

## LETTER

# Off-Line Keyword Guessing Attacks on Searchable Encryption with Keyword-Recoverability

Eun-Jun YOON<sup>†a)</sup> and Kee-Young YOO<sup>†b)</sup>, Members

**SUMMARY** In 2009, Jeong et al. proposed a new searchable encryption scheme with keyword-recoverability which is secure even if the adversaries have any useful partial information about the keyword. They also proposed an extension scheme for multi-keywords. However, this paper demonstrates that Jeong et al.'s schemes are vulnerable to off-line keyword guessing attacks, where an adversary (insider/outsider) can retrieve information of certain keyword from any captured query message of the scheme.  
**key words:** keyword search, keyword-recoverability, cryptanalysis, keyword guessing attacks

## 1. Introduction

The notion of searchable encryption was first suggested by Boneh et al. in [1]. With a searchable encryption scheme, a sender makes a ciphertext by encrypting a keyword with the public key of a receiver. The receiver can make a trapdoor for a keyword with a private key. Then any party can test whether or not the ciphertext and the trapdoor were made with the same keyword without knowing the keyword itself.

Bellare et al. [2] first proposed an SEKR (searchable encryption scheme with keyword-recoverability) in 2007. The SEKR scheme provides keyword-recoverability as well as keyword-testability. Keyword-testability means that a receiver of a ciphertext can test whether the ciphertext contains a specific keyword. Keyword-recoverability means that a receiver can extract the keyword from a ciphertext. Bellare et al.'s SEKR scheme provides only these two properties compared with the previous searchable encryption schemes.

In 2009, Jeong et al. [3] pointed out that Bellare et al.'s SEKR scheme does not provide IND-CKA (indistinguishability against chosen keyword attacks) since their SEKR scheme is constructed to be an "efficiently-searchable" encryption scheme. Furthermore, Jeong et al. proposed a new SEKR scheme which is secure even if the adversaries have any useful partial information about the keyword. They also proposed the mSEKR scheme for multi-keywords.

However, this paper demonstrates that Jeong et al.'s SEKR schemes [3] are not secure to off-line keyword guessing attacks [4], which an adversary (insider/outsider) can retrieve information of certain keyword from any captured query message of the scheme.

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<sup>†</sup>The authors are with the School of Electrical Engineering and Computer Science, Kyungpook National University, 1370 Sankyuk-Dong, Buk-Gu, Daegu 702-701, South Korea.

a) E-mail: ejyoon@knu.ac.kr

b) E-mail: yook@knu.ac.kr (corresponding author)

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## 2. Review of Jeong et al.'s Schemes

The following algorithms are used in the schemes [3].

- **Bilinear Map.** Let  $\mathbb{G}_1$  be a group of prime order  $q$ .  $e$  is a bilinear map  $e : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$  with the following properties: (1) For all  $u, v \in \mathbb{G}_1$  and  $a, b \in \mathbb{Z}$ ,  $e(u^a, v^b) = e(u, v)^{ab}$ . (2) If  $g$  is a generator of  $\mathbb{G}_1$ ,  $e(g, g)$  is a generator of  $\mathbb{G}_2$ .
- **Computational Diffie-Hellman (CDH) Assumption.** Given  $g, g^{u_1}, g^{u_2} \in \mathbb{G}_1$  as input, where  $u_1, u_2 \leftarrow [1, q]$ , compute  $g^{u_1 u_2} \in \mathbb{G}_1$ .
- **Message Authentication Code (MAC).** MAC consists of  $M = (\text{Mac}, \text{Vfy})$ . Given a random key  $k$ ,  $\text{Mac}$  computes a tag  $\tau$  for a message  $m$ ;  $\tau \leftarrow \text{Mac}_k(m)$ .  $\text{Vfy}$  verifies the message-tag pair using the key  $k$ , and returns 1 if the tag is valid or 0 otherwise;  $m, \text{Vfy}_k(m, \text{Mac}_k(m)) \stackrel{?}{=} 1$ .
- **Random Oracle Model.** Let  $H$  be a hash function such that  $H : \{0, 1\}^* \rightarrow \{0, 1\}^\theta$ , where  $\theta$  is the length of the results of the hash function.

### 2.1 SEKR Scheme

Let the keyword  $KW \in \{0, 1\}^l$ . Let  $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$ ,  $H_2 : \mathbb{G}_2 \rightarrow \{0, 1\}^{\log_2 q}$ ,  $H_3 : \mathbb{G}_1 \rightarrow \{0, 1\}^l$  and  $H_4 : \mathbb{G}_1 \rightarrow \{0, 1\}^{\log_2 q}$  be hash functions.

- **SEKR.key( $1^\theta$ ).** The algorithm picks a random  $\alpha \in \mathbb{Z}_q^*$  and a generator  $g$  of  $\mathbb{G}_1$ . It outputs a pair of public key  $pk = [g, h = g^\alpha]$  and private key  $sk = \alpha$ .
- **SEKR.enc( $pk, KW$ ).** The algorithm first computes  $a = e(H_1(KW), h^r)$  and  $k = H_4(h^r)$  for a random  $r \in \mathbb{Z}_q^*$ . Then it outputs

$$A = g^r, B = H_2(a), C = H_3(h^r) \oplus KW \\ D = \text{Mac}_k(A||B||C)$$

- **SEKR.td( $sk, KW$ ).** The algorithm outputs  $t_{KW} = H_1(KW)^\alpha \in \mathbb{G}_1$ .
- **SEKR.test( $pk, c, t_{KW}$ ).** Let  $c = [A, B, C, D]$ . The algorithm tests if

$$H_2(e(t_{KW}, A)) \stackrel{?}{=} B$$

If so, the algorithm outputs 1; if not, the algorithm outputs 0.

- $\text{SEKR.dec}(sk, c)$ . Let  $c = [A, B, C, D]$ . The algorithm calculates  $k = H_4(A^\alpha)$ . Then the algorithm tests if

$$\text{Vfy}_k(A\|B\|C, D) \stackrel{?}{=} 1$$

If so, the algorithm outputs  $KW \leftarrow C \oplus H_3(A^\alpha)$ . Otherwise, it outputs  $\perp$ .

## 2.2 mSEKR Scheme for Multi-Key Words

- $\text{mSEKR.key}(1^\theta)$ . The algorithm picks a random  $\alpha \in \mathbb{Z}_q^*$  and a generator  $g$  of  $\mathbb{G}_1$ . It outputs a pair of public key  $pk = [g, h = g^\alpha]$  and private key  $sk = \alpha$ .
- $\text{mSEKR.enc}(pk, \mathbf{KW})$ , where  $\mathbf{KW} = (KW_1, \dots, KW_n)$ . The algorithm first computes  $a_i = e(H_1(KW_i), h^r)$  and  $k = H_4(h^r)$  for a random  $r \in \mathbb{Z}_q^*$ , where  $1 \leq i \leq n$ . Then it outputs

$$A = g^r, B_i = H_2(a_i), C_i = H_3(h^r) \oplus KW_i \\ D = \text{Mac}_k(A\|B_1\|\dots\|B_n\|C_1\|\dots\|C_n)$$

for  $1 \leq i \leq n$ .

- $\text{mSEKR.td}(sk, \mathbf{KW})$ . The algorithm outputs  $t_{KW} = H_1(KW)^\alpha \in \mathbb{G}_1$ .
- $\text{mSEKR.test}(pk, c, t_{KW})$ . Let  $c = [A, B_1, \dots, B_n, C_1, \dots, C_n, D]$ . The algorithm tests if

$$H_2(e(t_{KW}, A)) \stackrel{?}{=} B_i$$

for some  $i$ . If so, the algorithm outputs 1; if not, the algorithm outputs 0.

- $\text{mSEKR.dec}(sk, c)$ . Let  $c = [A, B_1, \dots, B_n, C_1, \dots, C_n, D]$ . The algorithm calculates  $k = H_4(A^\alpha)$ . Then the algorithm tests if

$$\text{Vfy}_k(A\|B_1\|\dots\|B_n\|C_1\|\dots\|C_n, D) \stackrel{?}{=} 1$$

If so, the algorithm outputs  $KW_i \leftarrow C_i \oplus H_3(A^\alpha)$  for  $1 \leq i \leq n$ . Otherwise, it outputs  $\perp$ .

## 3. Off-Line Keyword Guessing Attacks

In general, keywords are chosen from much smaller space than passwords and users usually use well-known keywords (low entropy) for search of document [4]. For example, in an e-mail search system which is a major application area of keyword search scheme based on public key encryption, users are interested to search for their e-mails sent by ‘‘Supervisor’’ or ‘‘Lover’’ in the From field or they may concern well-known keywords such as ‘‘Urgent’’, ‘‘Exam’’, and ‘‘Hello’’ in the Title fields. Usually, when users fill in a title of e-mail, they use a simple and representative sentence composed of very short keywords to make receivers easily grasp the content of e-mail. Sufficiently, this fact can give rise to keyword guessing attacks where an malicious

adversary is able to guess some candidate keywords, and verify his/her guess is correct or not in an off-line manner. By performing this off-line keyword guessing attack, malicious outsider/insider adversary can get relevant information of encrypted e-mail, and intrude on a users’ e-mail privacy. The off-line keyword guessing attack on the Jeong et al.’s SEKR scheme [3] can be performed by an adversary  $Adv$  as follows.

Let  $\mathbb{D}$  be a dictionary of keywords whose size is bounded by some polynomial. Let  $pk = [g, h = g^\alpha]$  be a public key for a party. Assume that an adversary  $Adv$  is given  $t_{KW} = H_1(KW)^\alpha$  such that  $\text{SEKR.test}(pk, c, t_{KW}) = 1$ , and  $t_{KW}$  was made with keywords in  $\mathbb{D}$ . Then  $Adv$  can determine which keyword was used in  $t_{KW}$  as follows:

1.  $Adv$  guesses an appropriate keyword  $KW^*$  in  $\mathbb{D}$ , and computes  $H_1(KW^*)$ .
2.  $Adv$  tests if

$$e(H_1(KW^*), h) \stackrel{?}{=} e(t_{KW}, g) \quad (1)$$

If so, the guessed keyword is a valid keyword. Otherwise, go to Step 1.

We know that  $t_{KW}$  is equal to  $H_1(KW)^\alpha$  from the  $\text{SEKR.td}(sk, KW)$  algorithm of SEKR scheme. Therefore, if  $KW$  is equal to  $KW^*$ , then Eq. (1) always holds since

$$e(H_1(KW^*), h) = e(H_1(KW^*), g^\alpha) \\ = e(H_1(KW^*)^\alpha, g) \\ = e(t_{KW}, g)$$

Similarly, this off-line keyword guessing attack works on the Jeong et al.’s mSEKR scheme to the multi-keywords settings [3]. As a result, Jeong et al.’s schemes are not secure to off-line keyword guessing attacks.

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