LETTER

Optimized Median Lifting Scheme for Lossy Image Compression*

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SUMMARY In JPEG2000, the Cohen-Daubechies-Feauveau (CDF) 9/7-tap wavelet filter was implemented by using the conventional lifting scheme. However, the filter coefficients remain complex, and the conventional lifting scheme disregards image edges in the coding process. In order to solve these issues, we propose a lifting scheme in two steps. In the first step, we select the appropriate filter coefficients; in the second step, we employ a median operator to regard image edges. Experimental results show that the peak signal-to-noise ratio (PSNR) value of the proposed lifting scheme is significantly improved, by up to 0.75 dB on average, compared to that of the conventional lifting scheme in the CDF 9/7-tap wavelet filter of JPEG2000.

key words: *wavelet transforms, image coding, lifting scheme, filter design, JPEG2000*

1. Introduction

Grossmann and Morlet [1] proposed wavelets, which are known as the first-generation wavelets and were implemented by using the convolution scheme. We can use the lifting scheme to implement wavelets known as the second-generation wavelets [2]. By the lifting scheme, several wavelet filters with different coefficients have been proposed based on the CDF 9/7-tap wavelet filter.

Daubechies et al. [2] used the lifting scheme and a factorizing method with the irrational number $\alpha = -1.586134342\ldots$ to find the coefficients of a wavelet filter. This filter is referred to as the CDF 9/7-tap wavelet filter. Guangjun et al. [3] proposed a simple 9/7-tap wavelet filter based on the lifting scheme with $\alpha = -1.5$. Compared to the CDF 9/7-tap wavelet filter, their filter has similar performance in terms of PSNR values, but α is simplified into a rational number. The main concern is whether there is any other value of α and the corresponding coefficients of wavelet filters that can provide better performance with less complex coefficients than that of the CDF 9/7-tap wavelet filter.

In JPEG2000, both the first-generation and second-generation wavelets disregard image edges in the image coding operation. Some authors have proved that image edges need to be regarded [4], and they proposed methods to enhance coding efficiency by regarding image edges. Ding et al. [5] proposed a directional lifting scheme which combines the conventional lifting scheme of JPEG2000 with the idea of intra-prediction in H.264, and obtained doubled coding efficiency. Liu et al. [6] proposed a weighted adaptive lifting scheme based on the weighted functions of different directional predictions.

In this letter, we first design optimized 9/7-tap wavelet filters by examining the whole possible range of α, so that we can find the α which gives the best performance. Then, we propose a new lifting scheme which uses a median operator since median lifting is a nonlinear operation and it yields much better image quality for edge-dominant images [7]. Therefore, we obtain an efficient lifting scheme based on filter optimization and median operation.

2. Proposed Lifting Scheme

The proposed lifting scheme includes two parts: finding appropriate α value and applying this value into median lifting.

2.1 Optimized Wavelet Filters

In wavelets, the forward transform uses a low-pass filter, h, and a high-pass filter, g, followed by subsampling. In the z-domain, h can be represented as in [2]:

$$h(z) = \sum_{k=k_{\min}}^{k_{\max}} h_k z^{-k}, \tag{1}$$

where $k_{\min}$ and $k_{\max}$ are respectively the smallest and the largest integer number for which h_k is a nonzero coefficient.

The functions of h and g in the ω-domain are defined as

$$H(\omega) = h_0 + 2\sum_{n=1}^{L_1} h_n \cos n\omega, \tag{2}$$

$$G(\omega) = g_0 + 2\sum_{n=1}^{L_2} g_n \cos n\omega, \tag{3}$$

where $L_1 = 4$ and $L_2 = 3$ in the CDF 9/7-tap wavelet filter.

If $H(\omega)$ and $G(\omega)$ construct a bi-orthogonal wavelet, the normalized conditions must be satisfied

$$H(0) = \sqrt{2} \text{ and } G(0) = \sqrt{2}. \tag{4}$$

Combining (4) with (2) and (3), we have

Manuscript received July 8, 2010.
Manuscript revised October 19, 2010.
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*This work was supported in part by ITRC through RBRC at GIST (NIPA-2010-(C1090-1011-0003)).
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DOI: 10.1587/transinf.E94.D.721

$$\begin{cases} h_0 + 2(h_1 + h_2 + h_3 + h_4) = \sqrt{2} \\ g_0 + 2(g_1 + g_2 + g_3) = \sqrt{2} \end{cases} . \tag{5}$$

Substituting $\omega = \pi$ into $H(\omega)$ of (2) and into $G(\omega)$ of (3), we obtain

$$h_0 + 2(-h_1 + h_2 - h_3 + h_4) = 0, \tag{6}$$

$$g_0 + 2(-g_1 + g_2 - g_3) = 0. \tag{7}$$

Taking the second derivative of (2), we have

$$2(-g_1 + 4g_2 - 9g_3) = 0. \tag{8}$$

A poly-phase matrix $P_a(z)$ is represented by

$$\begin{aligned} P_a(z) &= \begin{bmatrix} h_e(z) & g_o(z) \\ h_o(z) & g_e(z) \end{bmatrix} \\ &= \begin{bmatrix} h_0 + h_2\left(z + z^{-1}\right) + h_4\left(z^2 + z^{-2}\right) & g_1\left(1 + z^{-1}\right) + g_3\left(z + z^{-2}\right) \\ h_1(z+1) + h_3\left(z^2 + z^{-1}\right) & -g_0 - g_2\left(z + z^{-1}\right) \end{bmatrix}, \end{aligned} \tag{9}$$

where h_e and g_e contain even-number coefficients, and h_o and g_o contain odd-number coefficients.

The poly-phase matrix $P_a(z)$ can be factorized into five elementary matrices as in [2]:

$$\begin{aligned} P_a(z) &= \begin{bmatrix} 1 & \alpha\left(1 + z^{-1}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta(1+z) & 1 \end{bmatrix} \\ &\times \begin{bmatrix} 1 & \gamma\left(1 + z^{-1}\right) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta(1+z) & 1 \end{bmatrix} \begin{bmatrix} \xi & 0 \\ 0 & \frac{1}{\xi} \end{bmatrix}. \end{aligned} \tag{10}$$

Using (5) to (8) and making an equivalence of matrix $P_a(z)$ parameters in (9) and (10), we have a relationship of filter parameters that β, γ, δ, and ξ can be expressed by a parameter α as

$$\begin{cases} \beta = -\frac{1}{4(1+2\alpha)^2} \\ \gamma = -\frac{(1+2\alpha)^2}{1+4\alpha} \\ \delta = \frac{1}{16}\left(4 + \frac{1-8\alpha}{(1+2\alpha)^2} - \frac{2}{(1+2\alpha)^3}\right) \\ \xi = \frac{2\sqrt{2}(1+2\alpha)}{1+4\alpha} \end{cases} . \tag{11}$$

In the CDF 9/7-tap wavelet filter, prediction, updating, and normalized parameters are represented $\alpha = -1.586134342\ldots$, $\beta = -0.05298\ldots$, $\gamma = 0.882911\ldots$, $\delta = 0.443506\ldots$, and $\xi = 1.230174\ldots$ [8].

In our proposed scheme, α is examined in the $(-\infty, +\infty)$ range to find the optimized value for the 9/7-tap wavelet filters in terms of complexity of filter coefficients and performance. However, in the real situation, we can limit the range of the α value by calculating the statistic distribution of the optimized α value for our experimental database. For each α, we determine the $(\beta, \gamma, \delta, \xi)$ set using (11), and we compute the coefficients of h and g using (9) and (10). In this way, we can find the optimized α based on the maximum value of PSNR, i.e., PSNR_{max}, of the corresponding 9/7-tap wavelet filter and the optimized wavelet filter for each experimental image and compression ratio. Algorithm 1 is an algorithm for finding the optimized α value and PSNR_{max}.

Algorithm 1 Algorithm to find PSNR_{max} and α_{opt}.

1: Examine in the positive side of α;
2: Obtain PSNR_{max1} and α_{opt1};
3:
4: Examine in the negative side of α;
5: Obtain PSNR_{max2} and α_{opt2};
6:
7: **if** $\text{PSNR}_{max2} < \text{PSNR}_{max1}$ **then**
8: $\quad\text{PSNR}_{max} \leftarrow \text{PSNR}_{max1}$;
9: $\quad\alpha_{opt} \leftarrow \alpha_{opt1}$;
10: **else**
11: $\quad\text{PSNR}_{max} \leftarrow \text{PSNR}_{max2}$;
12: $\quad\alpha_{opt} \leftarrow \alpha_{opt2}$;
13: **end if**

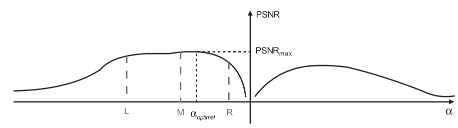


Fig. 1 Shape of PSNR as a function of α.

Algorithm 2 Bi-section algorithm

Require: L and R; PSNR(L) and PSNR(R)
1: s ← PSNR(L);
2: M ← (L+R)/2;
3: **if** PSNR(M) > s **then**
4: $\quad$L ← M;
5: **else**
6: $\quad$R ← M;
7: **end if**
8: Return to step 2 unless R-L is small enough;

Algorithm 1 shows our proposed method for designing the optimized wavelet filters. First, we examine α in both the negative and positive sides. In each side, we find a local PSNR_{max} and a locally optimized α. Then, we compare the value of the local PSNR_{max} in both sides. The higher PSNR_{max} is the final PSNR_{max} and we call its corresponding α as the final optimized α, denoted as α_{opt}. From α_{opt}, we set the optimized coefficients of the 9/7-tap wavelet filters.

The shape of PSNR curve for each 9/7-tap wavelet filter is a concave function of α, as shown in Fig. 1. This shape of a concave function will be confirmed by our experiments in the next section. In order to reduce the time spent in finding the optimized α, we employ a bi-section algorithm as shown in algorithm 2.

After selecting appropriate α, called the optimized α value, by using the above algorithms, we substitute α value into (11) to obtain other parameters, such as β, γ, δ, and ξ. Then, we use the optimized $(\alpha, \beta, \gamma, \delta, \xi)$ set to apply into the median lifting scheme as the following description.

2.2 Median Lifting Scheme

In image coding, the conventional lifting scheme disregards image edges. In this letter, we replace the conventional lifting scheme with the median lifting scheme and use the optimized $(\alpha, \beta, \gamma, \delta, \xi)$ set.

The N^{th}-order median filter is defined by

$$median_N\left(s\left(i,k\right)\right) = median\left(\left\{s\left(i,k-\left\lfloor \frac{N}{2} \right\rfloor\right),\ldots,\right.\right.$$
$$\left.\left. s\left(i,k+\left\lfloor \frac{N-1}{2} \right\rfloor\right)\right\}\right). \quad (12)$$

In order to show efficiency of median filtering for handling image edges, we extract nine pixel values from chin of "Grandma" image (row 313, columns 130-138). Then, we individually take the median and mean operations. While the mean filtering matches the wrong edges, we obtain the exact real edges by using the median operation. Figure 2 shows the result in detail. In proving the efficiency of median filtering for handling image edges by mathematics, some mathematical statisticians have successfully applied various models of median filters for images with edges [9].

The median lifting scheme consists of four steps. The splitting step is similar to that of the conventional lifting scheme. The prediction step is represented by

$$h\left(m,n\right) = s_o\left(m,n\right) - \sum_i p_i median_N\left(s_e\left(m,n+i\right)\right), \quad (13)$$

where p_i is the prediction parameter. We calculate the median of $s_e(m, n+i)$ and its $N-1$ neighboring even-number elements. Next, the updating step is represented by

$$l\left(m,n\right) = s_e\left(m,n\right) + \sum_j u_j median_M\left(h\left(m,n+j\right)\right), \quad (14)$$

where u_j is the updating parameter, and M denotes the order of the median updating filter. The (p_i, u_j) set is equivalent to the optimized $(\alpha, \beta, \gamma, \delta)$ set when we apply for a combination of filter optimization and median lifting. In the conventional lifting scheme used in the CDF 9/7-tap wavelet filter of JPEG2000, the (p_i, u_j) set is equivalent to the $(\alpha, \beta, \gamma, \delta)$ set. The last step is the normalization step which uses ξ parameter.

We use prediction, updating, and normalization parameters similarly to the conventional lifting scheme. We determine these parameters when we design the optimized wavelet filters in the decoding process as combination of filter optimization and median lifting. Hence, we can say that we encode and decode image using the identical lifting scheme.

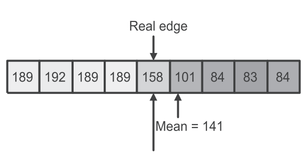


Fig. 2 Median operation for handling edges.

The median filtering will achieve some fixation in any type of problem due to distortion introduced in the transformation of the original information to the wavelet, because the median filtering is after all smoothing. Compared to the conventional lifting scheme, combination of the median operator and lifting scheme is robust because it is a nonlinear lifting scheme that regards neighboring elements. In addition, the median lifting schemes have been investigated for robust image de-noising [7]. Moreover, using both primal and dual lifting steps, the median lifting scheme produces much better image quality when the target image contains complex edges. Therefore, using the median operator and combination of filter optimization with median lifting, we can obtain improved results for edge-dominant images. This expectation will be confirmed by our experiments.

3. Experimental Results and Analysis

We conduct our experiments on the 8-bit edge-dominant images in our image database. For this purpose, we show "Grandma" image to represent the improvement of PSNR value and visual quality. Other images give the similar results. For comparison, we experiment on the conventional lifting scheme in the CDF 9/7-tap wavelet filter of JPEG2000 and the optimized median lifting scheme for lossy image compression.

We obtain a distribution of the optimized α for PSNR_{max} with α step size 0.1 in Fig. 3. By using this distribution, we limit the examination range of α. We calculate PSNR at four rates: 0.125 bits per pixel (bpp), 0.25 bpp, 0.5 bpp, and 1 bpp. In this figure, the maximum value of α, which is optimized in our proposed scheme, is observed that account for 21.88% of the experimental images in the database.

Figure 4 plots PSNR curves as concave functions of α for "Grandma" image. This image illustrates higher PSNR

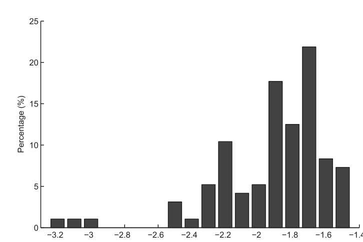


Fig. 3 Distribution of the optimized α for PSNR_{max}.

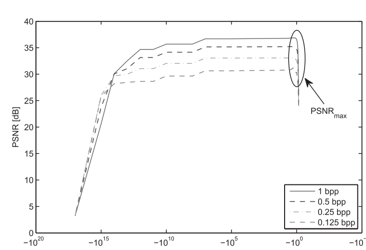
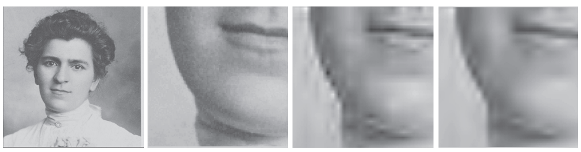


Fig. 4 PSNR curves as concave functions of α for "Grandma" image.

Table 1 Comparison of PSNR values.

Rate [bpp]	Scheme	PSNR values for test images [dB]						
		BIRD α_{opt}=-1.65	CLOWN α_{opt}=-1.55	GRANDMA α_{opt}=-1.68	LYNDA α_{opt}=-2.6	EINSTEIN α_{opt}=-3.2	ELAIN α_{opt}=-1.59	MRI α_{opt}=-1.52
0.125	Conventional	26.56	25.56	29.12	30.42	23.77	28.62	24.01
	Proposed	27.71	25.53	31.16	31.71	23.73	28.32	24.19
0.25	Conventional	29.49	27.56	32.2	33.8	25.49	29.87	26.61
	Proposed	30.87	27.63	32.98	35.56	26.23	30.14	26.76
0.5	Conventional	32.6	29.58	34.45	37.38	28.7	31.44	28.91
	Proposed	34.19	30.47	35.15	39.06	28.85	31.66	29.02
1	Conventional	36.27	32.5	36.35	40.38	30.6	32.47	31.46
	Proposed	37.77	33.72	36.89	41.48	31.18	32.78	32.37
Average PSNR gain [dB]		+1.41	+0.54	+1.02	+1.46	+0.36	+0.13	+0.34



(a) (b) (c) (d)

Fig. 5 Visual quality comparison for "Grandma" image at 0.125 bpp: (a) original image, (b) zoomed image, (c) conventional lifting scheme (29.12 dB), and (d) proposed lifting scheme (31.16 dB).

values and results in a better visual quality. Other test images show similar improvements. When using the bi-section algorithm to find PSNR$_{max}$ and α_{opt}, we assume that the PSNR values can be changed as a concave function of α. From Fig. 4, we find PSNR$_{max}$ and α_{opt}, as discussed in Sect. 2.

Table 1 lists the optimized α values for test images and PSNR values, obtained by different lifting schemes. As shown in Table 1, the optimized α values are rational numbers. For example, α_{opt} is -1.68 for "Grandma" image.

In order to compare coding performances of our proposed lifting scheme and that of the conventional lifting in the CDF-9/7 method of JPEG2000, we use PSNR gain. From Table 1, we observe that our proposed scheme obtains up to 0.75 dB of PSNR gain on average, compared to the conventional lifting scheme. The experimental results demonstrate that our proposed method outperforms the CDF-9/7 method of JPEG2000.

Compared to the CDF 9/7-tap wavelet filter of JPEG2000, the average PSNR gains of Ding et al. [5] and that of Liu et al. [6] methods are 0.57 dB and 0.69 dB, respectively. Therefore, our proposed method outperforms the existing methods in terms of the average PSNR gain.

Figure 5 compares the output picture qualities of different lifting schemes. For better visual comparison, we magnify some parts of "Grandma" image. From Fig. 5, we can observe significant visual quality improvement of our proposed scheme over the conventional lifting scheme in the CDF-9/7 of JPEG2000. In fact, the ringing artifact can be easily seen in the face, chin, and boundary of object in the image. Experimental results show that the ringing artifact is less severe in the output image of the proposed lifting scheme.

4. Conclusions

An optimized median lifting scheme has been proposed using the optimized wavelet filters and a median operator. The proposed lifting scheme outperforms existing schemes; especially, the PSNR value is significantly improved, by up to 0.75 dB on average, compared to that of the conventional lifting scheme in the CDF 9/7-tap wavelet filter of JPEG2000. Finally, the proposed lifting scheme also improves visual quality.

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