PAPER IEICE/IEEE Joint Special Section on Autonomous Decentralized Systems Technologies and Their Application to Networked Systems Decentralized Coverage Control for Multi-Agent Systems with Nonlinear Dynamics

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SUMMARY In this paper, we study the decentralized coverage control problem for an environment using a group of autonomous mobile robots with nonholonomic kinematic and dynamic constraints. In comparison with standard coverage control procedures, we develop a combined controller for Voronoi-based coverage approach in which kinematic and dynamic constraints of the actual mobile sensing robots are incorporated into the controller design. Furthermore, a collision avoidance component is added in the kinematic controller in order to guarantee a collision free coverage of the area. The convergence of the network to the optimal sensing configuration is proven with a Lyapunov-type analysis. Numerical simulations are provided approving the effectiveness of the proposed method through several experimental scenarios.

key words: networked control systems, sensor networks, coverage problem, nonholonomic mobile robots, collision avoidance

1. Introduction

Sensor networks have a broad application in environmental sampling, ecosystem monitoring, and military surveillance. One of the fundamental problems of multi-robot mobile sensor networks is how to deploy a group of autonomous robots to spread out over an environment in order to monitor some quantity of interest over an area, which is called coverage control. Researchers have proposed various solutions to a lot of interesting sensor network coverage problems. In [1] coverage controllers are categorized in three common kinds, a Voronoi controller, which is geometric in nature, a minimum variance controller, which has a probabilistic interpretation, and a potential field controller.

In [2], Du et. al. introduce the centroidal Voronoi tessellations as a comprehensive solution to a series of partition problems which sheds light on the sensor coverage problem, and also provide some of several centralized algorithms under deterministic and probabilistic domains. Cortes et. al. [3] propose a decentralized control law for multi-robot coverage of an area partitioned into Voronoi diagram, in the sense that continually driving the robots toward the centroids of their Voronoi cells. A recent text that presents much of this work in a cohesive fashion is [4] and an excellent overview is given in [5]. Different extensions of the framework devised in [3] have been proposed in the literature. In [6] the problem of limited-range interaction between agents was addressed. In [7] the basic approach was extended to deal with the agents with limited energy. The problem of the online learning of the distribution density function, while moving toward the optimal locations, was addressed in [8], [9]. In [1] the authors propose a cost function form for coverage problems that can be specialized to fit different distributed sensing and actuation scenarios. The cost function is shown to subsume several different kinds of existing coverage cost functions. There has been another extension to heterogeneous groups of finite size robots and non-convex environments in [10].

Standard approaches to Voronoi based coverage control assume simple integrator dynamics for the robots, yielding the ability of traversing both smooth and non-smooth trajectories for robots. They do not address the kinematic and dynamic constraints of physical nonholonomic mobile robots in developing coverage algorithms. However, most of the actual robots such as differential drive ones suffer from kinematic nonholonomic constraints confining the plausible motions of the robot. Once an appropriate feedback velocity control inputs are designed for kinematic steering system, one should take into account the specific dynamic vehicles to convert a steering system command into control inputs for the actual vehicle [11].

Stabilization and tracking control of nonholonomic mobile robots has been a subject of intense research in the past years [12]–[14]. Many approaches have been proposed to address this issue of nonholonomic stabilization. As pointed out by Kim and Tsiotras [15], the majority of nonholonomic control laws are based on kinematic models [16]–[18]. Stabilization of dynamic models for nonholonomic systems has also been addressed in [19]–[21]. A popular way of implementing a kinematic control law to a dynamic nonholonomic system is by backstepping the velocity control commands to acceleration input [23]. Backstepping has been used in translating kinematic controllers into equivalent dynamic ones in [11], [24].

In this paper, we extend the contributions in kinematic and dynamic control of single nonholonomic mobile robots to the Voronoi-based locational optimization framework introduced in [3], and propose a decentralized control law with the aim of coverage control problem. After including a kinematic velocity controller in the coverage problem, we seek to incorporate the dynamics of the robots into the coverage controller design based on the backstepping approach of [11]. Using Lyapunov stability theory, we prove that the

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control law causes the network to converge to an optimal sensing configuration. Then we propose a collision avoidance component be incorporated with the kinematic controller in order for the robots to avoid collision among them, and prove that the robots approach to their optimal configuration with bounded error.

The remainder of the paper is organized as follows. We describe problem setup along with some background on nonholonomic mobile robots and locational optimization problem in Sect. 2. In Sect. 3, we first present the proposed kinematic controller and prove its stability. Then we add a collision avoidance component so that no two robots get too close to each other. In Sect. 4 we present the proposed dynamic controller. Numerical simulation results are given in Sect. 5. Finally, we conclude the paper in Sect. 6.

2. Problem Setup

Consider we want to deploy a group of N nonholonomic mobile agents in a bounded, convex environment $\mathcal{D} \subset \mathbb{R}^2$. In the following, we first describe the characteristics of the sensing mobile robots, and then depict the Voronoi based coverage approach with some background on locational optimization problem.

2.1 Nonholonomic Mobile Agents

Let each of the agents be a two-wheeled mobile robot moving on a horizontal plane as shown in Fig. 1. Let $q_i \in Q \subset \mathbb{R}^3$, $i \in \mathcal{N} = [1, ..., N]$ be the configuration of the *i*'th robot described by generalized coordinates in the global frame as

$$q_i = \left[p_i^T, \theta_i\right]^T \tag{1}$$

where $p_i = (x_i, y_i) \in \mathcal{D}$ is the position of the point *C* of the *i*'th robot in the global coordinate frame O, X, Y and $\theta_i \in (-\pi, \pi]$ is the orientation of that, measured from X-axis of that frame. Each vehicle is subjected to nonholonomic kinematic constraints which can be expressed as:

$$A(q_i)\dot{q}_i = 0 \tag{2}$$

where $A(q_i) \in \mathbb{R}^{1 \times 3}$. Then the kinematic model for each robot can be written in the form of:

$$\dot{q}_i = S(q_i)\mathbf{v}_i \tag{3}$$

where, $S(q_i)$ is a matrix consisting of a set of linearly independent vector fields spanning the null space of $A(q_i)$,



Fig.1 Nonholonomic mobile robot configuration.

 $\mathbf{v}_i = [v_i, \omega_i]^T$ with $\omega_i = \dot{\theta}_i$ (the angular velocity) and $v_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2}$ (the linear velocity) of the *i*'th robot. It is easy to verify that the kinematic equations of motion of point C_i in terms of its linear velocity and angular velocity are [25]

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$
(4)

The Lagrange formalism is used to find the dynamic equations of the mobile robots. The dynamical equations of an n-dimensional mobile robot can be expressed in the matrix form [25]:

$$M_{i}(q_{i}) \ddot{q}_{i} + V_{m_{i}}(q_{i}, \dot{q}_{i}) \dot{q}_{i} + F_{i}(\dot{q}_{i}) + G_{i}(q_{i}) = B_{i}(q_{i}) \tau_{i} - A^{T}(q_{i}) \lambda_{i}$$
(5)

where $M(q_i) \in \mathbb{R}^{n \times n}$ is a symmetric, positive definite inertia matrix, $V_m(q_i, \dot{q}_i)$ is the centripetal and Coriolis matrix, $F_i(\dot{q}_i) \in \mathbb{R}^{n \times 1}$ denotes the surface friction, $G(q_i) \in \mathbb{R}^{n \times 1}$ is the gravitational vector, $B_i(q_i) \in \mathbb{R}^{n \times 1}$ is the input transformation matrix, $\tau_i \in \mathbb{R}^{n \times 1}$ is the input vector, and $\lambda_i \in \mathbb{R}^{m \times 1}$ is the vector of constraint forces.

One can rewrite the dynamic equations of the mobile base (i.e. (5)) by differentiating (3), substituting the result in (5), and multiplying by S^T as [11]:

$$\bar{M}_i(q_i)\,\dot{\mathbf{v}}_i + \bar{V}_{m_i}(q_i,\dot{q}_i)\,\mathbf{v}_i + \bar{F}_i(\dot{q}_i) = \bar{B}_i(q_i)\,\tau_i \tag{6}$$

$$\bar{\tau}_i = \bar{B}_i \tau_i \tag{7}$$

the parameters of which for the mobile base in Fig. 1 can be obtained as [11]:

$$\bar{M}_{i} = \begin{bmatrix} m_{i} & 0 & 0\\ 0 & m_{i} & 0\\ 0 & 0 & I_{i} \end{bmatrix}, \ \bar{B}_{i} = \frac{1}{r_{w_{i}}} \begin{bmatrix} 1 & 1\\ L_{i} & -L_{i} \end{bmatrix} \begin{bmatrix} \tau_{R_{i}}\\ \tau_{L_{i}} \end{bmatrix},$$
$$\bar{V}_{m_{i}} = 0, \tag{8}$$

in which m_i and I_i represent the mass and inertia of the robot, and L_i and r_{w_i} are shown in Fig. 1. It should be underlined that the matrix $\dot{M}_i - 2\bar{V}_{m_i}$ has a skew-symmetric property [11].

Once the desired velocity control inputs for the kinematic model, denoted by $\mathbf{v}_{d_i} = [v_{d_i}, \omega_{d_i}]^T$, are obtained, one should convert \mathbf{v}_{d_i} to the control torque inputs τ_i in order to incorporate the dynamics of the physical mobile robot platforms.

2.2 Locational Optimization

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Let an arbitrary point in \mathcal{D} be denoted by \tilde{p} . Let $\mathcal{V} = \{V_1, \ldots, V_N\}$ be the Voronoi partition of \mathcal{D} , for which the robots positions are the generator points. Specifically,

$$V_i = \left\{ \tilde{p} \in \mathcal{D} \mid \|\tilde{p} - p_i\| \leq \left\|\tilde{p} - p_j\right\|, \, \forall j \neq i \right\}$$
(9)

Let the unreliability of the sensor measurement be denoted by a quadratic function f(x) specifically, $f(||\tilde{p} - p_i||) =$ $1/2(||\tilde{p} - p_i||)^2$ describes how unreliable is the measurement of the information at \tilde{p} by a sensor at p_i . This form of $f(||\tilde{p} - p_i||)$ is physically appealing since it is reasonable that sensing will become more unreliable farther from the sensor [26]. Define the sensory function to be a continuous function $\phi : \mathcal{D} \to \mathbb{R}^+$ (where \mathbb{R}^+ is the set of strictly positive real numbers). The sensory function should be thought of as a weighting of importance over \mathcal{D} . As a measure of the system performance, we define the coverage functional as follows

$$\mathcal{H}(p_1, \dots, p_N) = \sum_{i=1}^n \int_{V_i} \frac{1}{2} \|\tilde{p} - p_i\|^2 \phi(\tilde{p}) d\tilde{p}$$
(10)

Qualitatively, a low value of \mathcal{H} corresponds to a good configuration for sensory coverage of the environment \mathcal{D} . The mass, first moment, and centroid of a Voronoi region V_i are respectively defined as

$$M_{V_{i}} = \int_{V_{i}} \phi(q) \, dq, \ L_{V_{i}} = \int_{V_{i}} q\phi(q) \, dq,$$

$$C_{V_{i}} = \frac{L_{V_{i}}}{M_{V_{i}}}.$$
 (11)

A standard result from locational optimization [3] is that

$$\frac{\partial \mathcal{H}}{\partial p_i} = -\int_{V_i} (q - p_i)\phi(q) \, dq = -M_{V_i}(C_{V_i} - p_i) \qquad (12)$$

Equation (12) implies that critical points of \mathcal{H} correspond to the configurations such that $p_i = C_{v_i}$ for all *i*, that is, each agent is located at the centroid of its Voronoi region. This brings us to the concept of optimal coverage as follows: A robot network is said to be in a (locally) optimal coverage configuration if every robot is positioned at the centroid of its Voronoi region, $p_i = C_{v_i}$ for all *i* [9].

3. Coverage with Nonholonomic Agents: Kinematic Control

In this section we investigate the decentralized coverage control for a group of nonholonomic mobile robots. Before we proceed, let suppose that the following assumptions hold:

Assumption 1: Every robot has complete knowledge of its own dynamics.

Assumption 2: The robots have the ability to compute their own Voronoi partitions in a distributed manner as given in [3].

Assumption 3: The robots have point dimensions, although they obey nonholonomic constraints. (This assumption will be relaxed in Sect. 3.1).

In order to design a kinematic controller, we also need to presume the following assumption, which will be relaxed in Sect. 4.

Assumption 4: Perfect velocity tracking holds such that $\mathbf{v}_i = \mathbf{v}_{d_i}, \forall i \in \mathcal{N}$



Fig. 2 Position and orientation errors in partition *i*.

Define position errors for the *i*'th robot as

$$x_{e_i} = x_i - C_{V_i,x} \tag{13}$$

$$y_{e_i} = y_i - C_{V_i,y} \tag{14}$$

the desired orientation of motion for $(x_{e_i}, y_{e_i}) \neq (0, 0)$ as

$$\theta_{d_i} = Atan2(-y_{e_i}, -x_{e_i}) \tag{15}$$

and the orientation error as

$$e_{\theta_i} = \theta_i - \theta_{d_i} \tag{16}$$

The kinematic error dynamics can be written independent of the inertial coordinate frame by Kanayama transformation [27]:

$$\begin{bmatrix} e_{x_i} \\ e_{y_i} \end{bmatrix} = \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} x_{e_i} \\ y_{e_i} \end{bmatrix}$$
(17)

where e_{x_i} and e_{y_i} are the error variables in mobile coordinate system which is attached to the *i*'th robot (Fig. 2).

We propose the following auxiliary velocity control law for the *i*'th robot:

$$v_{d_i} = -k_{v_i}\rho_i \cos(e_{\theta_i}) \tag{18}$$

$$\omega_{d_i} = -k_{\omega_i} e_{\theta_i} \tag{19}$$

where $\rho_i = \sqrt{e_{x_i}^2 + e_{y_i}^2}$, and k_{v_i} and k_{ω_i} are positive scalar gains.

Theorem 1: Consider a group of N nonholonomic mobile robots whose kinematic models are described by (4). Let the assumptions (1) through (4) hold. Under control laws (18) and (19), it is guaranteed that the whole system is asymptotically stable and the robots positions converge to a centroidal Voronoi configuration.

Proof: Consider the Lyapunov function candidate as $V = \mathcal{H}$. The time derivative of V along the trajectories of the error dynamics then can be obtained as follows

$$\dot{\mathcal{H}} = \sum_{i=1}^{N} \frac{\partial \mathcal{H}}{\partial q_i} \dot{q}_i$$

Using (12) and the fact that $\frac{\partial H}{\partial \theta_i} = 0$ one can write

$$\dot{\mathcal{H}} = \sum_{i=1}^{N} \frac{\partial \mathcal{H}}{\partial p_i} \dot{p}_i = \sum_{i=1}^{N} M_{V_i} (p_i - C_{V_i}) \dot{p}_i$$
$$= \sum_{i=1}^{N} M_{V_i} \mathbf{v}_i [x_{e_i} \quad y_{e_i}] [\cos \theta_i \quad \sin \theta_i]^T$$

$$= -\sum_{i=1}^{N} k_{v_i} \rho_i \cos(e_{\theta_i}) e_{x_i}$$

Using the fact that $\cos(e_{\theta_i}) = \frac{e_{x_i}}{\rho_i}$ one can conclude that

$$\dot{\mathcal{H}} = -\sum_{i=1}^{N} k_{v_i} \rho_i^2 \cos^2(e_{\theta_i})$$
⁽²⁰⁾

which is clearly non-positive. Due to the convexity of the region \mathcal{D} , each of the Voronoi centroids C_{V_i} lies in the interior of the *i*'th partition so in the interior of the region \mathcal{D} . Since control law (18) provides the robots with bidirectional linear velocities, the robots always move toward the interior of the region \mathcal{D} and never leave it. Therefore, \mathcal{D} is a positive invariant set for the trajectories of the closed loop system. Since this set is closed and bounded, one can make use of LaSalle's invariance principle to infer that the robots positions converge to the largest invariant subset of the set $S = \{(\rho_i = 0) \lor (\cos e_{\theta_i} = 0), \forall i \in \mathcal{N}\}$. For each robot, in the case that $\cos e_{\theta_i} = 0$ and e_{x_i} or $e_{y_i} \neq 0$, according to (19), $|\omega_i| = \pi/2$, so the set {cos $e_{\theta_i} = 0$ } is a non-invariant set except the case that the *i*'th robot is located on the centroid of its Voronoi partition. On the other hand, $\rho_i = 0$ only if both e_{x_i} and e_{y_i} are equal to zero. Therefore, the largest invariant set contained in S is the set $I = \{e_{x_i} = e_{y_i} = 0, \forall i \in \mathcal{N}\}$. Moreover, for every invariant set in *I*, it should be $\omega_i = 0$ which in turn yields $\theta_i = 0$. Therefore, under control laws (18) and (19), the closed loop system is asymptotically stable and the robots positions converge to the centroidal Voronoi configuration.

Remark 1: Convergence of θ_i to θ_{d_i} can be made arbitrarily exponentially fast by the selection of [28]:

$$\omega_i = -k_{\omega_i} e_{\theta_i} + \theta_{d_i} \tag{21}$$

where

$$\dot{\theta}_{d_i} = \frac{e_{x_i} \dot{e}_{y_i} - \dot{e}_{x_i} e_{y_i}}{\rho^2},$$
(22)

which results in

$$\dot{\theta}_i - \dot{\theta}_{d_i} = -k_{\omega_i}(\theta_i - \theta_{d_i}). \tag{23}$$

One can also make use of a sufficiently smooth estimate of $\dot{\theta}_{d_i}$, namely $\hat{\theta}_{d_i}$, which can be computed using the following estimations of \dot{e}_{x_i} and \dot{e}_{y_i} :

$$\hat{e}_{x_i} = \frac{e_{x_i}(t + \Delta t) - e_{x_i}(t)}{\Delta t}$$
(24)

$$\hat{e}_{y_i} = \frac{e_{y_i}(t + \Delta t) - e_{y_i}(t)}{\Delta t}$$
(25)

for some small $\Delta t > 0$ [22].

3.1 Collision Avoidance

Now we deal with the coverage problem for nonholonomic robots with finite size dimensions. The purpose is to design



Fig.3 Avoidance (radius 2r), repulsion (radius R_r), and detection (radius R_d) regions around the robot *i*.

a control law so that the robots approach their centroidal Voronoi configuration, while avoiding collision with each other. We assume that the robots all have circular shapes. For simplicity, we also assume that all robots have a common radius $r_i = r$; however, the proposed method is applicable to heterogeneous robots with different disk radii. If the robots start from a safe configuration, a sufficient condition to guarantee collision avoidance is that the center of each robot disk with radius r, maintains at least at a distance of 2r with respect to the center of the other robot disks. Hence, we deal with a constrained locational optimization problem by addressing a modified version of the cooperative avoidance function for robot i:

$$V_{a_i} = \sum_{j \in \mathcal{N}_i} V_{a_{ij}} \tag{26}$$

where

$$V_{a_{ij}} = \left(\min\left\{0, \frac{d_{ij}^2 - R_r^2}{d_{ij}^2 - 4r^2}\right\}\right)^2$$
(27)

in which $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, and r > 0, $2r < R_r < R_d$, and $R_d > 2r$ are the radii of the avoidance, repulsion, and detection regions for the *i*'th robot, respectively, depicted in Fig. 3. Accordingly one can define the set of *repulsing neighbors* for the robot *i* as $N_{r,i} = \{j \in N : ||p_j - p_i|| \le R_r\}$.

Now defining

$$X_{e_i} = x_{e_i} + \sum_{j \in N_i} \frac{\partial V_{aij}}{\partial x_i},$$
(28)

$$Y_{e_i} = y_{e_i} + \sum_{j \in N_i} \frac{\partial V_{aij}}{\partial y_i},$$
(29)

where

$$\frac{\partial V_{a_{ij}}}{\partial x_i} = \begin{cases} 0 & \text{if } d_{ij} \ge R_r, \\ 4 \frac{(R_r^2 - 4r^2)(d_{ij}^2 - R_r^2)}{(d_{ij}^2 - 4r^2)^3} (x_i - x_j), & \text{if } 2r \le d_{ij} \le R_r, \\ 0 & \text{if } 2r \le d_{ij} \le R_r. \end{cases}$$

and

$$\frac{\partial V_{a_{ij}}}{\partial y_i} = \begin{cases} 0 & \text{if } d_{ij} \ge R_r, \\ 4 \frac{(R_r^2 - 4r^2)(d_{ij}^2 - R_r^2)}{(d_{ij}^2 - 4r^2)^3} (y_i - y_j), & \text{if } 2r \le d_{ij} \le R_r, \\ 0 & \text{if } 2r \le d_{ij} \le R_r, \end{cases}$$

One can rewrite the error variables in mobile coordinate system as

$$\begin{bmatrix} E_{x_i} \\ E_{y_i} \end{bmatrix} = \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} X_{e_i} \\ Y_{e_i} \end{bmatrix}$$
(30)

and propose the following auxiliary velocity control law for the *i*'th robot:

$$v_{d_i} = -k_{v_i} \rho_i \cos(E_{\theta_i}) \tag{31}$$

$$\omega_{d_i} = -k_{\omega_i} E_{\theta_i} \tag{32}$$

where $\rho_i = \sqrt{E_{x_i}^2 + E_{y_i}^2}$, and $E_{\theta_i} = \theta_i - Atan^2 (-Y_{e_i}, -X_{e_i})$.

Theorem 2: Consider a group of N nonholonomic mobile robots whose kinematic models are described by (4). Let the assumptions (1), (2), and (4) hold. Under control laws (31) and (32), it is guaranteed that 1) the robots positions become ultimately bounded, i.e. they approach neighbourhoods of centroidal Voronoi configuration, and 2) the collision among the robots is avoided.

Proof: consider the following Lyapunov like function:

$$V = \sum_{i=1}^{N} \left[\int_{V_i} \frac{1}{2} (||\tilde{p} - p_i||)^2 \phi(\tilde{p}) d\tilde{p} + V_{ai} \right]$$

The time derivative of V along the trajectories of the error dynamics will then be

$$\dot{V} = -\sum_{i=1}^{N} \left[k_{v_i} \rho_i \cos(e_{\theta_i}) e_{x_i} + \frac{\partial V_{ai}}{\partial x_i} \dot{x}_i + \frac{\partial V_{ai}}{\partial y_i} \dot{y}_i \right].$$
(33)

In the case that $\mathcal{N}_{r,i} = \emptyset, \forall i \in \mathcal{N}$ one can see that $\frac{\partial V_{ai}}{\partial x_i} = \frac{\partial V_{ai}}{\partial y_i} = 0, \forall i$, and (33) reduces to (20); Therefore, \dot{V} is non-positive when all the robots are outside the repulsion region of each other. Now consider the case that for some robot *i*, there exists at least one robot *j* such that $||p_i - p_j|| \le R_r$, so the partial derivatives of repulsive avoidance function V_{ai} would be nonzero. In this case, if $k_{v_i}\rho_i \cos(e_{\theta_i})e_{x_i} > -\frac{\partial V_{ai}}{\partial x_i}\dot{x}_i - \frac{\partial V_{ai}}{\partial y_i}\dot{y}_i$ holds, the *i*'th component of \dot{V} would be non-positive. If this holds for all $i \in \mathcal{N}$, \dot{V} is non-positive. Now consider the case that for some *i*, $k_{v_i}\rho_i \cos(e_{\theta_i})e_{x_i} \leq -\frac{\partial V_{ai}}{\partial x_i}\dot{x}_i - \frac{\partial V_{ai}}{\partial y_i}\dot{y}_i$, in which the robot *i* tries to approach the centriod, while robot *j* pushes it away from its related centriod. In this situation, at the worst case for robot *i* where the centriod of V_i lies inside the semicircle region Λ_{ii}^{1} (See Fig. 4), the Euclidean norm of the position error will be a value less than $\delta = r + R_r/2$, according to the fact that the Voronoi centriod for each partition, lies in the interior of that partition. Note that although there exists a unique distance $\delta_i(q_i, q_{n_1}, \dots, q_{n_k}, R_r, k_i), n_i \in N_{r,i}$ at which $k_{v_i}\rho_i \cos(e_{\theta_i})e_{x_i}$ equals $-\frac{\partial V_{ai}}{\partial x_i}\dot{x}_i - \frac{\partial V_{ai}}{\partial y_i}\dot{y}_i$, the value of δ_i depends on the number of all repulsing neighbors $N_{r,i}$ and cannot be obtained straightforward. However, the most conservative value is δ . According to Fig. 4 if the centroid lies outside the semicircle region Λ_{ii}^{1} , robot *i* will move away from robot j according to bidirectional linear velocities, and



Fig. 4 The worst case for the position error.

the position error will decrease and even approach to zero if no other robot lies in the avoidance region of the robot *i*. Concluding the above discussion, one can see that provided that for all $i \in N$, $||[e_{x_i}, e_{y_i}]^T|| < \delta$, \dot{V} is non-positive and the proof completes.

Remark 2: As it can be seen in the proof, with the use the proposed kinematic controller for coverage with collision avoidance, the norm of the position error with respect to the centroid for each robot is bounded by $\delta = r + R_r/2$.

4. Coverage with Nonholonomic Agents: Dynamic control

Now we consider the case that the perfect velocity tracking assumption (assumption 4) does not hold. Considering \mathbf{u}_i as an auxiliary input, a suitable control input for velocity following is given by the computed-torque nonlinear feedback control input [11]

$$\bar{\tau}_i = \bar{B}_i^{-1}(\bar{M}_i(q_i)\mathbf{u}_i + \bar{V}_{m_i}(q_i, \dot{q}_i)\mathbf{v}_i + \bar{F}_i(\mathbf{v}_i)), \tag{34}$$

which converts the dynamic control problem into:

$$\dot{q}_i = S(q_i)\mathbf{v}_i \tag{35}$$

$$\dot{\mathbf{v}}_i = \mathbf{u}_i. \tag{36}$$

One can define the auxiliary velocity error as:

$$e_{d_i} = \mathbf{v}_{d_i} - \mathbf{v}_i. \tag{37}$$

which can be written as

$$e_{d_i} = \begin{bmatrix} e_{d_{i,1}} \\ e_{d_{i,2}} \end{bmatrix} = \begin{bmatrix} v_i + k_{v_i}\rho_i\cos(e_{\theta_i}) \\ \omega_i + k_{\omega_i}e_{\theta_i} \end{bmatrix}$$
(38)

Differentiating (38) and using (6) and (7), one can write the mobile robots dynamics in terms of velocity tracking error and its derivative:

$$\bar{M}_{i}(q_{i})\dot{e}_{d_{i}} = -\bar{V}_{m_{i}}(q_{i},\dot{q}_{i})e_{d_{i}} - \bar{\tau}_{i} + f_{i}(\mathbf{x}_{i})$$
(39)

where

$$f_i(\mathbf{x}_i) \stackrel{\Delta}{=} \bar{M}_i(q_i) \, \dot{\mathbf{v}}_{d_i} + \bar{V}_{m_i}(q_i, \dot{q}_i) \, \mathbf{v}_{d_i} + \bar{F}_i(\dot{q}_i) \tag{40}$$

is the nonlinear mobile robot function and the vector \mathbf{x}_i is defined as $\mathbf{x}_i = [\mathbf{v}_i^T \quad \mathbf{v}_{d_i}^T \quad \mathbf{v}_{d_i}^T]^T$. Proposing the auxiliary nonlinear control input to be

$$\mathbf{u}_i = \dot{\mathbf{v}}_{d_i} + K_i e_{d_i} \tag{41}$$

where K_i is a positive definite and diagonal matrix defined by $K = kI_2$, one can obtain the following torque input for the *i*'th robot

$$\tau_i = \bar{B}_i^{-1}(\bar{M}_i(q_i)Ke_{d_i} + f_i(\mathbf{x}_i)).$$
(42)

Theorem 3: Consider a group of *N* nonholonomic mobile robots whose kinematic and dynamic models are described through (4), (6) and (7). Let the assumptions (1) through (3) hold. Under control laws (18), (19) and (42), it is guaranteed that the whole system is asymptotically stable and the robots positions converge to a centroidal Voronoi configuration.

Proof: Pick the candidate Lyapunov function as

$$V = \mathcal{H} + \frac{1}{2} e_{d_i}{}^T \bar{M}_i e_{d_i}.$$

Differentiating V results in

$$\dot{V} = \dot{\mathcal{H}} + \frac{1}{2} e_{d_i}{}^T \dot{\bar{M}}_i e_{d_i} + e^T{}_{d_i} \bar{M}_i \dot{e}_{d_i}$$
(43)

 \mathcal{H} is shown to be non-positive in (20). Substituting (42) into (39) results in the closed loop error dynamics as

$$\bar{M}_i(q_i)\dot{e}_{d_i} = -(\bar{M}_i(q_i)K + \bar{V}_{m_i}(q_i, \dot{q}_i))e_{d_i}.$$
(44)

Substituting (44) into (43) and considering the skew symmetric property mentioned in Sect. 2, one can write:

$$\dot{V} = \dot{\mathcal{H}} - e_{d_i}{}^T (\bar{M}_i K) e_{d_i} \tag{45}$$

Since $-e^{T}_{d_{i}}(\bar{M}_{i}K)e_{d_{i}} \leq 0$ is negative semi-definite, considering the same argument as the preceding theorem, one can deduce that the closed loop system is asymptotically stable and the position and velocity errors asymptotically converge to the set $I = \{e_{x_{i}} = e_{y_{i}} = e_{d_{i,1}} = e_{d_{i,2}} = 0, \forall i \in N\}$.

5. Simulation Results

The proposed decentralized coverage controller has been demonstrated via numerical simulations in Matlab environment. A team of 20 mobile robots is waiting to be deployed into a $2 \text{ m} \times 2 \text{ m}$ square environment. The robots in the network start their motion from random initial positions with the angle of $\theta_{i_0} = \pi/2$. The Matlab numerical solver ode45 have been used to integrate the equations of motion of the group of robots, and the spatial integrals in (11) required for the computation of the centroids have been computed by discretizing each Voronoi region and summing contributions of the integrand over the grid. Voronoi regions can be computed using a decentralized algorithm similar to that of [3]. The parameters for the mobile robots are selected as $m_i = 1 \text{ kg}, I_i = 0.5 \text{ kg} - m^2, r_{w_i} = 0.03 m, L_i = 0.85 r_i$, and the ones for the controllers are selected as $k_{v_i} = 3$, $k_{\omega_i} = 6$, and $K_i = 10$.

The simulations are carried out via three scenarios with two examples for each one. In the first example of each scenario, the robots are to be deployed in an environment



Fig. 5 Scenario 1, example 1, (a) initial configuration, (b) sensory function, (c) robot trajectories and final configuration, and (d) mean square error.

with a Gaussian sensory function,

$$\phi(\tilde{p}) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\tilde{p}-\mu)^2}{2\sigma^2}\right)$$
(46)

where $\mu = (1, 1)^T$, $\sigma = 0.2$. In the case of the second example, a bimodal Gaussian distribution function is considered as

$$\phi(\tilde{p}) = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp\left(-\frac{(\tilde{p} - \mu_1)^2}{2\sigma_1^2}\right) + \frac{1}{\sigma_2 \sqrt{2\pi}} \exp\left(-\frac{(\tilde{p} - \mu_2)^2}{2\sigma_2^2}\right)$$
(47)

the parameters of which are selected as $\mu_1 = (1/3, 1/3)^T$, $\mu_2 = (5/3, 5/3)^T$, $\sigma_1 = \sigma_2 = 0.18$. In the first scenario, the physical (avoidance region) radii for all of the robots are considered as r = 0.08 m, and the kinematic and dynamic controllers (18), (19), and (42) are applied for each robot. Therefore, no collision avoidance strategy here is applied. The positions of the robots through the evolution in the first scenario are shown in Fig. 5 and Fig. 6. Figure 5 (a) depicts the initial configuration of the robots, and Fig. 5 (b) presents the sensory function ϕ for the first example; Fig. 5 (c) shows the initial and final configuration of the robots as well as the trajectories of them during the simulation run of the first example. Figure 5 (d) portrays the mean square error of the positions of all robots with respect to their optimal centroidal configuration. Figures 6 (a), (b), (c), and (d), depict the same properties but through the simulation of the second example. The centers of the contributing Gaussian functions and the centroids of the Voronoi partitions are marked with red o's and blue \times 's, respectively. The performance of the proposed controller is clearly demonstrated in the simulation results. It is clear that in both examples, the robots converge to their optimal configuration and the mean square error reaches zero at a finite time. Also, inasmuch as the



Fig.6 Scenario 1, example 2, (a) initial configuration, (b) sensory function, (c) robot trajectories and final configuration, and (d) mean square error.



Fig. 7 Scenario 2, robot trajectories and final configuration of (a) the first and (b) the second example.

size of the robots with respect to the area are selected small enough, no collision occurs although no avoidance control is enforced on the system. In the second scenario the same simulation set-up as the first one is taken into consideration with the difference that the physical radii of the robots are considered as r = 0.13 m. Here, the same controllers are applied (with no collision avoidance component). The trajectories of the robots positions together with the final configuration in this scenario are shown in Figs. 7 (a), and (b), for the first and second example, respectively. It is seen that the robots collide with each other since there exists no avoidance control. In the last scenario, we simulate the previous set-up with the kinematic controller in (31) and (32), designed for collision-free coverage control and dynamic control law (42). The repulsion and detection regions radii are selected as $R_r = 1.3r$, $R_d = 3r$, respectively. The trajectories of the robots as well as the final configuration and the mean square position errors of all the robots for the first example are presented in Fig. 8 (a) and (b), and for the second example in Figs. 8 (c) and (d), respectively. The plots clearly demonstrate the effect of the incorporation of the collision avoidance component to the controller. While the robots do not collide with each other, the mean square error of the robots positions with respect to the optimal configuration



Fig.8 Scenario 3, example 1, (a) robot trajectories and final configuration (b) mean square error, and example 2, (c) robot trajectories and final configuration (d) mean square error.

maintains at a non-zero level, which illustrates the trade-off between coverage and collision avoidance.

6. Conclusion and Future Work

In this study, we first considered the decentralized coverage problem of autonomous mobile sensing robots subject to nonholonomic kinematic and dynamic constraints, and introduced an extension to the standard Voronoi-based coverage problem for single integrator agents with point dimensions. A Lyapunov based stability was used to investigate the converge of the robots to their centroidal Voronoi configuration. The proposed method is decentralized in the sense of both coverage and collision avoidance, and it has been successfully verified in numerical simulations. Future work will focus on the extension of the proposed controller for coverage in unknown environments.

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