A Time and Situation Dependent Semantics for Ontological Property Classification*

Ken KANEIWA[†a)] *and* Riichiro MIZOGUCHI[††b)], *Members*

SUMMARY This paper proposes a new semantics that characterizes the time and/or situation dependencies of properties, together with the ontological notion of existential rigidity. For this purpose, we present order-sorted tempo-situational logic (OSTSL) with rigid/anti-rigid sorts and an existential predicate. In this logic, rigid/anti-rigid sorted terms enable the expressions for sortal properties, and temporal and situational operators suitably represent the ontological axioms of existential rigidity and time and/or situation dependencies. A specific semantics of OSTSL adheres to the temporal and situational behaviors of properties based on existential rigidity. As a result, the semantics guarantees that the ontological axioms of properties expressed by sorted tempo-situational formulas are logically valid.
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1. Introduction

Formal ontology deals with different types of entities in the real world, such as properties, events, processes, objects, and parts, as discussed in Smith's paper [12]. The results of formal ontology help us define the semantics of various concepts in information systems. In other words, ontological definitions describe the general features of knowledge, which guide us in representing taxonomic knowledge in many application domains.

In the field of formal ontology, Guarino and Welty [5] have defined meaningful property classifications as meta-ontologies, wherein the properties of individuals are rigorously classified as sortal/non-sortal, rigid/anti-rigid/non-rigid, and unity/anti-unity/non-unity using logical formalization. According to the definitions in Welty and Guarino's paper [17], a property is called *sortal* if it carries an Identity Condition (IC), and it is called *rigid* if it is essential to all its instances. Based on this notion of sortality and rigidity, an ontological property classification, as shown in Fig. 1 (presented by Guarino, Carrara, and Giaretta [4]), can be constructed with the following types of properties:

Substantial property: sortal and rigid
Non-substantial property: sortal and anti-rigid
Generic property: non-sortal and rigid

Characterizing property: non-sortal and non-rigid

For example, *apple* is a substantial property, *nurse* is a non-substantial property, *red* is a characterizing property, and *water* is a generic property. In order-sorted logic, a substantial property is a rigid sort and is also used as a type.

Following this notion, Welty and Andersen [16] exhaustively axiomatized the meta-properties of existential rigidity, actuality, and permanence by using modal formulas. In their axioms, a property is called *existentially rigid* if in any possible world that an instance of the property exists, it instantiates the property. Further, a property is called *actual* if the property holds for actually existing entities. The existential rigidity and actuality were formalized in a more sophisticated way by considering the behavior of a property with respect to time, and they are called *temporally* existential rigidity and actuality. In addition, a property is called *permanent* with respect to time if its instances exist for all times in worlds in which they exist at all.

The axiomatization of temporally existential rigidity and actuality was well established; however, this does not characterize the behavior of a property with respect to both time and situation. For example, the following time and situation dependencies of properties are conceivable.

Time and situation dependencies:
A property (e.g., a baby) holds depending only on time and is situationally stable, while another (e.g., a weapon) holds depending on its use in a situation and is temporally stable. For example, a knife can be a weapon in one situation but is more normally employed as a tool for eating.

In particular, these dependencies of properties are extremely important when a property holds in one situation but not in another at the same time. To the best of our knowledge, the existing formal ontology has not axiomatized the existential rigidity of properties with respect to time and/or situation dependencies.

In this paper, we present an ontological definition that characterizes the time and/or situation dependencies of properties in a time- and situation-dependent semantics of order-sorted logic. Specifically, we define a class of sorted structures that is a semantics of order-sorted logic, quantified modal logic, and temporal logic extended by ontological notions. The temporal and situational behaviors of properties under existential rigidity are appropriately interpreted using varying domains and two-dimensional modalities over
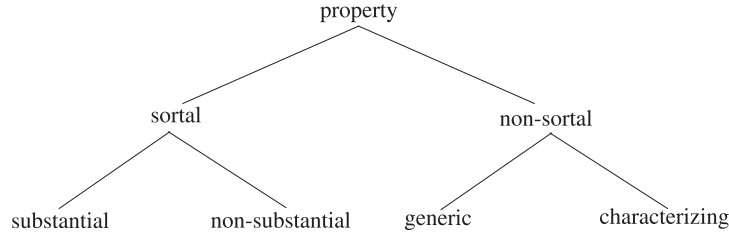
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                              property
                         /                \
                    sortal              non-sortal
                   /      \             /         \
          substantial  non-substantial  generic  characterizing
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Fig. 1 The ontological property classification [4].

the Cartesian product of the sets of times and situations. We use the semantics to declare ontologically and logically consistent models that validate the ontological axioms of properties.

Alternatively, order-sorted logic has been recognized as a tool for providing a logical knowledge representation language for taxonomic knowledge [1]–[3], [7], [11], [13]–[15]. This logical language contains a sort-hierarchy that is used to represent domain-specific ontologies; however, there is a gap between formal ontology and logical languages because the meta-properties axiomatized in formal ontology are not easily embedded in the specifications of logical languages, such as syntax and semantics. Kaneiwa and Mizoguchi [8], [10] observed that the ontological property classification [17] fits the syntax and semantics of order-sorted logic, and they enhanced order-sorted logic by means of the ontological notions of rigidity and sortality. In the semantics, *rigid* sort-predicates (unary-predicates indexed by sorts) are true across multiple knowledge bases, where each knowledge base can transfer rigid sort-predicates from other knowledge bases.

However, the standard order-sorted logic does not encompass the ontological notions of properties: existential rigidity and time and/or situation dependencies in the syntax and semantics. To represent the axioms of such ontological notions, we formalize order-sorted tempo-situational logic (OSTSL) that is extended using rigid/anti-rigid sorted terms and three types of modal operators, namely, $\blacksquare, \blacklozenge$ (tempo-situational), $\Box_{\text{Tim}}, \Diamond_{\text{Tim}}$ (temporal), and $\Box_{\text{Sit}}, \Diamond_{\text{Sit}}$ (situational). Next, existential rigidity and time and/or situation dependencies are axiomatized by sorted tempo-situational formulas.

The contributions this paper seeks to the field of formal ontology are as follows:

(i) To present an ontological definition of the time and/or situation dependences of properties together with existential rigidity, which can characterize the situational behaviors of sortal properties,

(ii) To define the syntax and semantics of OSTSL, which enhance order-sorted logic based on the ontological definition of time and/or situation dependencies.

OSTSL is not only a useful combination of order-sorted logic, quantified modal logic, and temporal logic; it has also been significantly extended to include tempo-situational operators over the Cartesian product of sets of times and situations. Our former work [9] addressed existential rigid-

ity and time and/or situation dependencies, but did not use the Cartesian product of the sets of times and situations in the semantics. This paper provides a more natural semantics of tempo-situational operators, where the rigidity and time and/or situation dependencies are redefined in the extended semantics. Order-sorted logic has the advantage that sorted terms and formulas adequately represent sortal properties based on formal ontology. Cialdea-Mayer and Cerrito's quantified modal logic provides us with a logical language for supporting varying domains and non-rigid terms, which can be enhanced by incorporating order-sorted terms/formulas and tempo-situational operators. Temporal logic contains time representation; however, it does not represent situation dependency and its combination with time dependency.

This paper is arranged as follows. In Sect. 2, we formalize the syntax and semantics of OSTSL by introducing rigid/anti rigid sorted terms and sorted tempo-situational formulas. In Sect. 3, we formally define a class of sorted structures in Kripke semantics that is constrained by the notions of existential rigidity and time and/or situation dependencies. Next, the ontological signatures and axioms of properties are described. Finally, in Sect. 4, we present our conclusion and discuss future work.

2. Order-Sorted Tempo-Situational Logic

In this section, we define the syntax and semantics of order-sorted tempo-situational logic (OSTSL).

2.1 Syntax

The alphabet of a sorted first-order tempo-situational language $\mathcal{L}$ with rigidity and sort predicates comprises the following symbols: a countable set T of type symbols (including the greatest type $\top$), a countable set S_A of anti-rigid sort symbols ($T \cap S_A = \emptyset$), a countable set C of constant symbols, a countable set F_n of n-ary function symbols, and a countable set P_n of n-ary predicate symbols (including the existential predicate symbol E; and the set $P_{T \cup S_A}$ of sort predicate symbols $\{p_s \mid s \in T \cup S_A\}$), the connectives $\wedge, \vee, \rightarrow, \neg$, the quantifiers $\forall, \exists$, the temporal, situational, and tempo-situational operators $\Box_{\text{Tim}}, \Diamond_{\text{Tim}}, \Box_{\text{Sit}}, \Diamond_{\text{Sit}}, \blacksquare, \blacklozenge$, and the auxiliary symbols (,).

We generally refer to type symbols τ or anti-rigid sort symbols σ as *sort symbols* s. $T \cup S_A$ is the set of sort symbols. V_s denotes an infinite set of variables x_s of sort s. We

abbreviate variables $x_\top$ of sort $\top$ as x. The set of variables of all sorts is denoted by $V = \bigcup_{s \in T \cup S_A} V_s$. The unary predicates $p_s \in P_1$ indexed by the sorts s (called *sort predicates*) are introduced for all sorts $s \in T \cup S_A$. In particular, the predicate p_τ indexed by a type τ is called a *type predicate*, and the predicate p_σ indexed by an anti-rigid sort σ is called an *anti-rigid sort predicate*. Hereafter, we assume that the language $\mathcal{L}$ contains all the sort predicates in $P_{T \cup S_A}$.

Definition 1 (Sorted Signatures): A signature of a sorted first-order tempo-situational language $\mathcal{L}$ with rigidity and sort predicates (called *sorted signature*) is a tuple $\Sigma = (T, S_A, \Omega, \leq)$ such that

1. $(T \cup S_A, \leq)$ is a partially ordered set of sorts where $T \cup S_A$ is the union of the set of type symbols and the set of anti-rigid sort symbols in $\mathcal{L}$ and each ordered pair $s_i \leq s_j$ is a subsort relation (implying that s_i is a subsort of s_j).
2. Ω is a set of sort declarations of constants, functions, and predicates where
   a. if $c \in C$, then there is a unique constant declaration $c \colon \to \tau \in \Omega$;
   b. if $f \in F_n$ ($n > 0$), then there is a unique function declaration $f \colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$;
   c. if $p \in P_n$, then there is a unique predicate declaration $p \colon s_1 \times \cdots \times s_n \in \Omega$; and
   d. if $p_s \in P_{T \cup S_A}$, then there is a unique sort predicate declaration $p_s \colon s' \in \Omega$ where $s \leq s'$).

In sorted signatures, the sorts of constants, functions, and predicates are declared by the following notions. The sort declarations of constants c and functions f are denoted by the forms $c \colon \to \tau$ and $f \colon \tau_1 \times \cdots \times \tau_n \to \tau$ where types τ_i, τ are used to declare the domains and ranges. Constants and functions are required to be rigidly sorted to avoid the anti-rigid domains and ranges of constants and functions. In contrast, the sort declarations of predicates are denoted by the form $p \colon s_1 \times \cdots \times s_n$ where types and anti-rigid sorts s_i can be used to set the domains of the predicates p. This is because the domains of predicates can be anti-rigid.

Following the sorted signature, we introduce the three types of terms: *typed term*, *anti-rigid sorted term*, and *sorted term* in a sorted first-order tempo-situational language $\mathcal{L}_\Sigma$.

Definition 2 (Typed Terms): Let $\Sigma = (T, S_A, \Omega, \leq)$ be a sorted signature. The set $\mathcal{T}_\tau^-$ of terms of type τ (called *typed terms*) is the smallest set such that

1. for every $x_\tau \in V_\tau$, $x_\tau \in \mathcal{T}_\tau^-$,
2. for every $c \in C$ with $c \colon \to \tau \in \Omega$, $c_\tau \in \mathcal{T}_\tau^-$,
3. if $t_1 \in \mathcal{T}_{\tau_1}^-, \ldots, t_n \in \mathcal{T}_{\tau_n}^-$, $f \in F_n$, and $f \colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$, then $f_{\tau^*, \tau}(t_1, \ldots, t_n) \in \mathcal{T}_\tau^-$ with $\tau^* = \tau_1, \ldots, \tau_n$, and
4. if $t \in \mathcal{T}_{\tau'}^-$ and $\tau' \leq \tau$, then $t \in \mathcal{T}_\tau^-$.

Definition 3 (Anti-Rigid Sorted Terms): Let $\Sigma = (T, S_A, \Omega, \leq)$ be a sorted signature. The set $\mathcal{T}_\sigma^-$ of terms of anti-rigid sort σ (called *anti-rigid sorted terms*) is the smallest set such that

1. for every $x_\sigma \in V_\sigma$, $x_\sigma \in \mathcal{T}_\sigma^-$ and
2. if $t \in \mathcal{T}_{\sigma'}^-$ and $\sigma' \leq \sigma$, then $t \in \mathcal{T}_\sigma^-$.

Definition 4 (Sorted Terms): Let $\Sigma = (T, S_A, \Omega, \leq)$ be a sorted signature. The set $\mathcal{T}_s$ of terms of sort s (called *sorted terms*) is the smallest set such that

1. $\mathcal{T}_s^- \subseteq \mathcal{T}_s$ and
2. if $t \in \mathcal{T}_{s'}$ and $s' \leq s$, then $t \in \mathcal{T}_s$.

We denote the set of ground terms of sort s by $\mathcal{T}_{s,0}$. Due to the rigidity of types and anti-rigid sorts, any anti-rigid sorted term (in $\mathcal{T}_\sigma^-$) must be a variable term whereas typed terms (in $\mathcal{T}_\tau^-$) can contain constants and functions. In other words, every anti-rigid sorted term is not rigid (e.g., $x_{student}$) and every typed term is rigid (e.g., c_{person}). We denote the function $sort(t)$ as the sort of a term t, i.e., $sort(t) = s$ if t is of the form x_s, c_s, or $f_{\tau^*, s}(t_1, \ldots, t_n)$.

Next, we define the set of sorted tempo-situational formulas in the language $\mathcal{L}_\Sigma$.

Definition 5 (Sorted Tempo-Situational Formulas): Let $\Sigma = (T, S_A, \Omega, \leq)$ be a sorted signature. The set $\mathcal{F}$ of formulas is the smallest set such that

1. if $t_1 \in \mathcal{T}_{s_1}, \ldots, t_n \in \mathcal{T}_{s_n}$, $p \in P_n$, and $p \colon s_1 \times \cdots \times s_n \in \Omega$, then $p(t_1, \ldots, t_n)$ is a formula,
2. if $t \in \mathcal{T}_{s'}$, $p \in P_{T \cup S_A}$, and $p_s \colon s' \in \Omega$, then $p_s(t)$ is a formula,
3. if $t \in \mathcal{T}_\top$, then $E(t)$ is a formula, and
4. if F, F_1, and F_2 are formulas, then $\neg F, (\forall x_s)F, (\exists x_s)F$, $\Box_{\text{Tim}}F, \Diamond_{\text{Tim}}F, \Box_{\text{Sit}}F, \Diamond_{\text{Sit}}F, \blacksquare F, \blacklozenge F, F_1 \wedge F_2, F_1 \vee F_2$, and $F_1 \to F_2$ are formulas.

The existential predicate formula $E(t)$ asserts the existence of entities denoted by a term t in a possible world. A sorted formula is called closed if it does not contain free variables.

2.2 Semantics

We define the semantics for a sorted first-order tempo-situational language $\mathcal{L}_\Sigma$. In the semantics, the sets W_{Tim} and W_{Sit} of times and situations and three accessibility relations R_{Tim}, R_{Sit}, and R over $W_{\text{Tim}} \times W_{\text{Sit}}$ are introduced to interpret the temporal, situational, and tempo-situational operators $\Box_{\text{Tim}}, \Diamond_{\text{Tim}}, \Box_{\text{Sit}}, \Diamond_{\text{Sit}}$, and $\blacksquare, \blacklozenge$, respectively. Note that we interpret each modal operator over the Cartesian product $W_{\text{Tim}} \times W_{\text{Sit}}$ (denoted as W). Let tm be a time in W_{Tim} and st be a situation in W_{Sit}. We simply denote an ordered pair $\langle tm, st \rangle$ of time and situation by $\vec{w}$ if no confusion arises.

Definition 6 (Sorted Σ-Structures with Times and Situations): Let Σ be a sorted signature. A sorted Σ-structure M is a tuple $(W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ such that

1. $W = W_{\text{Tim}} \times W_{\text{Sit}}$ where W_{Tim} is the set of times and W_{Sit} is the set of situations ($W_{\text{Tim}} \cap W_{\text{Sit}} = \emptyset$);
2. $R_{\text{Tim}} \subseteq \{\langle \langle tm, st \rangle, \langle tm', st' \rangle \rangle \in W \times W \mid st = st'\}$ where R_{Tim} is reflexive and transitive;

3. $R_{\mathbf{Sit}} \subseteq \{\langle\langle tm, st\rangle, \langle tm', st'\rangle\rangle \in W \times W \mid tm = tm'\}$ where $R_{\mathbf{Sit}}$ is reflexive and transitive;

4. $R \subseteq W \times W$ is a superset of $R_{\mathbf{Tim}} \cup R_{\mathbf{Sit}}$ where R is reflexive and transitive;

5. U is a superset of $\bigcup_{\langle tm, st\rangle \in W} U_{\langle tm, st\rangle}$ where $U_{\langle tm, st\rangle}$ is a non-empty set of individuals in world $\langle tm, st\rangle^{\dagger}$; and

6. $I = \{I_{\langle tm, st\rangle} \mid \langle tm, st\rangle \in W\}$ is the set of interpretation functions $I_{\langle tm, st\rangle}$ for all worlds $\langle tm, st\rangle \in W$ with the following conditions:

   a. if $s \in T \cup S_A$, then $I_{\langle tm, st\rangle}(s) \subseteq U_{\langle tm, st\rangle}$, (in particular, if $s = \top$, then $I_{\langle tm, st\rangle}(s) = U_{\langle tm, st\rangle}$). In addition, $I(s)$ is a superset of $\bigcup_{\langle tm, st\rangle \in W} I_{\langle tm, st\rangle}(s)$ such that $U_{\langle tm, st\rangle} \cap I(s) \subseteq I_{\langle tm, st\rangle}(s)^{\dagger\dagger}$,

   b. if $s_i \leq s_j$ with $s_i, s_j \in T \cup S_A$, then $I_{\langle tm, st\rangle}(s_i) \subseteq I_{\langle tm, st\rangle}(s_j)$,

   c. if $c \in C$ and $c \colon \to \tau \in \Omega$, then $I_{\langle tm, st\rangle}(c) \in I(\tau)$,

   d. if $f \in F_n$ and $f \colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$, then $I_{\langle tm, st\rangle}(f) \colon I(\tau_1) \times \cdots \times I(\tau_n) \to I(\tau)$,

   e. if $p \in P_n$ and $p \colon s_1 \times \cdots \times s_n \in \Omega$, then $I_{\langle tm, st\rangle}(p) \subseteq I_{\langle tm, st\rangle}(s_1) \times \cdots \times I_{\langle tm, st\rangle}(s_n)$ (in particular, if $p_s \in P_{T \cup S_A}$ and $p_s \colon s' \in \Omega$, then $I_{\langle tm, st\rangle}(p_s) \subseteq I_{\langle tm, st\rangle}(s'))$.

Unlike the above semantics, ordinal temporal logic may have more structural time than situation. We simplify time structures by defining them as reflexive and transitive relations. This is because temporal rigidity or anti-rigidity in a situation and situational rigidity or anti-rigidity in a time are similarly defined in nested combinations of temporal and situational modalities. In our ontological modeling, time and situation are treated equally where accessibility relations over the two-dimensional worlds are used to characterize the time and situation dependencies.

To define the denotation of sorted terms, we introduce the set C_U of new constants $\bar{d}$ for individuals d in U, where every new constant is interpreted by itself. In the following, we will adopt a sorted first-order tempo-situational language $\mathcal{L}_\Sigma$ extended by adding the set C_U of new constants.

Definition 7: Let $M = (W, \vec{w}_0, R_{\mathbf{Tim}}, R_{\mathbf{Sit}}, R, U, I)$ be a sorted Σ-structure. The denotation $[\![\ ]\!]_{\langle tm, st\rangle} \colon \mathcal{T}_{\top,0} \to U_{\langle tm, st\rangle}$ of ground terms is defined by the following rules:

1. $[\![c_\tau]\!]_{\langle tm, st\rangle} = I_{\langle tm, st\rangle}(c)$ for $c \in C$ where $c \colon \to \tau \in \Omega$,
2. $[\![\bar{d}]\!]_{\langle tm, st\rangle} = d$ for $\bar{d} \in C_U$, and
3. $[\![f_{\tau^*, \tau}(t_1, \ldots, t_n)]\!]_{\langle tm, st\rangle} = I_{\langle tm, st\rangle}(f)([\![t_1]\!]_{\langle tm, st\rangle}, \ldots, [\![t_n]\!]_{\langle tm, st\rangle})$ for $f \in F_n$ where $f \colon \tau_1 \times \cdots \times \tau_n \to \tau \in \Omega$.

In this definition, $\mathcal{T}_\top$ (the domain of $[\![\ ]\!]_{\langle tm, st\rangle}$) denotes the set of terms of all sorts since $\top$ is the greatest type and $\mathcal{T}_s$ denotes the set of terms of sort s and all subsorts of it. A closed formula is a formula without free variables. We define the set of subterms of a term t as follows:

1. if $t = c_\tau$, then $sub(t) = \{c_\tau\}$, and
2. if $t = f_{\tau^*, \tau}(t_1, \ldots, t_n) \colon s$, then $sub(t) = \{f_{\tau^*, \tau}(t_1, \ldots, t_n) \colon s\} \cup sub(t_1) \cup \cdots \cup sub(t_n)$.

We define the existence of terms in each world that is used in the satisfiability of sorted tempo-situational formulas.

Definition 8: Let $M = (W, \vec{w}_0, R_{\mathbf{Tim}}, R_{\mathbf{Sit}}, R, U, I)$ be a sorted Σ-structure, let $\langle tm, st\rangle \in W$, and let $[\![t]\!]_{\langle tm, st\rangle}$ be the denotation of a sorted ground term t in $\langle tm, st\rangle$. The set $Nex_{\langle tm, st\rangle}$ of closed formulas with sorted ground terms non-existing in $\langle tm, st\rangle$ is the smallest set such that

1. $p(t_1, \ldots, t_n) \in Nex_{\langle tm, st\rangle}$ iff for some ground term $t \in sub(t_1) \cup \cdots \cup sub(t_n)$, $[\![t]\!]_{\langle tm, st\rangle} \notin U_{\langle tm, st\rangle}$;
2. $\neg F, (\forall x_s)F, (\exists x_s)F \in Nex_{\langle tm, st\rangle}$ iff $F \in Nex_{\langle tm, st\rangle}$;
3. $\square_{\mathbf{Tim}}F, \diamondsuit_{\mathbf{Tim}}F, \square_{\mathbf{Sit}}F, \diamondsuit_{\mathbf{Sit}}F, \blacksquare F, \blacklozenge F \notin Nex_{\langle tm, st\rangle}$;
4. $F_1 \wedge F_2 \in Nex_{\langle tm, st\rangle}$ iff $F_1 \in Nex_{\langle tm, st\rangle}$ or $F_2 \in Nex_{\langle tm, st\rangle}$;
5. $F_1 \vee F_2 \in Nex_{\langle tm, st\rangle}$ iff $F_1 \in Nex_{\langle tm, st\rangle}$ and $F_2 \in Nex_{\langle tm, st\rangle}$; and
6. $F_1 \to F_2 \in Nex_{\langle tm, st\rangle}$ iff $\neg F_1 \in Nex_{\langle tm, st\rangle}$ and $F_2 \in Nex_{\langle tm, st\rangle}$.

We define the satisfiability of each sorted tempo-situational formula as follows.

Definition 9 (Σ-Satisfiability Relation):
Let $M = (W, \vec{w}_0, R_{\mathbf{Tim}}, R_{\mathbf{Sit}}, R, U, I)$ be a Σ-structure, let $F \in \mathcal{A} \cup \mathcal{G} \cup C$ be a closed formula, and let $\langle tm, st\rangle \in W$. The Σ-satisfiability relation $\langle tm, st\rangle \models F$ is defined inductively as follows:

1. $\langle tm, st\rangle \models p(t_1, \ldots, t_n)$ iff $([\![t_1]\!]_{\langle tm, st\rangle}, \ldots, [\![t_n]\!]_{\langle tm, st\rangle}) \in I_{\langle tm, st\rangle}(p)$;
2. $\langle tm, st\rangle \models E(t)$ iff there exists $d \in U_{\langle tm, st\rangle}$ such that $[\![t]\!]_{\langle tm, st\rangle} = d$;
3. $\langle tm, st\rangle \models \neg F$ iff $\langle tm, st\rangle \not\models F$;
4. $\langle tm, st\rangle \models F_1 \wedge F_2$ iff $\langle tm, st\rangle \models F_1$ and $\langle tm, st\rangle \models F_2$;
5. $\langle tm, st\rangle \models F_1 \vee F_2$ iff $\langle tm, st\rangle \models F_1$ or $\langle tm, st\rangle \models F_2$;
6. $\langle tm, st\rangle \models F_1 \to F_2$ iff $\langle tm, st\rangle \not\models F_1$ or $\langle tm, st\rangle \models F_2$;
7. $\langle tm, st\rangle \models (\forall x_s)F$ iff for all $d \in I_{\langle tm, st\rangle}(s)$, $\langle tm, st\rangle \models F[x_s/\bar{d}]$;
8. $\langle tm, st\rangle \models (\exists x_s)F$ iff for some $d \in I_{\langle tm, st\rangle}(s)$, $\langle tm, st\rangle \models F[x_s/\bar{d}]$;
9. $\langle tm, st\rangle \models \square_{\mathbf{Tim}}F$ iff for all $tm' \in W_{\mathbf{Tim}}$ with $\langle\langle tm, st\rangle, \langle tm', st\rangle\rangle \in R_{\mathbf{Tim}}$, $\langle tm', st\rangle \models F$ or $F \in Nex_{\langle tm', st\rangle}$;
10. $\langle tm, st\rangle \models \diamondsuit_{\mathbf{Tim}}F$ iff for some $tm' \in W_{\mathbf{Tim}}$ with $\langle\langle tm, st\rangle, \langle tm', st\rangle\rangle \in R_{\mathbf{Tim}}$, $\langle tm', st\rangle \models F$ and $F \notin Nex_{\langle tm', st\rangle}$;
11. $\langle tm, st\rangle \models \square_{\mathbf{Sit}}F$ iff for all $st' \in W_{\mathbf{Sit}}$ with $\langle\langle tm, st\rangle, \langle tm, st'\rangle\rangle \in R_{\mathbf{Sit}}$, $\langle tm, st'\rangle \models F$ or $F \in Nex_{\langle tm, st'\rangle}$;
12. $\langle tm, st\rangle \models \diamondsuit_{\mathbf{Sit}}F$ iff for some $st' \in W_{\mathbf{Sit}}$ with $\langle\langle tm, st\rangle, \langle tm, st'\rangle\rangle \in R_{\mathbf{Sit}}$, $\langle tm, st'\rangle \models F$ and $F \notin Nex_{\langle tm, st'\rangle}$;
13. $\langle tm, st\rangle \models \blacksquare F$ iff for all $\vec{w}' \in W$ with $\langle\langle tm, st\rangle, \vec{w}'\rangle \in R$, $\vec{w}' \models F$ or $F \in Nex_{\vec{w}'}$; and
14. $\langle tm, st\rangle \models \blacklozenge F$ iff for some $\vec{w}' \in W$ with $\langle\langle tm, st\rangle, \vec{w}'\rangle \in R$, $\vec{w}' \models F$ and $F \notin Nex_{\vec{w}'}$;

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†Each world can have a different domain (possibly $U_{\vec{w}_1} \neq U_{\vec{w}_2}$).
††If an individual in $I(s)$ exists in a world $\langle tm, st\rangle$, then it must belong to the interpretation $I_{\langle tm, st\rangle}(s)$ in $\langle tm, st\rangle$. That is, $I(s)$ may be constructed by $\bigcup_{\langle tm, st\rangle \in W} I_{\langle tm, st\rangle}(s)$ and individuals non-existing in any world.

Note that the tempo-situational formula $\blacksquare F$ is satisfied in a pair $\langle tm, st \rangle$ of time tm and situation st if for any world $\vec{w}'$ accessible from $\langle tm, st \rangle$, F is satisfied in $\vec{w}'$ ($\vec{w}' \models F$) or some ground terms in F do not exist in $\vec{w}'$ ($F \in Nex_{\vec{w}'}$). Let F be a formula. It is Σ-true in M if $\vec{w}_0 \models F$ (M is a Σ-model of F). If F has a Σ-model, it is Σ-satisfiable, otherwise, it is Σ-unsatisfiable. F is Σ-valid if every sorted Σ-structure is a Σ-model of F.

3. A Property Classification in Semantics

In this section, we define a specific semantics of OSTSL to characterize the time and/or situation dependencies of properties under existential rigidity.

3.1 Specific Sorted Structures

In this study, we focus on the temporal and situational behaviors of properties where other specific possible worlds (e.g., beliefs and locations that are neither times nor situations) are not introduced in the Kripke semantics. The time and/or situation dependencies of properties are semantically defined by constraining a class of sorted Σ-structures $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$, i.e., the ontological conditions are added to the interpretation of properties over the Cartesian product $W = W_{\text{Tim}} \times W_{\text{Sit}}$. In particular, the two sets W_{Tim} and W_{Sit} of times and situations and the three accessibility relations R, R_{Tim}, and R_{Sit} over $W \times W$ are used to define existential rigidity and the time and/or situation dependencies of properties.

In the specific semantics, the tempo-situational formulas constructed by the three types of modal operators, namely, $\blacksquare$, $\blacklozenge$ (tempo-situational), $\square_{\text{Tim}}$, $\diamondsuit_{\text{Tim}}$ (temporal), and $\square_{\text{Sit}}$, $\diamondsuit_{\text{Sit}}$ (situational) are well interpreted. To axiomatize existential rigidity and time and/or situation dependencies, we use the temporal, situational, and tempo-situational operators to assert the statement that a formula F holds for any accessible time, situation, or world, whenever individuals exist. For example, the sorted tempo-situational formula

$$\square_{\text{Tim}} p_{male}(bob_{person})$$

implies that for any time accessible from a world, Bob is a male person *as long as he exists*.

3.2 Rigidity

We semantically define the *rigidity* of properties expressed by sorts in sorted Σ-structures. Let τ be a type (i.e., substantial sort). The rigidity of types is defined by the following statement:

- For all possible worlds $\vec{w}_i, \vec{w}_j \in W$, $I_{\vec{w}_i}(\tau) = I_{\vec{w}_j}(\tau)$.

This leads to the rigidity of individuals denoted by constants and functions. Let c be a constant and f be a function. Then, the following statement holds:

- For all possible worlds $\vec{w}_i, \vec{w}_j \in W$, $I_{\vec{w}_i}(c) = I_{\vec{w}_j}(c)$, and $I_{\vec{w}_i}(f) = I_{\vec{w}_j}(f)$.

However, the constant domains do not appear to be realistic since there may be different entities (individuals) in each possible world. For example, every instance of the property *person* ceases to exist at some time since no person can live forever. In light of this, we use varying domains such that $U_{\vec{w}}$ is the set of individuals existing in a possible world $\vec{w} = \langle tm, st \rangle$. The varying domains enable us to consider the case where $U_{\vec{w}_1}$ and $U_{\vec{w}_2}$ do not coincide for some possible worlds $\vec{w}_1, \vec{w}_2 \in W$.

The rigidity of sorts, constants, and functions is redefined by supporting individual existence in the following manner.

Definition 10 (Existential Rigidity):
Let $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ be a sorted Σ-structure. Then, the following conditions hold:

1. for every constant c, if $I_{\vec{w}_i}(c), I_{\vec{w}_j}(c) \in U_{\vec{w}_i} \cap U_{\vec{w}_j}$, then $I_{\vec{w}_i}(c) = I_{\vec{w}_j}(c)$,
2. for every n-ary function f, if $d_1, \ldots, d_n \in U_{\vec{w}_i} \cap U_{\vec{w}_j}$ and $\{I_{\vec{w}_i}(f)(d_1, \ldots, d_n), I_{\vec{w}_j}(f)(d_1, \ldots, d_n)\} \subseteq U_{\vec{w}_i} \cap U_{\vec{w}_j}$, then $I_{\vec{w}_i}(f)(d_1, \ldots, d_n) = I_{\vec{w}_j}(f)(d_1, \ldots, d_n)$,
3. for every type τ and for every world $\vec{w}$, if $d \in I_{\vec{w}}(\tau)$ and $\langle \vec{w}, \vec{w}' \rangle \in R$, then $d \in U_{\vec{w}'}$ implies $d \in I_{\vec{w}'}(\tau)$, and
4. for every anti-rigid sort σ and for every world $\vec{w}$, if $d \in I_{\vec{w}}(\sigma)$, then there exists $\vec{w}_j \in W$ with $\langle \vec{w}, \vec{w}_j \rangle \in R$ such that $d \notin I_{\vec{w}_j}(\sigma)$ with $d \in U_{\vec{w}_j}$.

Note that the sorted Σ-structures (in Definition 6) guarantee that each $U_{\vec{w}}$ is a non-empty set of individuals.

3.3 Time and Situation Dependencies

The semantics can be further elaborated in terms of time and/or situation dependencies. The two-dimensional modalities of time and situation are used to define distinctions among anti-rigid sorts (as non-substantial properties). We show some examples of time and/or situation dependencies that classify anti-rigid sorts as follows:

Time dependent: baby, child, youth, adult, elderly
Situation dependent: teacher, student, nurse
Time-situation dependent: novice teacher

In Fig. 2, these distinctions are added to the non-substantial properties in the ontological property classification. Time dependency implies that the meaning of a property depends only on time or is decided essentially by time. For example, the property *baby* is time dependent, so that each instance has the denoting property in a particular time or period.

Situation dependency indicates that a situation establishes whether a property holds but time does not. Moreover, the situation dependency obtained from extending types (such as *weapon* and *table*, but not *student*) involves a complex idea, as described below. We can regard the property *weapon* as a substantial sort (type); however, it is anti-rigid and situation-dependent if it is used as a role expressed by the sort predicate p_{weapon}. For instance, the properties *weapon* and *table* have the following two types of instances:
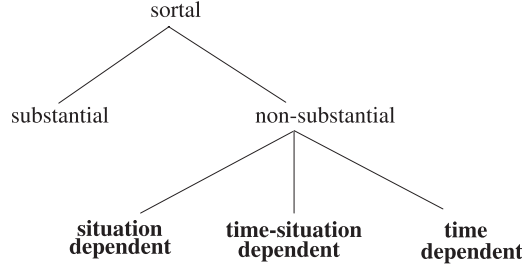
Fig. 2　Time and/or situation dependencies in ontological property classification.

(i) Guns and dining tables that innately possess the property of *weapon* and *table*

(ii) Knives and boxes that play the roles of *weapon* and *table*, respectively

In the latter case, the properties are not really the aforementioned artifacts and are just referred to as *weapon* and *table*. Thus, knives play the role of a *weapon* only when they are used to attack or kill someone. In the language of OSTSL, the former case is an instantiation of a sort (e.g., constant c_{knife} of sort *knife*), and the latter case is an individual characterized by a sort predicate (e.g., $p_{weapon}(c_{knife})$). In other words, the type *weapon* is rigid, but the property p_{weapon} is anti-rigid (situation dependent). It guarantees that all types are still rigid even if an individual additionally belongs to the predicate denoted by a type (e.g., a knife is being used as a weapon but not essentially as a weapon). We consider such different interpretations for a property to be appropriate because we also interpret a natural language word (e.g., weapon) as having an essential property and a role.

Here, we define these dependencies semantically in possible worlds over the Cartesian product of W_{Tim} and W_{Sit}. The basic notion of interpreting time dependency is that for every time-dependent property p and for every individual $d \in U$, if $d \in I_{\langle tm,st \rangle}(p)$ with $tm \in W_{\text{Tim}}$, then another time $tm_j \in W_{\text{Tim}}$ exists such that $d \notin I_{\langle tm_j,st \rangle}(p)$. This simple definition is based on the constant domains, which can be refined by considering the existence of individuals in each world. In the following, time, situation, and time-situation dependencies with individual existence are defined over accessibility relations over $W \times W$.

Definition 11 (Time Dependency):
Let $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ be a sorted Σ-structure. A unary predicate p is time-dependent if the following statements hold:

1. (temporally existential anti-rigid)
   for all $\langle \vec{w}, \langle st, tm \rangle \rangle \in R_{\text{Tim}}$ and for all $d \in U_{\langle st,tm \rangle}$,

   - if $d \in I_{\langle st,tm \rangle}(p)$, then there exists $tm_j \in W_{\text{Tim}}$ with $\langle \langle st, tm \rangle, \langle st, tm_j \rangle \rangle \in R_{\text{Tim}}$ such that $d \notin I_{\langle st,tm_j \rangle}(p)$ with $d \in U_{\langle st,tm_j \rangle}$.

2. (situationally existential rigid at a time)
   for all $\vec{w} \in W$ and for all $tm \in W_{\text{Tim}}$,

   - if $d \in I_{\langle tm,st \rangle}(p)$ with $\langle \vec{w}, \langle tm, st \rangle \rangle \in R_{\text{Tim}}$, then for all situations $st' \in W_{\text{Sit}}$ with $\langle \langle tm, st \rangle, \langle tm, st' \rangle \rangle \in$

R_{Sit}, $d \in U_{\langle tm,st' \rangle}$ implies $d \in I_{\langle tm,st' \rangle}(p)$.

Temporally existential anti-rigid implies that for every time tm accessible from a world $\vec{w}$, if an individual d has the property p at the time tm, we can find a time tm_j accessible from tm where the individual does not have the property. Situationally existential rigid at a time defines the fact that for every time tm accessible from a world $\vec{w}$, if an individual d has the property p at the time tm, then the individual has this property in any situation st accessible from the time tm as long as the individual exists.

Similar to the above time dependency, situation dependency can be defined as follows:

Definition 12 (Situation Dependency):
Let $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ be a sorted Σ-structure. A unary predicate p is situation-dependent if the following statements hold:

1. (situationally existential anti-rigid)
   for all $\langle \vec{w}, \langle tm, st \rangle \rangle \in R_{\text{Sit}}$ and for all $d \in U_{\langle tm,st \rangle}$,

   - if $d \in I_{\langle tm,st \rangle}(p)$, then there exists $st_j \in W_{\text{Sit}}$ with $\langle \langle tm, st \rangle, \langle tm, st_j \rangle \rangle \in R_{\text{Sit}}$ such that $d \notin I_{\langle tm,st_j \rangle}(p)$ with $d \in U_{\langle tm,st_j \rangle}$.

2. (temporally existential rigid in a situation)
   for all $\vec{w} \in W$ and for all $st \in W_{\text{Sit}}$,

   - if $d \in I_{\langle tm,st \rangle}(p)$ with $\langle \vec{w}, \langle tm, st \rangle \rangle \in R_{\text{Sit}}$, then for all times $tm' \in W_{\text{Tim}}$ with $\langle \langle tm, st \rangle, \langle tm', st \rangle \rangle \in R_{\text{Tim}}$, $d \in U_{\langle tm',st \rangle}$ implies $d \in I_{\langle tm',st \rangle}(p)$.

Further, time-situation dependency is defined such that the meaning of a property depends on both the time and the situation. For example, the property *novice_teacher* is time-situation dependent. Since each novice teacher will become a veteran teacher after a certain number of years, the property *novice_teacher* holds only at a particular time in a situation. In other words, time-situation dependency implies time dependency under a situation, while situation dependency bears no relationship to time. In the semantics with varying domains, time-situation dependency can be defined as a more complex dependency, as shown below.

Definition 13 (Time-Situation Dependency):
Let $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ be a sorted Σ-structure. A unary predicate p is time-situation dependent if the following statements hold:

1. (situationally existential anti-rigid) the same as in Definition 12.
2. (temporally existential anti-rigid in a situation) for all $\vec{w} \in W$ and for all $st \in W_{\text{Sit}}$,

   - if $d \in I_{\langle tm, st \rangle}(p)$ with $\langle \vec{w}, \langle tm, st \rangle \rangle \in R_{\text{Sit}}$, then there are some $tm_i, tm_j \in W_{\text{Tim}}$ with $\langle \langle tm, st \rangle, \langle tm_i, st \rangle \rangle, \langle \langle tm, st \rangle, \langle tm_j, st \rangle \rangle \in R_{\text{Tim}}$ such that $d \in I_{\langle tm_i, st \rangle}(p)$ and $d \notin I_{\langle tm_j, st \rangle}(p)$ with $d \in U_{\langle tm_i, st \rangle} \cap U_{\langle tm_j, st \rangle}$.

In addition to situational anti-rigidity, *temporally existential anti-rigid in a situation* implies that for every situation *st* accessible from a world $\vec{w}$, if an individual *d* has the property *p* in the situation *st*, then there are times tm_i, tm_j accessible from *st* such that the individual has the property *p* at tm_i but not at tm_j. Time-situation dependency is enhanced by the fact that neither time nor situation dependency can explain time-situation-dependent properties (such as novice teachers). This is because time-situation dependency is different from both time and situation dependencies holding.

3.4   Ontological Signatures and Axioms for Properties

We consider whether each sort in OSTSL is time, situation, or time-situation dependent. Types can be situation dependent (i.e., no type has time dependency or time-situation dependency), while anti-rigid sorts can be either time, situation, or time-situation dependent. For example, the type *weapon* is situation dependent, and the anti-rigid sort *adult* is time dependent. Each sort predicate p_s has the same dependency as its sort *s*.

A partially ordered set $(T \cup S_A, \leq)$ constructs a sort-hierarchy by suitably ordering different types of sorts where the subsort relation is assumed to be transitive and reflexive. In other words, the sort-hierarchy is restricted by the following conditions.

- No subsort of anti-rigid sorts is a type.
- No subsort of situation dependent sorts or time-situation dependent sorts is time dependent.
- No subsort of time dependent sorts or time-situation dependent sorts is situation dependent.

They are ontologically guaranteed by the fact that each sort inherits (temporal and situational) anti-rigidity from its supersorts. For example, the sort *novice_teacher* must be situationally existential anti-rigid (as time-situation dependent) if the supersort *teacher* is situation dependent.

The restrictions of the sort-hierarchy define an ontological sorted signature, as follows.

Definition 14 (Ontological Sorted Signatures):   An ontological sorted signature is a signature $\Sigma = (T, S_A, \Omega, \leq)$ such that

1. every subsort of types is a type or an anti-rigid sort ($s \leq \tau$) and every subsort of anti-rigid sorts is an anti-rigid sort ($\sigma \leq \sigma'$);

2. every subsort of time dependent sorts is time or time-situation dependent;
3. every subsort of situation dependent sorts is situation or time-situation dependent; and
4. every subsort of time-situation dependent sorts is time-situation dependent.

We describe ontological axioms in the class of sorted Σ-structures $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ that assert existential rigidity, time dependency, situation dependency, and time-situation dependency (in Definitions 10 - 13). The rigidity and time and/or situation dependencies in sorted Σ-structures validate the ontological axioms of properties, as follows.

Proposition 1 (Ontological Axioms):   Let p_τ be a type predicate with $p_\tau : \tau_0$, $\tau \leq \tau_0 \in \Omega$ and p_σ be an anti-rigid sort predicate with $p_\sigma : \tau \in \Omega$. The following axioms are Σ-valid.

1. **Rigid predicate axiom:**
   $(\forall x_\tau)(p_\tau(x_\tau) \rightarrow \blacksquare p_\tau(x_\tau))$ if p_τ is not situation dependent

2. **Anti-rigid predicate axiom:**
   $(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \blacklozenge(\neg p_\sigma(x_\tau)))$

3. **Time dependency axiom:**
   $\square_{\text{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \lozenge_{\text{Tim}}(\neg p_\sigma(x_\tau)))$
   $\square_{\text{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \square_{\text{Sit}} p_\sigma(x_\tau))$

4. **Situation dependency axiom:**
   $\square_{\text{Sit}}(\forall x_\tau)(p(x_\tau) \rightarrow \lozenge_{\text{Sit}}(\neg p(x_\tau)))$
   $\square_{\text{Sit}}(\forall x_\tau)(p(x_\tau) \rightarrow \square_{\text{Tim}} p(x_\tau))$ where p is p_τ or p_σ

5. **Time-situation dependency axiom:**
   $\square_{\text{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \lozenge_{\text{Sit}}(\neg p_\sigma(x_\tau)))$
   $\square_{\text{Sit}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow (\lozenge_{\text{Tim}} p_\sigma(x_\tau) \wedge \lozenge_{\text{Tim}}(\neg p_\sigma(x_\tau))))$

Proof.   Let $M = (W, \vec{w}_0, R_{\text{Tim}}, R_{\text{Sit}}, R, U, I)$ be any sorted Σ-structure. We prove that it satisfies the axioms.

(rigid predicate axiom)

Let *d* be any individual in $I_{\vec{w}_0}(\tau)$ ($\subseteq U_{\vec{w}_0}$). If p_τ is an inextensible type predicate with $p_\tau : \tau \in \Omega$ and $\vec{w}_0 \models p_\tau(\bar{d})$, then $d \in I_{\vec{w}_0}(\tau)$ (due to the condition $I_{\vec{w}}(\tau) = I_{\vec{w}}(p_\tau)$ for every $\vec{w} \in W$). So, by Condition (2) of Definition 6, for all $\vec{w}'$ with $\langle \vec{w}_0, \vec{w}' \rangle \in R$, $d \in U_{\vec{w}'}$ implies $d \in I_{\vec{w}'}(\tau)$ ($= I_{\vec{w}'}(p_\tau)$). So, $p_\tau(\bar{d}) \in Nex_{\vec{w}'}$ (i.e., $[\![\bar{d}]\!]_{\vec{w}'} \notin U_{\vec{w}'}$) or $\vec{w}' \models p_\tau(\bar{d})$. Hence, $\vec{w}_0 \models \blacksquare p_\tau(\bar{d})$.

(anti-rigid predicate axiom)

Let $d \in I_{\vec{w}_0}(\tau)$ ($\subseteq U_{\vec{w}_0}$). By Condition (2) of Definition 6, if $d \in I_{\vec{w}_0}(\sigma)$, then there exists $\vec{w}_j \in W$ with $\langle \vec{w}_0, \vec{w}_j \rangle \in R$ such that $d \notin I_{\vec{w}_j}(\sigma)$ with $d \in U_{\vec{w}_j}$. Thus, for every $d' \in I_{\vec{w}_0}(\tau)$, $\vec{w}_0 \models p_\sigma(\bar{d}')$ implies that for some $\vec{w}_j \in W$ with $\langle \vec{w}_0, \vec{w}_j \rangle \in R$, $\vec{w}_j \models \neg p_\sigma(\bar{d}')$ and $\neg p_\sigma(\bar{d}') \notin Nex_{\vec{w}_j}$ according to $I_{\vec{w}}(\sigma) = I_{\vec{w}}(p_\sigma)$ for every $\vec{w} \in W$. Therefore, for every $d' \in I_{\vec{w}_0}(\tau)$, $\vec{w}_0 \models p_\sigma(\bar{d}') \rightarrow \blacklozenge(\neg p_\sigma(\bar{d}'))$.

(time dependency axiom)

Let *tm* be any time in W_{Tim} with $\langle \vec{w}_0, \langle tm, st \rangle \rangle \in R_{\text{Tim}}$, and let $d \in I_{\langle tm, st \rangle}(\tau)$ ($\subseteq U_{\langle tm, st \rangle}$). By the temporal unstability, for every time-dependent predicate p_σ with $p_\sigma : \tau \in$

Ω, if $d \in I_{\langle tm, st \rangle}(p_\sigma)$, then there exists $\langle tm_j, st \rangle \in W_{\text{Tim}}$ with $\langle \langle tm, st \rangle, \langle tm_j, st \rangle \rangle \in R_{\text{Tim}}$ such that $d \notin I_{\langle tm_j, st \rangle}(p_\sigma)$ with $d \in U_{\langle tm_j, st \rangle}$. So, for every $d' \in I_{\langle tm, st \rangle}(\tau)$, if $\langle tm, st \rangle \models p_\sigma(\bar{d}')$, then $\langle tm, st \rangle \models \Diamond_{\text{Tim}}(\neg p_\sigma(\bar{d}'))$. Thus, $\langle tm, st \rangle \models (\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Diamond_{\text{Tim}}(\neg p_\sigma(x_\tau)))$. Therefore, $\vec{w}_0 \models \Box_{\text{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Diamond_{\text{Tim}}(\neg p_\sigma(x_\tau)))$. Moreover, by the situational stability, if $d \in I_{\langle tm, st \rangle}(p_\sigma)$ with $\langle \vec{w}_0, \langle tm, st \rangle \rangle \in R_{\text{Tim}}$, then for all situations $st' \in W_{\text{St}}$ with $\langle \langle tm, st \rangle, \langle tm, st' \rangle \rangle \in R_{\text{St}}$, $d \in U_{\langle tm, st' \rangle}$ (iff $p_\sigma(\bar{d}) \notin Nex_{\langle tm, st' \rangle}$) implies $d \in I_{\langle tm, st' \rangle}(p_\sigma)$. Then, for every $d' \in I_{\langle tm, st' \rangle}(\tau)$, $\langle tm, st' \rangle \models p_\sigma(\bar{d}') \rightarrow \Box_{\text{St}} p_\sigma(\bar{d}')$, so that $\vec{w}_0 \models \Box_{\text{Tim}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow \Box_{\text{St}} p_\sigma(x_\tau))$.

(situation dependency axiom)

Similar to the above proof of the time dependency axiom.

(time-situation dependency axiom)

Let p_σ be a time-situation dependent predicate with $p_\sigma: \tau \in \Omega$. The first formula is Σ-valid because it is the same as the situation dependency axiom. Let st be any situation in W_{St} with $\langle \vec{w}_0, \langle tm, st \rangle \rangle \in R_{\text{St}}$, and let $d \in I_{\langle tm, st \rangle}(\tau) (\subseteq U_{\langle tm, st \rangle})$. If $\langle tm, st \rangle \models p_\sigma(\bar{d})$, then by the situational unstability under time, for some $tm_i, tm_j \in W_{\text{Tim}}$ with $\langle \langle tm, st \rangle, \langle tm_i, st \rangle \rangle, \langle \langle tm, st \rangle, \langle tm_j, st \rangle \rangle \in R_{\text{Tim}}$, $d \in I_{\langle tm_i, st \rangle}(p_\sigma)$ and $d \notin I_{\langle tm_j, st \rangle}(p_\sigma)$ with $d \in U_{\langle tm_i, st \rangle} \cap U_{\langle tm_j, st \rangle}$. Thus, for every $d' \in I_{\langle tm, st \rangle}(\tau)$, $st \models p_\sigma(\bar{d})$ implies $\langle tm_i, st \rangle \models p_\sigma(\bar{d}')$ and $\langle tm_j, st \rangle \models \neg p_\sigma(\bar{d}')$ where $p_\sigma(\bar{d}') \notin Nex_{\langle tm_i, st \rangle}$ and $\neg p_\sigma(\bar{d}') \notin Nex_{\langle tm_j, st \rangle}$. So, we have $st \models (\forall x_\tau)(p_\sigma(x_\tau) \rightarrow (\Diamond_{\text{Tim}} p_\sigma(x_\tau) \land \Diamond_{\text{Tim}}(\neg p_\sigma(x_\tau))))$. Therefore, this derives the conclusion that $\vec{w}_0 \models \Box_{\text{St}}(\forall x_\tau)(p_\sigma(x_\tau) \rightarrow (\Diamond_{\text{Tim}} p_\sigma(x_\tau) \land \Diamond_{\text{Tim}}(\neg p_\sigma(x_\tau))))$. $\qquad\square$

We denote the set of axioms by $\mathcal{A}_{\mathcal{L}_\Sigma}$. OSTSL takes into account the notion of individual existence, and therefore these axioms suitably assert the time and/or situation dependencies under existential rigidity. The first axiom implies that if a rigid predicate holds, then it holds in any world as long as x exists. The second axiom implies that if an anti-rigid predicate holds, then there exists another world wherein it does not hold. The third axiom indicates the anti-rigidity at a particular time and the rigidity in any situation within the time that the property holds. Similarly, the fourth axiom implies the anti-rigidity in a particular situation and the rigidity at any time within the situation that the property holds. The axiom of time-situation dependency expresses the anti-rigidity in a situation, and if the property holds in a situation, then the property holds at a time but not at another time.

We now provide an example of sorted tempo-situational formulas, as follows.

Example 1: Let p_{apple} be a type predicate and $p_{nov\_teacher}$ be an anti-rigid sort predicate (time-situation dependent) where $p_{apple}: fruit$ and $p_{nov\_teacher}: person$ in Ω. Then, the two sorted temporal and situational formulas:

$$p_{apple}(c_{fruit}) \rightarrow \blacksquare p_{apple}(c_{fruit})$$

and

$$\Box_{\text{St}}(p_{nov\_teacher}(john_{person}) \rightarrow (\Diamond_{\text{Tim}} p_{nov\_teacher}(john_{person}) \land$$
$$\Diamond_{\text{Tim}} \neg p_{nov\_teacher}(john_{person})))$$

are Σ-valid. The subformula $\blacksquare p_{apple}(c_{fruit})$ expresses rigidity with *individual existence*. This implies "c_{fruit} is an apple in any world as long as it exists," (for any accessible world $\vec{w} \in W$, if $[\![c_{fruit}]\!]_{\langle tm, st \rangle} \in U_{\vec{w}}$ (it exists), then $[\![c_{fruit}]\!]_{\langle tm, st \rangle} \in I_{\vec{w}}(p_{apple})$), but this does not imply "c_{fruit} is an apple forever" or "c_{fruit} exists forever." Moreover, the subformula:

$$\Diamond_{\text{Tim}} \neg p_{nov\_teacher}(john_{person})$$

indicates "there is a time where the person $john_{person}$ exists but is not a novice teacher."

3.5 Ontological Property Description in Ontology Languages

We briefly discuss how rigid sorts and time and/or situation dependent sorts can be represented in a formal ontology language like RDF (Resource Description Language) and OWL (Web Ontology Language). Unfortunately, RDF and OWL have no modal operators to represent the rigidity and time and/or situation dependences.

We can describe rigid sorts and time and/or situation dependent sorts using the meta-modeling of RDF, as follows. Let `rigidClass` and `situationDependentClass` be new meta-classes, i.e., the following subclass relations are additionally declared.

```
rigidClass rdfs:subClass rdfs:Class.
    situationDependentClass
            rdfs:subClass rdfs:Class.
```

Then, the RDF triple

```
person rdf:type rigidClass.
```

indicates that `person` is a rigid class. Also, the RDF triple

```
student rdf:type situationDependentClass.
```

indicates that `student` is a situation dependent class.

In addition, each instance of anti-rigid sorts is represented using an extension of RDF with time labeling [6] as follows.

```
Tom rdf:type student [st1].
```

This triple states that Tom is a student in situation `st1`.

We can describe such an ontology, including rigid sorts and time and/or situation dependent sorts, that is semantically guaranteed in our time and situation-dependent semantics. Our semantics can be used to enhance the RDF entailment rules by adding inference rules for the rigidity and time and/or situation dependencies.

4. Conclusion and Future Work

We have presented a specific semantics based on the ontological notions of sortality, rigidity, and time and/or situation dependencies in OSTSL. The specific semantics is defined by a class of sorted structures where the ontological property classification is embedded in the extended order-sorted logic. We have shown that the specific semantics validates the ontological axioms of properties for existential rigidity and time and/or situation dependencies.

Future work in this area concerns considering the temporal operators "always in the future," "always in the past," "until," and "since" in the ontological property classification. This extension gives rise to more complex time structures than is represented by the situation in the semantics. If these temporal operators are refined by existential rigidity, we can more precisely characterize the temporal features of properties in the real world. For example, although defining the feature that the property *person* holds from the time of birth of a given person until the time of his/her death is possible, our work does not yet cover such a broad property.

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Ken Kaneiwa is an Associate Professor of the Department of Electrical Engineering and Computer Science, Iwate University. He worked for Fujitsu, Ltd. from 1993 to 1996, and received his M.S. and Ph.D. degrees from Japan Advanced Institute of Science and Technology in 1998 and 2001, respectively, majoring in Information Science in both cases. His research interests include Knowledge Representation and Reasoning, Order-Sorted Logic, and Semantic Web. He is a member of the Japan Society for Software Science and Technology, the Information Processing Society of Japan, the Japanese Society for Artificial Intelligence, and the Association for Logic Programming.

Riichiro Mizoguchi received Ph.D. degree from Osaka University in 1977 and is currently Professor of the Institute of Scientific and Industrial Research, Osaka University. His research interests include Non-parametric data analyses, Knowledge-based systems, Ontological engineering and Intelligent learning support systems. Dr. Mizoguchi was President of International AI in ED Society, Asia-Pacific Society for Computers in Education from 2001 to 2003 and of Japanese Society for Artificial Intelligence (JSAI) from 2006 to 2008, respectively. He received honorable mention for the Pattern Recognition Society Award, the Institute of Electronics, Information and Communication Engineers Award, 10th Anniversary Paper Award of JSAI, Best paper Awards of ICCE99 and ICCE2006, Okawa publication award and best paper award of JSAI in 1985, 1988, 1996, 1999, 2006, 2005 and 2006, respectively. He is currently vice-president of Semantic Web Science Association (SWSA).