{f} + \boldsymbol{D}\boldsymbol{B}\boldsymbol{d}_i + \boldsymbol{n}_i$$
(7)

Defining $e_i = DBd_i$ gives

$$\boldsymbol{e}_i = \boldsymbol{D}\boldsymbol{B}\boldsymbol{d}_i = \boldsymbol{D}\boldsymbol{B}\boldsymbol{M}_i\boldsymbol{f} - \boldsymbol{D}\boldsymbol{B}\hat{\boldsymbol{M}}_i\boldsymbol{f} = \boldsymbol{H}_i\boldsymbol{f} - \hat{\boldsymbol{H}}_i\boldsymbol{f} \qquad (8)$$

Adding \boldsymbol{e}_i to the AWGN \boldsymbol{n}_i , a new noise \boldsymbol{n}'_i can be denoted as

$$\mathbf{n}_i = \mathbf{e}_i + \mathbf{n}_i \tag{9}$$

Since \hat{H}_i , H_i , and f are determined variant matrix or vectors, e_i is determined correspondingly, which represents the error information caused by the inaccurate image registration. e_i is called the registration error and it is different in each pixel. Its value of its elements is small corresponding to the flat areas of the LR image, and large to the vicinity of image edges.

The observation process of *i*-th LR images with the registration error e_i is illustrated in Fig. 2.



Fig.2 The observation process of *i*-th LR images with the registration error e_i .

Without loss of generality, n_i is assumed to be Gaussian noise. So, the probability density function of the new noise n'_i per pixel can be expressed as

$$p_{\mathbf{n}_{i}(m)}(x) = \left(1 / \sqrt{2\pi\sigma_{i}}\right) \exp\left(-[x - \mathbf{e}_{i}(m)]^{2} / 2\sigma_{i}^{2}\right) \quad (10)$$

where $m \in [1, \dots, N]$. It is different from the zero-mean AWGN n_i . Its mean is the registration error e_i .

The aim of the SR problem is to obtain the desired HR image. That is, e_i is not obtained by (8) explicitly. There are some properties helpful to approximate the registration error e_i .

Using the property that the matrices **B** and \hat{M}_i commute [24], [25], (8) can be rewritten as

$$\boldsymbol{e}_i = \boldsymbol{H}_i \boldsymbol{f} - \boldsymbol{D} \boldsymbol{\tilde{M}}_i \boldsymbol{B} \boldsymbol{f} \tag{11}$$

While image registration is implemented between LR images, \hat{M}_i is obtained by mapping \hat{M}_{iLR} onto the HR grid, where \hat{M}_{iLR} denotes the estimated motion matrix on the LR grid.

Finally, we change the order of \hat{M}_i and D, and get

$$e_{i} \cong H_{i}f - \hat{M}_{iLR}DBf$$

$$\cong (g_{i} - n_{i}) - \hat{M}_{iLR}(g_{1} - n_{1})$$

$$\cong g_{i} - \hat{M}_{iLR}g_{1} + n_{1} - n_{i}$$
(12)

Some approaches for eliminating noise can be used to remove n_1 and n_i in $g_i - \hat{M}_{iLR}g_1$ here.

4. SR Image Restoration with Inaccurate Image Registration

SR image restoration is an ill-posed problem due to various factors, such as insufficient number of LR images, illconditioned blur operators, inaccurate image registration, and the AWGN. Regularization is often used to stabilize the inversion of the ill-posed problem by combining some prior knowledge about the desired HR image. In this section, the ill-posed SR problem is formulated in terms of Miller's regularization to construct a cost function, an adaptive gradient descent optimization is used to minimize the cost function, and regularization parameters are selected in terms of the registration error and the AWGN.

4.1 Cost Function

According to Miller [22], two kinds of constraints are regulated for solving the problem of the classic image restoration [23]:

$$\|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|_2^2 \le \varepsilon^2 \tag{13}$$

$$\|\boldsymbol{C}\boldsymbol{f}\|_2^2 \le E^2 \tag{14}$$

where ε and *E* are two defined limits. ε^2 is related to the energy of the noise *n*, E^2 is related to the high-frequency energy of *f*. $||g - Hf||_2^2$ denotes a measure of the fidelity of *f*, and $||Cf||_2^2$ denotes a measure of the smoothness of *f*. Here *C* denotes a regularization matrix formed by the twodimensional 3×3 Laplacian kernel defined as

$$\left[\begin{array}{rrrr} 0 & -1/4 & 0 \\ -1/4 & 1 & -1/4 \\ 0 & -1/4 & 0 \end{array}\right]$$

Connecting (13) and (14) into a cost function by regularization parameter gives

$$J(f) = \|g - Hf\|_{2}^{2} + \lambda \|Cf\|_{2}^{2}$$
(15)

where $\lambda = \varepsilon^2 / E^2$ denotes the regularization parameter that can be used to compromise between fidelity and smoothness.

In classical image restoration, the noise n is generally assumed i.i.d. AWGN with zero mean and variance σ^2 , and ε^2 is set a global bound, whereas in the SR problem, the noise n'_i has non-zero mean, and the value of the registration error is different in each pixel. Based on this consideration, two kinds of constraints in each pixel are regulated here. The two constraints can be expressed as

$$\left(\boldsymbol{g}_{i}(m) - \hat{\boldsymbol{H}}_{i}(m, n)\boldsymbol{f}(n)\right)^{2} \le \varepsilon_{i}(m)^{2}$$
(16)

$$\left(\boldsymbol{C}(n,n)\boldsymbol{f}(n)\right)^2 \le E^2 \tag{17}$$

where $n \in [1, \dots, L_1 L_2 N]$, and $\varepsilon_i^2(m)$ is related to the energy of the noise $\mathbf{n}'_i(m)$, that is

$$\varepsilon_i^2(m) = \left(\sigma_i^2 + (\boldsymbol{e}_i(m))^2\right) \tag{18}$$

where $(\sigma_i^2 + (e_i(m))^2)$ denotes the energy of the noise $n'_i(m)$. Combining the two inequalities (16) and (17), we get

$$J_i(n) = \lambda_i(m) \left(\boldsymbol{g}_i(m) - \hat{\boldsymbol{H}}_i(m,n) \boldsymbol{f}(n) \right)^2 + \left(\boldsymbol{C}(n,n) \boldsymbol{f}(n) \right)^2$$
(19)

where $\lambda_i(m) = E^2 / (\sigma_i^2 + (e_i(m))^2)$ is the regularization parameter.

We join $J_i(n) \forall n, i$ into a simultaneous cost function

$$J(\boldsymbol{f}) = \sum_{i=1}^{p} \left(\left\| \sqrt{\lambda_i} (\boldsymbol{g}_i - \hat{\boldsymbol{H}}_i \boldsymbol{f}) \right\|_2^2 + \|\boldsymbol{C}\boldsymbol{f}\|_2^2 \right)$$
(20)

where λ_i is the diagonal regularization parameter matrix.

$$\lambda_i = diag(\lambda_i(1), \lambda_i(2), \cdots, \lambda_i(m), \cdots, \lambda_i(N)) \quad (21)$$

When the AWGN and the registration error are trivial, $\lambda_i(m)$ will tend to zero and is not useful to the SR problem. So in order to avoid this case, we map the regularization parameter from the LR grid to the HR grid by backward

motioning the up-sampled e_i , and change the order of the regularization parameter matrix to the smoothness term. We rewrite (20) as

$$J(f) = \sum_{i=1}^{p} \left\| (g_i - \hat{H}_i f) \right\|_2^2 + \left\| \sqrt{\bar{\lambda}_i} C f \right\|_2^2$$
(22)

$$\bar{\lambda}_i = diag(\bar{\lambda}_i(1), \bar{\lambda}_i(2), \cdots, \bar{\lambda}_i(n), \cdots, \bar{\lambda}_i(L_1L_2N))$$
(23)

$$\bar{\boldsymbol{\lambda}}_{i}(n) = \left(\sigma_{i}^{2} / E^{2}\right) + \left(\left(\hat{\boldsymbol{M}}_{i}(n,n)^{T} \boldsymbol{U}(n,m)\boldsymbol{e}_{i}(m)\right)^{2} / E^{2}\right) \quad (24)$$

where U denotes the up-sampling operation of the image, and $\bar{\lambda}_i$ is the diagonal regularization parameter matrix.

The conformation of (22) is similar to the idea of multiparameter regularization for image restoration in [27]. In the case that $\forall i, n, \bar{\lambda}_i(n)$ is same, (22) is reduced to the counterpart in CLS. In the case that $\forall n, \bar{\lambda}_i(n)$ is same in each observation channel, (22) is reduced to the counterparts in [20], [21].

Finally, the third constraint is also imposed as a priori knowledge to threshold f due to the fact that image intensity is finite and nonnegative.

$$0 \le \min \le f(n) \le \max \le \infty \tag{25}$$

where min and max denote lower and upper limits.

4.2 Adaptive Gradient Descent Optimization

The desired HR image can be obtained by minimizing the cost function J(f), i.e.

$$\hat{f} = \arg\min_{f} J(f) \tag{26}$$

A gradient descent optimization procedure is used to minimize J(f). For each observation channel, the registration noise and the AWGN are different and thus have a different influence on single-channel gradient descent. Therefore, the iteration step should be adjusted adaptively in terms of the tendency of gradient descent with respect to each observation channel.

$$\hat{f}^{l} = \hat{f}^{l-1} + \frac{1}{p} \sum_{i=1}^{p} \beta_{i}^{l} \nabla J_{i}^{l}(f)$$
(27)

$$\nabla J_i^l(f) = \hat{\boldsymbol{H}}_i^T \boldsymbol{g}_i - \left(\hat{\boldsymbol{H}}_i^T \hat{\boldsymbol{H}}_i + \boldsymbol{C}^T \bar{\boldsymbol{\lambda}}_i \boldsymbol{C}\right) \hat{\boldsymbol{f}}^{l-1}$$
(28)

where β_i^l is the step at the *l*-th iteration with respect to *i*-th observation channel and expressed as

$$\mathcal{B}_{i}^{l} = \left(\nabla J_{i}^{l}(\boldsymbol{f})^{T} \nabla J_{i}^{l}(\boldsymbol{f}) \right) \left| \left(\nabla J_{i}^{l}(\boldsymbol{f})^{T} \left(\hat{\boldsymbol{H}}_{i}^{T} \hat{\boldsymbol{H}}_{i} + \boldsymbol{C}^{T} \bar{\boldsymbol{\lambda}}_{i} \boldsymbol{C} \right) \nabla J_{i}^{l}(\boldsymbol{f}) \right)$$
(29)

Then, we introduce the third constraint (25).

$$\hat{f}^{l}(n) = \begin{cases} \max & \hat{f}^{l}(n) \ge \max \\ \hat{f}^{l}(n) & \min \le \hat{f}^{l}(n) \le \max \\ \min & \hat{f}^{l}(n) \le \min \end{cases}$$
(30)

In the iteration implementation, matrix-matrix and matrix-vector multiply are performed by translation, convolution and sampling operations. Application of these operations in the image domain avoids constructing the matrices explicitly. In [25], [28], the operations of the down-sampling matrix \boldsymbol{D} and its transpose (up-sampling) matrix \boldsymbol{D}^T are implemented by pulse sampling and zero-padding operations, which are not in accordance with the detector model. In this paper, we deal with the pixel averaging and sub-sampling operation respectively. The sub-sampling operation S and its transpose S^T are implemented by pulse sampling operation and nearest interpolation shown in Fig. 3.

The adaptive gradient descent optimization of the proposed regularization algorithm is now summarized as following:

- Step 1. Begin at l = 0 with initial estimation \hat{f}^0 of desired HR image and select regularization parameters;
- Step 2. Let l = l + 1;
- Step 3. Compute the gradient $\nabla J_i^l(f)$ given in (28) for i = $1, 2, \cdots, p;$
- Step 4. Compute the step size β_i^l by using (29) for i =1, 2, \cdots , p; Step 5. Update \hat{f}^l given in (27);
- Step 6. Constrain \hat{f}^l using (30);
- Step 7. If the stopping criterion is satisfied, the iteration is stopped, otherwise go to Step2.

4.3 **Regularization Parameter Selection**

The first term $\left(\sigma_i^2/E^2\right)$ of (24) is related to the variance of the AWGN n_i . A channel with larger variance is smoother, whereas a channel with lower variance is less smooth. The signal-to-noise (SNR) based method proposed in [29] can be used to determine the term.

Firstly, an LR image is bilinear interpolated to get an interpolated image. In the interpolated image, an interpolated pixel is a weighted average of its four nearest neighbors [30]. Due to the bilinear weights, the noise variance of the interpolated pixel is

$$\bar{\sigma}_i^2 = \left(1 - 2x + 2x^2\right) \left(1 - 2y + 2y^2\right) \sigma_i^2 \tag{31}$$

where *x*, *y* denote the distance to the nearest neighbor.

Let $\alpha = (1 - 2x + 2x^2)(1 - 2y + 2y^2)$ expresses the ratio, which depends on the distances $0 \le x \le 1, 0 \le y \le 1$.



Fig.3 Up-sampling operation S^T on a 3×3 image and sub-sampling operation S on the corresponding 6×6 up-sampled image (down-sampling factors are $L_1 = L_2 = 2$).

$$\alpha = (1 - 2x + 2x^2)(1 - 2y + 2y^2) \to 0.25 \le \alpha \le 1 \quad (32)$$

Then in terms of the SNR-based selection scheme [29] of a regularization parameter in classical image restoration, we use the ratio between the maximum and minimum local variances of the interpolated image to obtain its SNR approximately. The calculation of the local variance refers to [31]. The SNR can be computed as mentioned in [29].

$$SNR_i = 1.04 \times 10 \times \log(\max var / \min var) - 7$$
 (33)

where max var and min var are the maximum and minimum local variances of the interpolated image, respectively.

Finally, the first term of (24) is given by the snr_i

$$\sigma_i^2 / E^2 = 10^{-(SNR_i/10)} / \alpha \tag{34}$$

In the second term of (24), we implement up-sampling operation U by bilinear interpolation. Since the registration error is caused by the DFD and is gray error information in each pixel, we take the square of upper limit in a priori gray range, i.e. \max^2 , as the approximate to E^2 . Thus, a pixel with less registration error is more fused. On the other hand, a pixel with more registration error is less fused.

Thus, the regularization parameters can adaptively balance fidelity to the original LR data and smoothness of the solution according to information from the within-channel AWGN and the cross-channel registration error. These parameters ultimately reduce distortion in the restored HR image and make the restored HR image closer to the original one.

5. **Experiments**

To verify the proposed regularization algorithm, we compare with four other algorithms: bilinear interpolation, the algorithm of Hardie [8] with the best visual reconstruction (the regularization parameter was obtained via trial and error), the algorithm of Hu He (the regularization algorithm for different error levels per frame in [21]) and the adaptive M-estimation framework [19]. In the proposed regularization algorithm, we make use of local mean to remove the noise. For equitable comparison, every algorithm runs until the 20-th iteration arrived.

5.1 Experiments with Synthetic Images

We use 256×256 'Barbara' for synthetic tests. The downsampling factors are $L_1 = L_2 = 2$. Four LR images are generated by translating, blurring and down-sampling the test images. i.i.d. AWGN with different variance is added to each LR image, corresponding to a signal-to-noise ratio (SNR) of 30-40 dB. Defocus blur, a kinds of familiar blurring models are selected as Point Spread Functions (PSF). The defocus radius is set 2.

Two tests are performed. In Test 1, motion parameters are known, which means that the noise only includes the AWGN. Test 2 is for inaccurate image registration, as well as for the AWGN. The accurate translation parameters for the four LR frames are [0, 0], [0, 1], [1, 0] and [1, 1] in the HR grid respectively. We assume that inaccurate registration parameters are biased to [0.6, 0.4], [-0.4, 1.2] and [1.2, 0.8], respectively corresponding to [0, 1], [1, 0] and [1, 1].

The restored HR images by the five algorithms are shown in Fig. 4 and Fig. 5 respectively. The SNR improvement is used as the quantitative measure of SR image restoration performance, which is given by

$$\Delta SNR = 10 \times \log_{10} \left(\left\| \boldsymbol{f}^{0} - \boldsymbol{f} \right\|_{2}^{2} / \left\| \boldsymbol{\hat{f}}^{\prime} - \boldsymbol{f} \right\|_{2}^{2} \right)$$
(35)

where f^0 is initial estimation of the desired HR image. In this paper, f^0 is given by bilinear interpolating the first reference LR frame.

Since bilinear interpolation only considers an LR image and makes the interpolated images poorest, Fig. 4 (a) obviously show the poor result.

In Test 1, it is easy to be seen from Fig. 4 that although the restored HR images from the other three methods expect bilinear interpolation is similar in the subjective evaluation, the proposed regularization algorithm gives greater SNR improvement.

In Test 2, the fixed regularization parameter in Hardie's algorithm makes the edge of the restored HR image smoothed globally as shown in Fig. 5 (b). The restored HR image by Hu He's algorithm has texture confusion that can be observed from the area of scarf in Fig. 5(c). The adaptive M-estimation framework without regularization gives the restored HR image with ring artifacts in the vicinity of the edges shown in Fig. 5 (d). Via trail and error, a fit regularization parameter for the adaptive M-estimation framework with regularization is selected to make ring artifacts suppressed in Fig. 5 (e). Whereas in our proposed algorithm, the consideration of the registration error and the SNR regularizes the distortion caused by inaccurate image registration in each pixel, and the AWGN in each observation channel respectively, and retains the fusion of useful information adaptively. Since sub-sampling operation and sampling aliasing are dealt with respectively, the restored HR image approaches the original one further. So it can be seen from Fig. 5 (f) and (g) that the proposed algorithm can provide the restored HR image with sharper edges as well as greater SNR improvement.

5.2 Experiments with Real Images

In the real-image test, we cut the region of interest from different frames of an image sequence [32] to get four LR images shown in Fig. 6, with the enhanced resolution factor set to 2. We first register LR images by edge projection based on the registration method. Here we neglect the trivial rotation between the left-most LR frame and the others in Fig. 6. Then, different algorithms are used to restore HR images, shown in Fig. 7. We assume the blurring operation is defocus, and the defocus radius is 0.7 as identified by error-parameter analysis. We can observe from



Fig. 4 Restored HR images in Test 1. (a) bilinear interpolation (PSNR=20.8301 dB); (b) Hardie's algorithm (PSNR=21.7810 dB); (c) Hu He's algorithm (PSNR=21.8154 dB); (d) the proposed regularization algorithm (PSNR=23.2507 dB); (e) SNR improvement versus the number of iterations.

Fig. 7 that bilinear interpolation and Hu He's algorithm give poorest restored results. Similar with Test 2, Hardie's algorithm gives a smoothed HR image. Severe ring artifacts can be observed form Fig. 7 (d) restored by the adaptive M-



Fig.5 Restored HR images in Test 2. (a) bilinear interpolation (PSNR=20.8301 dB); (b) Hardie's algorithm (PSNR=21.4504 dB, regularization parameter=0.01); (c) Hu He's algorithm (PSNR=21,7184 dB); (d) the adaptive M-estimation framework without regularization (PSNR=21.5468 dB); (e) the adaptive M-estimation framework with regularization (PSNR=21.4945 dB, regularization parameter=0.01); (f) the proposed regularization algorithm (PSNR=23.0136 dB); (g) SNR improvement versus the number of iteration.



Fig. 6 Low-Resolution images.

estimation framework without regularization. Although the adaptive M-estimation framework with regularization gives an improved HR image in Fig. 7 (e), ring artifacts can't be fully suppressed. From Fig. 7 (f), the proposed algorithm produces sharper restored HR images compared to the other four algorithms.

6. Conclusion

Due to inaccurate image registration, the DFD causes crosschannel registration error. In this paper, the registration error is considered as the noise mean and is added to the withinchannel AWGN. Within the framework of Miller's regularization, two constraints are regulated pixel by pixel and connected by regularization parameters to construct a cost function. The regularization parameters are adaptively selected in each pixel in terms of the registration error, as well as in each observation channel in terms of the AWGN. The consideration of the registration error and the AWGN regularizes the distortion in SR restoration and makes the solution closer to the original HR image. In the iterative implementation of the proposed algorithm, sub-sampling and sampling aliasing in the detector model are dealt with respectively to make the restored HR image approach the original one further. The transpose of the sub-sampling operation is implemented by nearest interpolation. Thus, the proposed regu-



Fig.7 Restored HR images in Test 2. (a) bilinear interpolation; (b) Hardie's algorithm (regularization parameter=0.1); (c) Hu He's algorithm; (d) the adaptive M-estimation framework without regularization; (e) the adaptive M-estimation framework with regularization (regularization parameter=0.01); (f) the proposed regularization algorithm.

larized algorithm gives results that are more satisfactory in both objective and subjective measurements.

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