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PAPER

Impact of Channel Estimation Errors in Cooperative Transmission over Nakagami-*m* Fading Channels

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SUMMARY In this paper, we analyze the impact of channel estimation errors for both decode-and-forward (DF) and amplify-and-forward (AF) cooperative communication systems over Nakagami-*m* fading channels. Firstly, we derive the exact one-integral and the approximate expressions of the symbol error rate (SER) for DF and AF relay systems with different modulations. We also present expressions showing the limitations of SER under channel estimation errors. Secondly, in order to quantify the impact of channel estimation errors, the average signal-to-noise-ratio (SNR) gap ratio is investigated for the two types of cooperative communication systems. Numerical results confirm that our theoretical analysis for SER is very efficient and accurate. Comparison of the average SNR gap ratio shows that DF model is less susceptible to channel estimation errors than AF model.

key words: *cooperative communication systems, channel estimation errors, average symbol error rate (SER), average signal-noise-ratio (SNR) gap ratio*

1. Introduction

Recently, cooperative relaying techniques have gained increasing interest for its ability to provide spatial diversity, enhance the performance and mitigate fading in wireless networks. There are various protocols to obtain the benefits of user cooperation at a reasonable complexity [1]–[3], including decode-and-forward (DF) protocols and amplify-and-forward (AF) protocols. In DF protocols, the relays decode the received signals and only when no error occurs will the relay participate into the cooperative transmission [4]–[6]. While in AF protocols, the relays simply amplify the received signals and forward them to the destination, where the amplification factors can be fixed or variable [7]–[11]. Lots of works have been done on the performance analysis of these cooperative communication systems. It should be noted that, most of these research efforts are based on coherent detection, where fading channel coefficients need to be firstly estimated before being used in the detection process. The quality of channel estimation inevitably affects the overall performance of the cooperative communication systems. However, an indispensable assumption of these systems is the perfect knowledge of the channel state information, which is probably unavailable in practice and imperfect channel estimation always occurs as a result of imperfect estimation algorithm or the instability of the channel.

Therefore, investigating the performance of cooperative systems with imperfect channel estimation will be significatively important to practice. There have been increasing studies on this topic recently [12]–[17]. Imperfect channel estimation problem in DF systems consists of individual estimation of source-to-relay and relay-to-destination channels. In [12], the outage probability of DF systems with channel estimation errors was well studied, where performance analysis could be explained by a similar approach as point-to-point networks. On the other hand, in AF systems, a cascaded channel consisting of source-to-relay and relay-to-destination links needs to be estimated and the effect of channel estimation errors in the two links should be jointly considered. Both composite and separated channel estimation schemes can be employed. The composite channel estimation means that the overall channel from the source to the destination is estimated at the destination. In [13], considering mismatched-coherent and partially-coherent receivers at the destination terminal, pairwise error probability (PEP) expressions under imperfect channel estimation in AF relaying systems were derived. The SNR expression was derived in [14] with three different approximate assumptions and the outage probability was obtained for a fixed gain AF system in the presence of channel estimation errors. There are also some researches for AF cooperative systems employing the separated channel estimation scheme. Yi Wu and Pätzold in [16] and [17] gave an accurate symbol-error-rate (SER) expression for variable-gain AF cooperative communication systems, although an iterated integral was still involved. Besides, the works in [13]–[17] were limited for a single relay scenario. During the time of the review process of this paper, there are some new-issued researches on this topic for multi-relay cases. Considering both conventional cooperative systems (i.e., all relays participate in the relaying phase) and opportunistic cooperative systems (i.e., only the best relay participates in the relaying phase), the expressions of the error and outage probabilities taking channel estimation errors were obtained over Rayleigh and Rician fading channels in [18] and [19]. In [20], a framework for evaluating the bit-error-rate (BER) performance and asymptotic bounds for both the variable- and fixed-gain AF cooperative systems impacted by channel estimation errors were presented. However, the aforementioned performance analysis works have been confined to Rayleigh or Rician fading channels. Nakagami-*m* fading, which covers a broader va-

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riety of fading scenarios, needs to be paid more attention to. Although Yao Ma et al. have developed channel estimation errors model for Nakagami-m fading channels and given the BER performance with M-QAM modulation formats in [21]. But his work was only valid for point-to-point networks, and yet, to the best of our knowledge, accurate analytical performance results for cooperative communication systems taking into account the channel estimation errors in Nakagami-m fading channels are sparse.

In this paper, based on the separated channel estimation scheme, we analyze the impact of channel estimation errors for both DF and AF cooperative communication systems over Nakagami-m fading channels. The main contributions of this paper are summarized as follows.

- For the AF case, we derive the probability distribution function (PDF) of the SNR at the receiver for each cooperative link in the presence of channel estimation errors. Exact closed-form expressions of the SER are obtained using the moment-generating function (MGF) approach for both DF and AF relaying types. Besides, motivated by a modified approximation of the Q-function, we derive accurate approximations of SER for a wide variety of M-ary commonly used modulations, where no integration is involved.
- Based on the SER analysis for the cooperative communication systems with channel estimation errors, we find that the SER performance will not behave better as the transmit SNR goes larger in high SNR region. In other words, due to the existence of channel estimation errors, SER will converge and we define this convergent value as limitation of SER. This value actually shows the system SER when only considering the channel estimation errors, and reflects the degree of the performance loss due to imperfect channel estimation from a certain angle. Therefore, limitations of SER for both DF and AF cooperative systems are investigated.
- The average SNR gap ratio is employed by our system to quantify the impact of channel estimation errors, which was proposed in [12] and shows a new method to quantify the reduction in the SNR due to the channel estimation errors. We derive the exact expressions of the average SNR gap ratio for both DF and AF systems and present the comparison between them to investigate different degrees of the impact. Interesting results show that DF model is less susceptible to channel estimation errors than AF model.

The remainder of this paper is organized as follows. Section 2 describes the system model for both DF and AF cooperative communication systems. In Sect. 3, we derive the exact and the approximate SER expressions for the two types of systems by using the moment-generating function (MGF) method. In Sect. 4 the average SNR gap ratio caused by channel estimation errors is presented. Section 5 shows the simulation results. Finally, conclusions are drawn in Sect. 6.

2. System Model

The cooperative wireless communication system includes the source s, the destination d, and a set of N relays r_k, $k = 1, \cdots, N$, as shown in Fig. 1. We assume that all terminals are equipped with a single antenna and cannot transmit and receive simultaneously. The fading channel coefficients are denoted by h_ζ, $\zeta = \{sd, sr_k, r_kd\}$, and the envelop $|h_\zeta|$ is modeled as Nakagami-m distributed random variable with parameters m_ζ, Ω_ζ. Thus, the effective power channel gain $|h_\zeta|^2$ follows Gamma distribution and its p.d.f. is given by

$$p_{|h_\zeta|^2}(a) = \frac{a^{m_\zeta - 1} e^{-a/\lambda_\zeta}}{\Gamma(m_\zeta)\lambda_\zeta^{m_\zeta}} \tag{1}$$

where $a = |h_\zeta|^2$, and $\lambda_\zeta = \Omega_\zeta / m_\zeta$ is the scale parameter. In addition, zero-mean additive white Gaussian noise with a fixed variance N_0 is assumed on each communication link.

2.1 Channel Estimation Error Model

We adopt the channel estimation error model in [18] to our system, which is valid for pilot symbol assisted modulation (PSAM) based near-minimum mean-square error (MMSE) channel estimators. The model is described as

$$h = \hat{h} + e \tag{2}$$

where h and $\hat{h}$ are the real and the estimated value of the channel gain, and the channel estimation error e can be approximated as a Gaussian random variable, independent of $\hat{h}$. Besides, the Nakagami-m parameter of $\hat{h}$, denoted by $\hat{m}$, has been derived as that $\hat{m} \simeq m$ in [21], when the average SNR or the number of the pilot symbols of the channel estimator is large. Thus, throughout this paper, we have the following assumptions: the imperfect channel estimation causes channel estimation errors for each communication link, which are zero-mean complex Gaussian variables, denoted by e_ζ with variance α_ζ, $\zeta = \{sd, sr_k, r_kd\}$. Imperfect estimated channel coefficients denoted by $\hat{h}_\zeta$ are known to the receiver. From the channel estimation error model in (2), we may conclude that $|\hat{h}_\zeta|^2$ will also be Gamma distributed with parameters m_ζ and $\hat{\lambda}_\zeta = (\Omega_\zeta - \alpha_\zeta)/m_\zeta$.

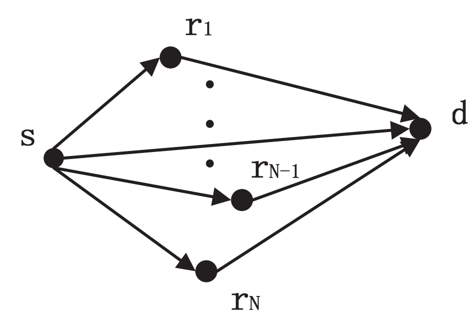

Fig. 1 A cooperative communication system with N relays.

2.2 Decode-and-Forward Model

A DF cooperative protocol proposed in [12] is adopted to our system, where the availability of a relay node assisting the source-to-destination communication depending on the quality of the received signal. In this protocol, a transmission of one symbol is implemented in $N + 1$ phases. In the first phase, s broadcasts its information to d and N relays with transmit power P_s. So at the destination d and the relay r_k, the received symbols can be written as

$$\begin{aligned} r_{sd}^{DF} &= \sqrt{P_s} h_{sd} x + n_{sd} \\ &= \sqrt{P_s} \hat{h}_{sd} x + \left(\sqrt{P_s} e_{sd} x + n_{sd} \right) \\ r_{sr_k}^{DF} &= \sqrt{P_s} h_{sr_k} x + n_{sr_k} \\ &= \sqrt{P_s} \hat{h}_{sr_k} x + \left(\sqrt{P_s} e_{sr_k} x + n_{sr_k} \right) \end{aligned} \tag{3}$$

where x is the source's transmitted signal with unit average energy. Then all relays will decode the received signal and check if it is correct, which can possibly be done through examing the included cyclic redundancy check (CRC) digits. If the symbol is correctly decoded, the relay will forward it to the destination; otherwise, the relay will remain silent. We assume that there are R relay nodes correctly decoding the signal and define $C_R = \{\widehat{r}_1, \cdots, \widehat{r}_i, \cdots, \widehat{r}_R\}$, $R = 0, \cdots, N$ as the set of these relay nodes. Then, the relay $\widehat{r}_i$ in C_R, will transmit the same signal x to d with power $P_{\widehat{r}_i}$ in the matched phase. So at the destination, the received signal from $\widehat{r}_i$ is given by

$$\begin{aligned} r_{\widehat{r}_i d}^{DF} &= \sqrt{P_{\widehat{r}_i}} h_{\widehat{r}_i d} x + n_{\widehat{r}_i d} \\ &= \sqrt{P_{\widehat{r}_i}} \hat{h}_{\widehat{r}_i d} x + \left(\sqrt{P_{\widehat{r}_i}} e_{\widehat{r}_i d} x + n_{\widehat{r}_i d} \right) \end{aligned} \tag{4}$$

Finally, at the destination d, with the knowledge of the estimates $\hat{h}_{sd}$ and $\hat{h}_{\widehat{r}_i d}$, the received signals will be jointly combined with corresponding MRC weights.

2.3 Amplify-and-Forward Model

The AF protocol considered here is the fixed gain type. Different from the analysis in [16] and [20], where the composite channel estimation scheme was employed to reduce the burden of the channel estimation at the relay, we assume that the channel estimation work here is implemented in a separated way. Because, for the MRC receiver, due to the existence of channel estimation errors, it still needs the knowledge of individual channel gain h_ζ in the MRC weights for detection.

In this AF protocol, the transmission of one symbol is also implemented in $N + 1$ phases. In the first phase, the source broadcasts its information. We have

$$\begin{aligned} r_{sd}^{AF} &= \sqrt{P_s} h_{sd} x + n_{sd} \\ &= \sqrt{P_s} \hat{h}_{sd} x + \left(\sqrt{P_s} e_{sd} x + n_{sd} \right) \\ r_{sr_k}^{AF} &= \sqrt{P_s} h_{sr_k} x + n_{sr_k} \\ &= \sqrt{P_s} \hat{h}_{sr_k} x + \left(\sqrt{P_s} e_{sr_k} x + n_{sr_k} \right) \end{aligned} \tag{5}$$

Then, all the relays will forward the scaled versions of the received signal to d in the matched phases. So at the destination terminal, the received signals from the relay r_k can be written as

$$\begin{aligned} r_{r_k d}^{AF} &= \beta_k h_{r_k d} r_{sr_k}^{AF} + n_{r_k d} \\ &= \sqrt{P_s} \beta_k \hat{h}_{sr_k} \hat{h}_{r_k d} x + \Big(\sqrt{P_s} \beta_k (\hat{h}_{sr_k} \hat{e}_{r_k d} + \hat{h}_{r_k d} \hat{e}_{sr_k} \\ &\quad + \hat{e}_{sr_k} \hat{e}_{r_k d}) x + \beta_k (\hat{h}_{r_k d} + \hat{e}_{r_k d}) n_{sr_k} + n_{r_k d} \Big) \end{aligned} \tag{6}$$

where $\beta_k = \sqrt{P_k / (P_s \Omega_{sr_k} + N_0)}$ is the amplification factor and P_k is the transmit power of the k-th relay.

3. SER Analysis

The MGF method is an effective way to give a closed-form SER expression. Therefore, it is vital to compute the output SNR at the MRC receiver, and the SER performance evaluation for a wide variety of M-ary modulations can be obtained using the MGF-based approach [22]. Table 1 gives integral form SER for commonly used modulations, which is $P_{SER} = \mathcal{F}_\varsigma(M, \mathcal{M}_\varsigma(s))$, $\varsigma = \{MAM, MPSK, MQAM\}$. In Table 1, $g_{MAM} = 3/(M^2 - 1)$, $g_{MPSK} = \sin^2(\pi/M)$, $g_{MQAM} = 3/2(M - 1)$, and $\mathcal{M}(s)$ is MGF of the output SNR, defined as $\mathcal{M}(s) = E\{e^{sx}\}$, where $E\{\cdot\}$ denotes the statistical expectation of a particular random variable.

Therefore, we will mainly focus on obtaining an expression of the output SNR for both the DF and AF models in the following contents. Besides, we should clarify that without loss of generality, we will take M-QAM modulation type as an example for the SER analysis, while other types of modulation will also be considered in the simulation part.

3.1 Decode-and-Forward Model

In the first phase, at the relay r_k, with the knowledge of the estimated value $\hat{h}_{sr_k}$, from (3), we can calculate the instantaneous output SNR as

$$\gamma_{sr_k}^{DF} = \frac{P_s}{P_s \alpha_{sr_k} + N_0} |\hat{h}_{sr_k}|^2 \tag{7}$$

and its MGF can be obtained as

$$\mathcal{M}_{sr_k}^{DF}(s) = \left(1 - \frac{P_s \hat{\lambda}_{sr_k} s}{P_s \alpha_{sr_k} + N_0} \right)^{-m_{sr_k}} \tag{8}$$

Table 1 MGF-based SER evaluations.

Modulation	Integral Form for SER
M-AM	$P_{SER}^{MAM} = \mathcal{F}_{MAM}\left(M, \mathcal{M}(s)\right) = \frac{2(M-1)}{M\pi} \int_0^{\frac{\pi}{2}} \mathcal{M}(-\frac{g_{MAM}}{\sin^2\theta}) d\theta$
M-PSK	$P_{SER}^{MPSK} = \mathcal{F}_{MPSK}\left(M, \mathcal{M}(s)\right) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \mathcal{M}(-\frac{g_{MPSK}}{\sin^2\theta}) d\theta$
M-QAM	$P_{SER}^{MQAM} = \mathcal{F}_{MQAM}\left(M, \mathcal{M}(s)\right) = \frac{4}{\pi}(1 - \frac{1}{\sqrt{M}}) \int_0^{\frac{\pi}{2}} \mathcal{M}(-\frac{g_{MQAM}}{\sin^2\theta}) d\theta - \frac{4}{\pi}(1 - \frac{1}{\sqrt{M}})^2 \int_0^{\frac{\pi}{4}} \mathcal{M}(-\frac{g_{MQAM}}{\sin^2\theta}) d\theta$

Then with the help of Table 1, the probability that relay r_i decodes the received signal in error could be regarded as the SER, and it can be given as

$$\varepsilon_{r_k} = \mathcal{F}_{MQAM}(M, \mathcal{M}^{DF}_{sr_k}(s)) \tag{9}$$

When m_{sr_k} is an integer, this integral for SER has a closed-form formulation in [22], which is

$$\begin{aligned}\varepsilon_{r_k} = {} & 2\left(\frac{\sqrt{M}}{\sqrt{M}-1}\right)\left[1-\mu_{sr_k}\sum_{n=0}^{m_{sr_k}-1}\binom{2n}{n}\left(\frac{1-\mu^2_{sr_k}}{4}\right)^n\right] \\ & -\left(\frac{\sqrt{M}}{\sqrt{M}-1}\right)^2\left[1-\frac{4}{\pi}\mu_{sr_k}\left\{\left(\frac{\pi}{2}-\tan^{-1}\mu_{sr_k}\right)\right.\right. \\ & \sum_{n=0}^{m_{sr_k}-1}\binom{2n}{n}\frac{1}{(4(1+c_{sr_k}))^n}-\sin(\tan^{-1}\mu) \\ & \left.\left.\sum_{n=1}^{m_{sr_k}-1}\sum_{l=1}^{n}\frac{T_{\ln}}{(1+c_{sr_k})^n}[\cos(\tan^{-1}\mu_{sr_k})]^{2(n-l)+1}\right\}\right]\end{aligned} \tag{10}$$

where

$$\begin{aligned} c_{sr_k} &= \frac{1.5P_s\hat{\lambda}_{sr_k}}{(M-1)(P_s\alpha_{sr_k}+N_0)} \\ \mu_{sr_k} &= \sqrt{\frac{c_{sr_k}}{1+c_{sr_k}}} \\ T_{\ln} &= \frac{\binom{2n}{n}}{\binom{2(n-l)}{n-l}4^l[2(n-l)+1]}\end{aligned} \tag{11}$$

As stated in Sect. 2.2, the set of relays which successfully decode and participate into the transmission to the destination is $C_R = \{\widehat{r}_1, \widehat{r}_2, \cdots, \widehat{r}_R\}$. From (4), we can compute the total output effective SNR of the MRC receiver at the destination node d as

$$\gamma^{DF}_{C_R} = \frac{P_s}{P_s\alpha_{sd}+N_0}|\hat{h}_{sd}|^2 + \sum_{i=1}^{R}\frac{P_{\widehat{r}_i}}{P_{\widehat{r}_i}\alpha_{\widehat{r}_i d}+N_0}|\hat{h}_{\widehat{r}_i d}|^2 \tag{12}$$

Then MGF can be given as

$$\begin{aligned}\mathcal{M}^{DF}_{C_R}(s) = {} & \left(1-\frac{P_s\hat{\lambda}_{sd}s}{P_s\alpha_{sd}+N_0}\right)^{-m_{sd}} \\ & \times\prod_{i=1}^{R}\left(1-\frac{P_{\widehat{r}_i}\hat{\lambda}_{\widehat{r}_i d}s}{P_{\widehat{r}_i}\alpha_{\widehat{r}_i d}+N_0}\right)^{-m_{\widehat{r}_i d}}\end{aligned} \tag{13}$$

Similar to getting the SER at the relay node, with the help of Table 1, for one symbol and a specific relay set C_R, the average error probability is

$$\xi^{DF}_{C_R} = \mathcal{F}_{MQAM}(M, \mathcal{M}^{DF}_{C_R}(s)) \tag{14}$$

Then, the final expression of the SER at d for this DF relaying system can be calculated by

$$P^{DF}_{SER} = \sum_{C_R, R=0,\cdots,N}\xi^{DF}_{C_R}Prob\{C_R\} \tag{15}$$

where $Prob\{C_R\}$ is the probability of the given cooperative node set C_R, which is

$$Prob\{C_R\} = \prod_{\widehat{r}_i\in C_R}(1-\varepsilon_{\widehat{r}_i})\prod_{r_k\notin C_R}\varepsilon_{r_k} \tag{16}$$

Note that for each number of R, there will be $\binom{N}{R}$ possible relay node permutations for C_R. In (15) if we assume identical Nakagami-m fading and channel estimation error conditions for all links, then (15) can be simplified as

$$P^{DF}_{SER} = \sum_{R=0}^{N}\binom{N}{R}(1-\varepsilon_{r_k})^R\varepsilon_{r_k}^{N-R}\xi^{DF}_{C_R} \tag{17}$$

3.2 Amplify-and-Forward Model

In this section, we will give our output SNR analysis based on the separate channel estimation model in our fixed gain AF cooperative system.

The case of the direct source-to-relay link can be taken from the DF model and the main efforts should be targeted on the cooperative link. From (6), we can compute the output SNR for one of the cooperative link as

$$\begin{aligned}\gamma^{AF}_k &= \frac{P_s\beta^2|\hat{h}_{sr_k}|^2|\hat{h}_{r_kd}|^2}{\left(\begin{array}{l}P_s\beta^2(|\hat{h}_{sr_k}|^2\alpha_{r_kd}+|\hat{h}_{r_kd}|^2\alpha_{sr_k}+\alpha_{sr_k}\alpha_{r_kd})\\ +\beta^2(|\hat{h}_{r_kd}|^2+\alpha_{r_kd})N_0+N_0\end{array}\right)} \\ &= \frac{P_sP_k|\hat{h}_{sr_k}|^2|\hat{h}_{r_kd}|^2}{\left(\begin{array}{l}P_sP_k\alpha_{r_kd}|\hat{h}_{sr_k}|^2+(P_sP_k\alpha_{sr_k}+P_kN_0)|\hat{h}_{r_kd}|^2\\ +P_sP_k\alpha_{sr_k}\alpha_{r_kd}+P_k\alpha_{r_kd}N_0+P_s\Omega_{sr_k}N_0+N_0^2\end{array}\right)}\end{aligned} \tag{18}$$

To simplify the presentation, we define $\gamma_{sr_k} = P_s|\hat{h}_{sr_k}|^2/N_0$, $\gamma_{r_kd} = P_k|\hat{h}_{r_kd}|^2/N_0$, as Gamma distributed with parameters $\hat{\eta}_{sr_k} = P_s|\hat{\lambda}_{sr_k}|^2/N_0$, m_{sr_k} and $\hat{\eta}_{r_kd} = P_k|\hat{\lambda}_{r_kd}|^2/N_0$, m_{r_kd} respectively. Note that it is really difficult to give an accurate result for the probability distribution function (PDF) of γ^{AF}_k. Reasonably, if we substitute $|\hat{h}_{sr_k}|^2$ with its statistic average value $\Omega_{sr_k}-\alpha_{sr_k}$, this problem may be solved. Then (18) can be rewritten as

$$\gamma^{AF}_k = \frac{\gamma_{sr_k}\gamma_{r_kd}}{\rho_k\gamma_{r_kd}+C_k} \tag{19}$$

where $\rho_k = \dfrac{P_s\alpha_{r_kd}}{N_0}+1$, $C_k = \dfrac{P_sP_k\alpha_{r_kd}}{N_0^2}+\dfrac{P_k\alpha_{r_kd}}{N_0}+\dfrac{P_s\Omega_{sr_k}}{N_0}+1$. Then, the total output effective SNR of the MRC receiver at the destination node d is

$$\gamma^{AF} = \frac{P_s}{P_s\alpha_{sd}+N_0}|\hat{h}_{sd}|^2+\sum_{k=1}^{N}\gamma^{AF}_k \tag{20}$$

Fortunately, with the help of [10, eq. 9], we can obtain the cumulative distribution function of γ^{AF}_k, given by

$$F_k^{AF}(\gamma) = 1 - \sum_{i=0}^{m_{sr_k}-1} \sum_{j=0}^{i} \delta(i,j) e^{-\rho_k \gamma / \hat{\eta}_{sr_k}} \times (\rho_k \gamma)^{\mu_k} K_{2\nu_k}\left(2\sqrt{\frac{C_k \gamma}{\hat{\eta}_{sr_k}\hat{\eta}_{r_k d}}}\right) \quad (21)$$

and

$$\delta(i,j) = \frac{2}{\Gamma(m_{r_k d}) i!} \binom{i}{j} \hat{\eta}_{sr_k}^{-\mu_k} \left(\frac{\hat{\eta}_{r_k d}}{\rho_k C_k}\right)^{-\nu_k - j} \quad (22)$$

where $K_v(\cdot)$ is the v-th order modified Bessel function of the second kind, $\mu_k = \dfrac{2i + m_{r_k d} - j}{2}$, $\nu_k = \dfrac{m_{r_k d} - j}{2}$. Then the PDF of γ_k^{AF}, can be easily obtained by differentiating (21) with respect to γ as

$$f_k^{AF}(\gamma) = \sum_{i=0}^{m_{sr_k}-1} \sum_{j=0}^{i} \delta(i,j) e^{-\rho_k \gamma / \hat{\eta}_{sr_k}} \gamma^{\frac{2i + m_{r_k d} - j}{2} - 1} \times \left[\left(\frac{\rho_k}{\hat{\eta}_{sr_k}}\gamma - i\right) K_{m_{r_k d} - j}\left(2\sqrt{\frac{C_k \gamma}{\hat{\eta}_{sr_k}\hat{\eta}_{r_k d}}}\right) + \sqrt{\frac{C_k \gamma}{\hat{\eta}_{sr_k}\hat{\eta}_{r_k d}}} K_{m_{r_k d} - j - 1}\left(2\sqrt{\frac{C_k \gamma}{\hat{\eta}_{sr_k}\hat{\eta}_{r_k d}}}\right)\right] \quad (23)$$

Using [24, eq. 6.643.3], the MGF of γ_k^{AF} can be computed as

$$\begin{aligned}\mathcal{M}_k^{AF}(s) = &\sum_{i=0}^{m_{sr_k}-1} \sum_{j=0}^{i} \frac{1}{\Gamma(m_{r_k d}) i!} \binom{i}{j} \left(\frac{C_k}{\rho_k \hat{\eta}_{r_k d}}\right)^{\nu_k + j} \left(1 - \frac{\hat{\eta}_{sr_k} s}{\rho_k}\right)^{-\mu_k} \\ &\times e^{\frac{\omega_k(s)}{2}} \Bigg[\Gamma(i + m_{r_k d} - j + 1)\Gamma(i+1) \\ &\times \frac{\rho_k \hat{\eta}_{r_k d} \sqrt{\omega_k(s)}}{C_k} W_{-\mu_k - 1/2, \nu_k}(\omega_k(s)) \\ &+ \Gamma(i + m_{r_k d} - j)\Gamma(i+1) W_{-\mu_k, \nu_k - 1/2}(\omega_k(s)) \\ &- i\Gamma(i + m_{r_k d} - j)\Gamma(i) \sqrt{\frac{1}{\omega_k(s)}} W_{-\mu_k + 1/2, \nu_k}(\omega_k(s))\Bigg]\end{aligned} \quad (24)$$

where $\omega_k(s) = \dfrac{C_k}{\hat{\eta}_{r_k d}(\rho_k - \hat{\eta}_{sr_k} s)}$ and $W_{\mu,\nu}(\cdot)$ is the Whittaker function, which can be evaluated using popular symbolic software such as MATHEMATICA and MATLAB.

Thus, we obtain the MGF of the total output effective SNR of the MRC receiver.

$$\mathcal{M}^{AF}(s) = \left(1 - \frac{P_s \hat{\lambda}_{sd} s}{P_s \alpha_{sd} + N_0}\right)^{-m_{sd}} \prod_{k=1}^{N} \mathcal{M}_k^{AF}(s) \quad (25)$$

Finally, for our fixed gain AF cooperative system, the average SER with M-QAM modulation can be obtained from Table 1, written as

$$P_{SER}^{AF} = \mathcal{F}_{MQAM}(M, \mathcal{M}^{AF}(s)) \quad (26)$$

3.3 Simple and Accurate Approximations

From the discussions above, we can find that there is still an unavoidable integral operation in the closed-form expressions of the SER. Although we get the MGF of the output SNR, it is not easy to calculate the value of the final SER. Puzzled by this, we resort to another way to obtain a simpler expression of the SER. Gaussian Q-function can also be used to express the SER performance for a wide variety of M-ary modulations [22]. We packed up the SER expressions in terms of desired form of Q-function given in Table 2, although some approximations may be contained. Table 2 is described as follows, where Q-function is defined as $Q(x) = \int_0^{\infty} \frac{1}{2\pi} e^{-\frac{y^2}{2}} dy$.

Interestingly, in [23], MinChul Ju et al. proposed a modified method for an approximation of the Q-function. It is described as

$$Q_{appro}(x, \sigma_1, \sigma_2) \approx \frac{1}{12} e^{-\sigma_1 x^2} + \frac{1}{4} e^{-\sigma_2 x^2} \quad (27)$$

With the help of (27), we can obtain the relationship between a commonly used form in Table 2 and the MGF. The relation is shown by

$$\begin{aligned}E_\gamma\{Q\sqrt{g\gamma}\} &= E_\gamma\left\{\frac{1}{12} e^{-g\sigma_1 \gamma} + \frac{1}{4} e^{-g\sigma_2 \gamma}\right\} \\ &= \frac{1}{12}\mathcal{M}(-g\sigma_1) + \frac{1}{4}\mathcal{M}(-g\sigma_2)\end{aligned} \quad (28)$$

where g is the coefficient of the SNR in the Q-function and (σ_1, σ_2) can be elaborately adjusted for different purpose to match the final expressions. For our cooperative communication systems experiencing Nakagami-m fading channels, we find two appropriate values $(\sigma_1, \sigma_2) = (0.47, 0.88)$ from numerical simulation results according to the method proposed in [23]. Section 5 illuminates that the approximation by (σ_1, σ_2) approaches the exact result well, and more appropriate values with little improvement are not included here.

Therefore, using this approximation in (27) and with the help of Table 2, the SER for our DF and AF cooperative systems can be simplified as follows:
M-QAM modulation is chosen as a representative example.

Table 2 Q-function SER form.

Modulation	Q-function Form for SER
M-AM	$P_{SER}^{MAM} = \frac{2(M-1)}{M} E_\gamma\left\{Q\left(\sqrt{\frac{6\gamma}{(M^2-1)}}\right)\right\}$
BPSK	$P_{SER}^{BPSK} = E_\gamma\left\{Q\left(\sqrt{2\gamma \sin^2\frac{\pi}{M}}\right)\right\}$
M-PSK	$P_{SER}^{MPSK} \approx 2E_\gamma\left\{Q\left(\sqrt{2\gamma \sin^2\frac{\pi}{M}}\right)\right\}$
M-QAM	$P_{SER}^{MQAM} \approx 4(1 - M^{-1/2}) E_\gamma\left\{Q\left(\sqrt{\frac{3\gamma}{(M-1)}}\right)\right\}$

For the AF model, we have

$$P_{SER}^{AF,appro} = (1 - M^{-1/2})\Bigg[\frac{1}{3}\left(1 + \frac{1.41 P_s \hat{\lambda}_{sd}}{(P_s\alpha_{sd} + N_0)(M-1)}\right)^{-m_{sd}} \times \prod_{k=1}^{N} \mathcal{M}_k^{AF}\left(-\frac{1.41}{M-1}\right) + \prod_{k=1}^{N} \mathcal{M}_k^{AF}\left(-\frac{2.64}{M-1}\right) \times \left(1 + \frac{2.64 P_s \hat{\lambda}_{sd}}{(P_s\alpha_{sd} + N_0)(M-1)}\right)^{-m_{sd}}\Bigg] \tag{29}$$

and for the DF model

$$\varepsilon_{r_k}^{appro} = (1 - M^{-1/2})\Bigg[\frac{1}{3}\left(1 + \frac{1.41 P_s \hat{\lambda}_{sr_k}}{(P_s\alpha_{sr_k} + N_0)(M-1)}\right)^{-m_{sr_i}} + \left(1 + \frac{2.64 P_s \hat{\lambda}_{sr_k}}{(P_s\alpha_{sr_k} + N_0)(M-1)}\right)^{-m_{sr_k}}\Bigg] \tag{30}$$

$$\xi_{C_R}^{DF,appro} = \left(1 - \sqrt{\frac{1}{M}}\right) \times \left[\frac{1}{3}\mathcal{M}_{C_R}^{DF}\left(-\frac{1.41}{M-1}\right) + \mathcal{M}_{C_R}^{DF}\left(-\frac{2.64}{M-1}\right)\right] \tag{31}$$

And using (17), when the parameters on each link are the same, the final expression of the SER, can be written as

$$P_{SER}^{DF,appro} = \sum_{R=0}^{N}\binom{N}{K}(1 - \varepsilon_r^{appro})^R(\varepsilon_r^{appro})^{N-R}\xi_{C_R}^{DF,appro} \tag{32}$$

Eventually, it is obvious that there is no complex integral in all the approximated SER expressions, and the calculation for SER will be laconic and facile.

3.4 Limitations for SER

Before switching to the limitation analysis of SER, we introduce a concept of transmit SNR. As the source's transmitted symbol x has unit average energy, the total power of the transmitted signal from the source and the relays is $P = P_s + \sum_k P_k$. We define the transmit SNR as P/N_0 in our systems. Generally speaking, in the circumstance of perfect channel estimation, the SER at the destination will approach zero when the transmit SNR $P/N_0 \to \infty$, or when $N_0 \to 0$ with fixed transmission power P. However, in the case that there are channel estimation errors, the SER may converge to a nonzero value as it is mainly determined by the channel estimation errors when $N_0 \to 0$. We define this convergent value as limitation of SER and present its expression for both the DF and AF cooperative systems.

$$\varepsilon_{r_k,\lim}^{appro} = \lim_{N_0 \to 0} \varepsilon_{r_k}^{appro} = (1 - M^{-1/2})\Bigg[\frac{1}{3}\left(1 + \frac{1.41\hat{\lambda}_{sr_k}}{\alpha_{sr_k}(M-1)}\right)^{-m_{sr_k}} + \left(1 + \frac{2.64\hat{\lambda}_{sr_k}}{\alpha_{sr_k}(M-1)}\right)^{-m_{sr_k}}\Bigg] \tag{33}$$

and

$$\mathcal{M}_{C_R,\lim}^{DF}(s) = \lim_{N_0 \to 0} \mathcal{M}_{C_R}^{DF}(s) = \left(1 - \frac{\hat{\lambda}_{sd}s}{\alpha_{sd}}\right)^{-m_{sd}} \prod_{i=1}^{R}\left(1 - \frac{\hat{\lambda}_{\hat{r}_i d}s}{\alpha_{\hat{r}_i d}}\right)^{-m_{\hat{r}_i d}} \tag{34}$$

Substituting (33) and (34) into (15), we can obtain $P_{SER,\lim}^{DF,appro}$. Then, for the AF case, we have

$$\mathcal{M}_{k,\lim}^{AF}(s) = \lim_{N_0 \to 0} \mathcal{M}_k^{AF}(s) = \mathcal{M}_k^{AF}(s)\Big|_{\frac{C_k}{\rho_k \hat{\eta}_{r_k d}} = \frac{1}{\hat{\lambda}_{r_k d}},\ \frac{\hat{\eta}_{sr_k}}{\rho_k} = \frac{\hat{\lambda}_{sr_k}}{\alpha_{r_k d}},\ \omega_k(s) = \frac{\alpha_{r_k d}}{\hat{\lambda}_{r_k d}(\alpha_{r_k d} - \hat{\lambda}_{sr_k}s)}} \tag{35}$$

Thus,

$$P_{SER,\lim}^{AF,appro} = \lim_{N_0 \to 0} P_{SER}^{AF,appro} = (1 - M^{-1/2}) \times \Bigg[\frac{1}{3}\left(1 + \frac{1.41\hat{\lambda}_{sd}}{\alpha_{sd}(M-1)}\right)^{-m_{sd}} \prod_{k=1}^{N} \mathcal{M}_{k,\lim}^{AF}\left(-\frac{1.41}{M-1}\right) + \left(1 + \frac{2.64\hat{\lambda}_{sd}}{\alpha_{sd}(M-1)}\right)^{-m_{sd}} \prod_{k=1}^{N} \mathcal{M}_{k,\lim}^{AF}\left(-\frac{2.64}{M-1}\right)\Bigg] \tag{36}$$

From the discussions about the SER limitation, we can find that this convergent value is independent of the transmit powers P_s and P_k. It is only related with the channel parameters and the variance of the channel estimation errors. This conclusion is not only valid for our DF relay systems, but also applicable in fixed gain AF relay systems. Thus, in the simulation results section, we would like to investigate the impact of different channel estimation error variances on the SER limitation.

4. Average SNR Gap Ratio

Noted that the SER analysis above is only an important performance evaluation for all communication systems, and for our system under imperfect channel estimation, a pertinent evaluation is needed to examine the degree of the impact of channel estimation errors on the system. In this section, we adopt a concept, called the SNR gap ratio, proposed in [12], to our cooperative transmission scenarios to give a scale to quantify the SNR loss caused by the channel estimation errors. The SNR gap ratio in [12] is defined as follows

$$\Upsilon = \frac{\gamma\big|_{\alpha=0} - \gamma\big|_{\alpha\neq 0}}{E\left\{\gamma\big|_{\alpha=0}\right\}} \tag{37}$$

where γ is the output SNR at the receiver. From (37), we can find that the average SNR gap ratio $E\{\Upsilon\}$ reflects the degree of the SNR loss caused by channel estimation errors. The larger the average SNR gap ratio is, the greater the effect of channel estimation errors is. In the following text, we will present the average SNR gap ratio analysis for both DF and AF models.

4.1 Decode-and-Forward Model

As described in Sect. 2, the relays participate in the transmission to the destination depending on whether the signal is correctly decoded or not. In other words, using the SER at the relay r_k obtained above, we can consider that the event that the relay r_k transmit signals to d is a Bernoulli random event with approximate probability $\varepsilon_{r_k}^{appro}$. Therefore, the final output SNR at the MRC receiver for the DF system can be written as

$$\gamma^{DF} = \frac{P_s}{P_s\alpha_{sd}+N_0}|\hat{h}_{sd}|^2 + \sum_{k=1}^{N}\frac{P_k(1-\varepsilon_{r_k}^{appro})}{P_k\alpha_{r_kd}+N_0}|\hat{h}_{r_kd}|^2 \tag{38}$$

With the help of (29), we can obtain the SER at the relays in the first phase, which is

$$\varepsilon_{r_k}^{\alpha=0} = \left(1-\sqrt{\frac{1}{M}}\right) \times\left[\frac{1}{3}\left(1+\frac{1.41P_s\hat{\lambda}_{sr_k}}{N_0(M-1)}\right)^{-m_{sr_k}} + \left(1+\frac{2.64P_s\hat{\lambda}_{sr_k}}{N_0(M-1)}\right)^{-m_{sr_k}}\right] \tag{39}$$

and $\varepsilon_{r_k}^{\alpha\neq0} = \varepsilon_{r_k}^{appro}$.

By substituting (38) into (37), and calculating the statistical expectation of the SNR gap ratio with $E\{\Upsilon\}$, we have

$$E\{\Upsilon^{DF}\} = \frac{\left(\begin{array}{l}\dfrac{P_s^2\alpha_{sd}}{P_s\alpha_{sd}+N_0}\hat{\Omega}_{sd}\\ +\displaystyle\sum_{k=1}^{N}\frac{P_k^2(1-\varepsilon_{r_k}^{\alpha=0})\alpha_{r_kd}+P_k(\varepsilon_{r_k}^{\alpha=0}-\varepsilon_{r_k}^{\alpha\neq0})N_0}{P_k\alpha_{r_kd}+N_0}\hat{\Omega}_{r_kd}\end{array}\right)}{P_s\hat{\Omega}_{sd}+\displaystyle\sum_{k=1}^{N}P_k(1-\varepsilon_{r_k}^{\alpha=0})\hat{\Omega}_{r_kd}} \tag{40}$$

where $\hat{\Omega}_{sd} = \Omega_{sd} - \alpha_{sd}$ and $\hat{\Omega}_{r_kd} = \Omega_{r_kd} - \alpha_{r_kd}$.

4.2 Amplify-and-Forward Model

From (20), we need to calculate the statistical expectation of the output SNR γ_k^{AF} for each of the cooperative link. With the PDF expressions in (23), we can obtain

$$\begin{aligned}E\{\gamma_k^{AF}\} = &\sum_{i=0}^{m_{sr_k}-1}\sum_{j=0}^{i}\frac{\hat{\eta}_{sr_k}}{\Gamma(m_{r_kd})i!}\binom{i}{j}\left(\frac{C_k}{\rho_k\hat{\eta}_{r_kd}}\right)^{\nu_k} e^{\frac{C_k}{2\rho_k\hat{\eta}_{r_kd}}}\\ &\times\left[\Gamma(i+m_{r_kd}-j+2)\Gamma(i+2)\sqrt{\frac{\hat{\eta}_{r_kd}}{\rho_kC_k}}W_{-\mu_k-3/2,\nu_k}\left(\frac{C_k}{\rho_k\hat{\eta}_{r_kd}}\right)\right.\\ &+\Gamma(i+m_{r_kd}-j+1)\Gamma(i+2)\rho_k^{-1}W_{-\mu_k-1,\nu_k-1/2}\left(\frac{C_k}{\rho_k\hat{\eta}_{r_kd}}\right)\\ &\left.-i\Gamma(i+m_{r_kd}-j+1)\Gamma(i+1)\sqrt{\frac{\hat{\eta}_{r_kd}}{\rho_kC_k}}W_{-\mu_k-1/2,\nu_k}\left(\frac{C_k}{\rho_k\hat{\eta}_{r_kd}}\right)\right]\end{aligned} \tag{41}$$

We have $\rho_k^{\alpha=0} = 1$ and $C_k^{\alpha=0} = P_s\Omega_{sr_k}/N_0 + 1$, when $\alpha_{sd} = \alpha_{r_kd} = \alpha_{sr_k} = 0$. Similarly, by substituting (41) into (20) and (37), and calculating the statistical expectation, we have

$$E\{\Upsilon^{AF}\} = \frac{\dfrac{P_s^2\alpha_{sd}}{N_0(P_s\alpha_{sd}+N_0)}\hat{\Omega}_{sd} + \displaystyle\sum_{k=1}^{N}(E\{\gamma_k^{AF}\}\big|_{\rho_k^{\alpha=0},C_k^{\alpha=0}} - E\{\gamma_k^{AF}\})}{P_s\hat{\Omega}_{sd}/N_0 + \displaystyle\sum_{k=1}^{N}E\{\gamma_k^{AF}\}\big|_{\rho_k^{\alpha=0},C_k^{\alpha=0}}} \tag{42}$$

The expressions of the average SNR gap ratio for DF and AF relay systems in (39) and (42) are very complex. If we consider the i.i.d. case, i.e. $\alpha_{sd} = \alpha_{r_kd} = \alpha_{sr_k} = \alpha$, and $\Omega_{sd} = \Omega_{r_kd} = \Omega_{sr_k} = \Omega$, we can find that the main factors that will impact on the average SNR gap ratio are the transmit powers of each node P_k and the number of relays N. We tend to investigate the effects of these factors on the average SNR gap ratio from numerical results in Sect. 5.

5. Simulation Results and Discussions

This section presents some numerical results showing the impact of channel estimation errors on the system performance of our cooperative communication systems, including the DF and AF cooperative transmission schemes. In order to prove that our analysis is very accurate and effective, we compare the exact one-integral expressions, the approximate formulas and the Monte Carlo simulation results of the SER at the destination. Limitations of SER are also illustrated versus the channel estimation errors' variances. Besides, the average SNR gap ratio caused by channel estimation errors is utilized to characterize and quantify their impact.

Nakagami-m fading channels are employed by all the communication links in our system. Similarly, for the consideration of fairness, we assume a power allocation policy proposed in [12], where $P_s = P/2$ and $P_k = P/(2N)$, and without loss of generality, we define the total power as $P = 1$. The Nakagami-m fading channel parameters are set as $\Omega_{sd} = \Omega_{sr_k} = \Omega_{r_kd} = 1$, $m_{sd} = m_{r_kd} = 1.5$, and $m_{sr_k} = 2$. The variances of the channel estimation errors are assumed the same for all links, that is $\alpha_{sd} = \alpha_{sr_k} = \alpha_{r_kd} = \alpha$.

Figures 2 and 3 show the theoretical and the Monte Carlo simulation results of the SER performance for the DF and AF cooperative communication systems with 4-QAM modulation. Exemplarily, we consider the cooperative communication systems employing one relay node. We assume a simple channel estimation error model, where the errors are independent of the channels, and the variance of the errors varies from 0.01 to 0.1 in Figs. 2 and 3. Besides,

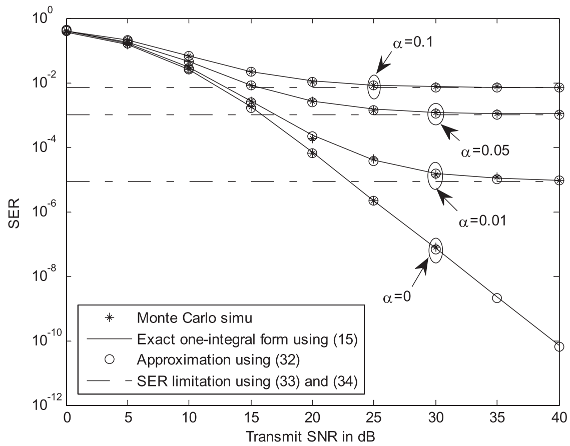

Fig. 2 Theoretical and simulation results for the SER of the DF cooperative system with one relay using 4-QAM signals. The variance of channel estimation errors is fixed as $\alpha \in \{0.1, 0.05, 0.01, 0\}$.

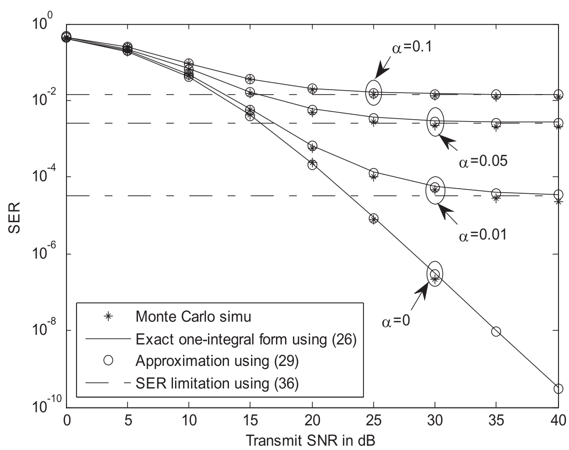

Fig. 3 Theoretical and simulation results for the SER of the AF cooperative system with one relay using 4-QAM signals. The variance of channel estimation errors is fixed as $\alpha \in \{0.1, 0.05, 0.01, 0\}$.

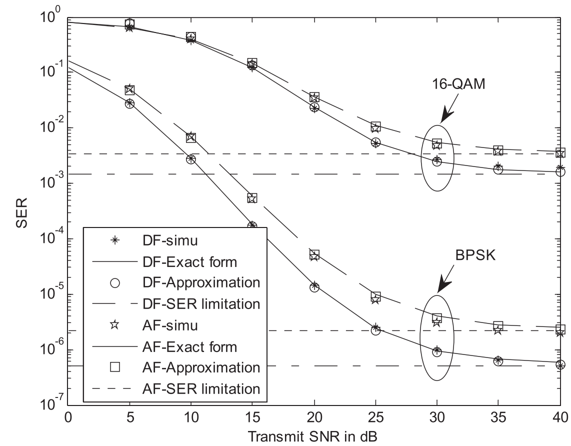

Fig. 4 Theoretical and simulation results of the SER for both DF and AF cooperative system with one relay. Different modulations are considered and $\alpha = 0.01$.

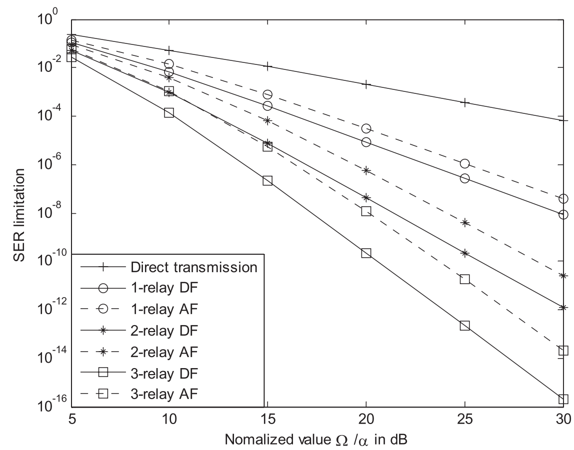

Fig. 5 Limitation of the SER when $P/N_0 \to \infty$ for the DF and AF cooperative systems with different number of relays versus the normalized value Ω/α.

the SER performances with perfect channel estimation are also presented, which degrade dramatically as the channel estimation error variance grows larger. Figure 2 illustrates that the exact SER expressions perfectly match the simulation results, and the approximate expression of the SER is very accurate in the DF relay systems. The limitations of the SER with channel estimation errors when transmit SNR $P/N_0 \to \infty$, are stressed by dashdotted lines. It is clear that the theoretical and simulation results of the SER approach this limitation asymptotically. In Fig. 3, for the AF relay systems, the same investigations are performed, as well as similar conclusion can be obtained. However, there's small difference in the performance between the theoretical analysis and the Monte Carlo simulation results, especially in high SNR regions and larger channel estimation error variances. This is probably because we made a simplification in the output SNR analysis, where we consider a random variable with its statistic average value.

Figure 4 presents the theoretical and the Monte Carlo simulation results of the SER performance for the DF and AF cooperative communication systems with other commonly used modulation types. BPSK and 16-QAM are considered with fixed channel estimation error variance $\alpha = 0.01$. The exact and approximate theoretical results are from Tables 1 and 2. Figure 4 also proves that our analysis is very accurate and matches simulation results perfectly.

Figure 5 shows the limitation of the SER performance with 4-QAM modulation for various values of Ω/α, when the transmit SNR $P/N_0 \to \infty$. Both DF and AF cases are considered against different number of relays, given by (32) and (29), respectively. Intelligibly, as the variance of the channel estimation errors α becomes smaller, the impact of the imperfect channel estimation will be reduced, and the SER performance will become better. Besides, the limitation of the SER will also decrease as more relays participate into the transmission.

In addition to the SER performance, the average SNR gap ratio is also of great interest as it provides an efficient

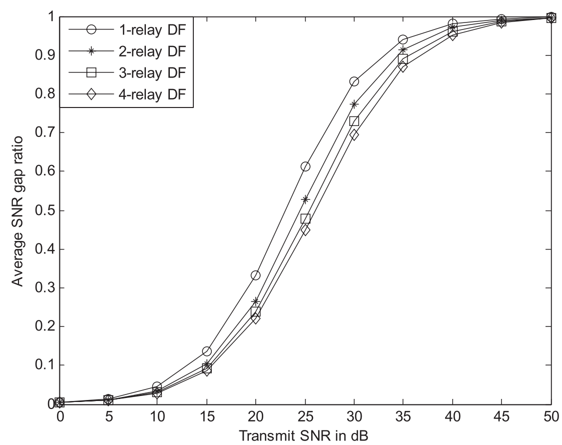

Fig. 6 Average SNR gap ratio of the DF cooperative system with different number of relays versus the transmit SNR. The variance of channel estimation errors is fixed as $\alpha = 0.01$.

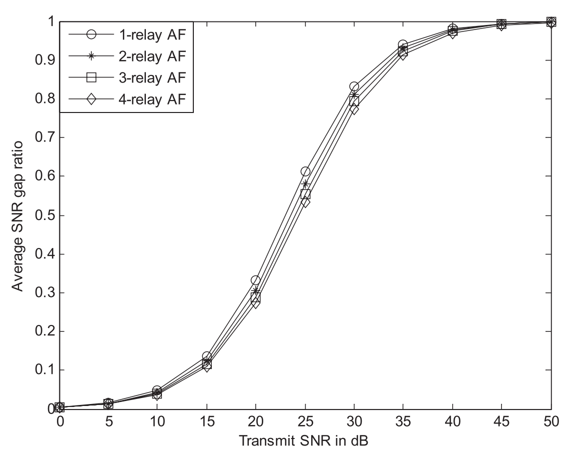

Fig. 7 Average SNR gap ratio of the AF cooperative system with different number of relays versus the transmit SNR. The variance of channel estimation errors is fixed as $\alpha = 0.01$.

way to quantify the impact of channel estimation errors. Figures 6 and 7 depict the average SNR gap ratio versus the transmit SNR P/N_0 for the DF and AF communication scenarios at a fixed error variance $\alpha = 0.01$. We can observe that the average SNR gap ratio increases with the transmit SNR, which is probably because channel estimation errors will dominate the output SNR when the noise variance is small. At high SNR region, the average SNR gap ratio even approaches to 1, which means that the effect of the channel estimation errors is so great that the system noise can be neglected and the SER at the destination may approaches the limitation discussed above. Moreover, increasing the number of relays will reduce the average SNR gap ratio. This is due to the fact that we adopt a power location scheme that as N increases, the channel estimation error portions in (40) and (42), αP_{r_i} and αP_k will be reduced.

In order to demonstrate how much the channel estimation errors influence the average SNR gap ratio, we further investigate it for various values of Ω/α with fixed transmit SNR $P/N_0 = 20$ dB in Fig. 8. Different cooperative com-

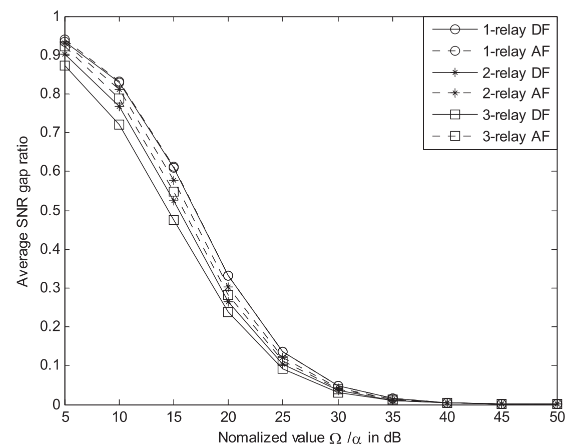

Fig. 8 Comparison of the average SNR gap ratio between the DF and AF cooperative systems with different number of relays versus the normalized value Ω/α. The transmit SNR is fixed as $P/N_0 = 20$ dB.

munication sceneries for DF and AF models are considered for the sake of comparison. Figure 8 shows that the SNR gap ratio will reduce dramatically as α becomes smaller for DF and AF relay systems, which is congruous with the fact, that when the variance of the channel estimation errors becomes smaller, its effect will correspondingly go smaller. Besides, we can find that the results of the average SNR gap ratio are almost the same for the DF and AF models with one relay. When increasing the relay numbers, the average SNR gap ratio for the AF model will overcome that of the DF model, which means that the DF model is less susceptible to the channel estimation errors compared with the AF model. Since errors are simultaneously amplified with the relay signals in the AF model, the effect of the channel estimation errors will be deteriorated and the average SNR gap ratio will be larger than that of the DF model.

6. Conclusion

In this paper, we have investigated the impact of channel estimation errors on the DF and AF cooperative communication systems over Nakagami-m fading channels. The SER and the average SNR gap ratio are utilized to demonstrate the effect of the imperfect channel estimation. We derive the exact one-integral, the approximate and the limitation of the SER expressions for both the DF and AF relay systems. Numerical simulation results have verified that our theoretical analysis is accurate and efficient. Besides, the average SNR gap ratios for the two types of relay systems are also presented to quantify the impact of the channel estimation errors. Results lead us to the conclusion that DF model is less susceptible to channel estimation errors than AF model and the average SNR gap ratio of DF model is smaller. Furthermore, the SER and the average SNR gap ratio results revealed that the channel estimation errors have a large effect on the cooperative system and deserve more attention when designing the system in realistic wireless networks.

Acknowledgments

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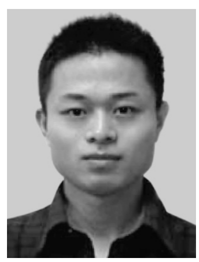

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