LETTER 3D Mesh Segmentation Based on Markov Random Fields and Graph Cuts*

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SUMMARY 3D Mesh segmentation has become an important research field in computer graphics during the past few decades. Many geometry based and semantic oriented approaches for 3D mesh segmentation has been presented. However, only a few algorithms based on Markov Random Field (MRF) has been presented for 3D object segmentation. In this letter, we present a definition of mesh segmentation according to the labeling problem. Inspired by the capability of MRF combining the geometric information and the topology information of a 3D mesh, we propose a novel 3D mesh segmentation model based on MRF and Graph Cuts. Experimental results show that our MRF-based schema achieves an effective segmentation.

key words: mesh segmentation, markov random field, graph cuts, SDF

1. Introduction

With the development of hardware and computer graphics, 3D Mesh segmentation has attracted many research works. Segmenting a complex 3D mesh into simpler sub-parts has become an important issue and challenging research. Approaches for 3D mesh segmentation can be distinguished into geometry based and semantic oriented [1]. In the former, the goal is to identify parts of the object that are homogeneous with respect to a criterion based on geometric properties. They usually identify homogeneous patches by using clustering techniques, such as region growing, iterative clustering or hierarchical clustering [2]. Geometry based approaches segment the 3D mesh into patches of homogeneous geometric properties and are typically used as pre-process for other geometry processing tasks. Curvature information is exploited by many works in this category. The actual principal curvatures and directions [3] are used to improve the segmentation results. In the latter, the goal is to identify parts of the object that are meaningful under some semantic or perceptive criteria. Many of these semantic oriented approaches introduce a bias which is assumed to

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help the segmentation algorithm to provide significant segmentations. Some examples of bias are feature points [4] and shape diameter function (SDF) [5].

Many research topics in digital image processing, such as image segmentation, image restoration, and image denoising, can be defined as a labeling problem. Any labeling problem can be regarded as a random field and each initial element is associated with a random variable [6]. There is a equivalence relation between labeling problem and random field. Therefore, solving a random field can be the same as solving a labeling problem. Markov Random Field (MRF) can be used to state and analysis the spatial or temporal properties of physical phenomenon. It provides a convenient and consistent way of modeling context-dependent entities and correlated features. A lot of approaches in 3D mesh segmentation have been made in the last few years. To our knowledge, only a few research works have used MRF model for 3D mesh processing: surface deformation [7], mesh smoothing [8]. However, only a few papers investigate random field model for mesh segmentation. Lavoué used MRF model to cluster the vertices with the roughness feature first, and then used the region growing method to segment the mesh [9]. Zouhar addressed the problem of 3D mesh segmentation for categories of objects and modeled the label distribution using Conditional Random Field to ensure semantic consistency in segmentation [10].

Inspired by their previous works, according to the MRF's theory, 3D mesh segmentation can be implemented with a labeling problem equivalent to MRF. We can introduce some priori information into the MRF, such as the features of segmentation boundary in 3D mesh. In this paper, we regard a mesh segmentation problem as a labeling problem whose solution is a set of labels assigned to faces, vertices or edges of the mesh according to the correlated features (e.g. SDF[5] etc) and the connectivity of the mesh. The SDF of faces is used as the correlated feature in our schema. To find the optimal label configuration, we propose a new mesh segmentation model based on MRF theory and Graph cuts algorithm. Experimental results show that our proposed schema is very effective in 3D mesh segmentation and over segmentation is also avoided.

2. Definition of Mesh Segmentation

A labeling problem is specified in term of a set of sites and a set of labels. A site often represents a point or a region in the Euclidean space such as a vertex, an edge, or a face of the

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mesh, and a label set can be categorized as being continuous or discrete. We use the discrete case of a label set for mesh segmentation in this paper.

Let a tuple $M = \{V, E, T\}$ be a 3D mesh M with vertices $V = \{v_i | v_i \in \mathbb{R}^3, 1 \le i \le n\}$, edges $E = \{e_{ij} =$ $(i, j)|v_i, v_i \in V, i \neq j$, and faces which are usually triangles $T = \{t_{ij} = (i, j, k) | v_i, v_j, v_k \in V, i \neq j, j \neq k, k \neq i\}$ [2]. Supposed that $S = \{1, 2, 3, \dots, m\}$ index a discrete set of sites, in which 1, 2, 3, \cdots , *m* are indices, $L = \{1, 2, \cdots, K\}$ be a set of K labels. Then mesh segmentation can be regarded as the process to assign a label $f_i \in L$ to the site $i \in S$. The set $f = \{f_1, f_2, \dots, f_m\}$ is called a labeling or a configuration (in random field theory) of the sites indexed S in terms of the labels in L, and the set $F = L \times L \times \cdots \times L$ is called configuration space which is the set of all possible configurations. A configuration *finF* segments the mesh model into different components. Specially, in some cases two of these components might be assigned the same label, but are disjoint in spatial.

Because each site (either a vertex, edge or face) is assigned a unique label in mesh segmentation, the assigned process $f_i = f(i)$ can be regarded as a mapping from S to L, i.e. $f : S \to L$, and f(S) is a configuration in F. In this paper, the mesh segmentation is defined as follows:

Definition 1: Let $M = \{V, E, T\}$ be a 3D mesh model, $S = \{1, 2, 3, \dots, m\}$ be the set of the either indices of vertices, edges or faces, and $L = \{1, 2, \dots, K\}$ be the set of labels. A segmentation of M is a configuration in F obtained by $f: S \rightarrow L$.

To evaluate the configurations $f \in F$, we define an energy function of f as $E(f) : F \to R$. Then we deem a mesh segmentation problem as an optimal problem in the following manner:

Definition 2: Given a 3D mesh model $M = \{V, E, T\}$, a set of sites $S = \{1, 2, 3, \dots, m\}$ and a set of labels $L = \{1, 2, \dots, K\}$, then mesh segmentation can be viewed as the optimization problem finding out a configuration $f \in F$ to minimize (or maximize) the energy function E(f).

3. MRF-Based Segmentation Model

3.1 Algorithm Overview

As the sites of a 3D mesh model are spatial dependent, if only the feature information is considered, the segmentation induced by the configuration may be noisy and oversegmentation. Previous research shows that the edge with a concave dihedral angle across it is a better candidate for the boundary between two components. Therefore, we must take into account some geometric constraints in our mesh segmentation model, including feature and spatial dependency, smoothness of the boundaries between components.

Here, we summarize the procedure of the algorithm as Fig. 1 in order to more comprehensively understand it, and we focus on the situation of S = T.



Fig. 1 Workflow of our proposed shcema.

3.2 Higher-Level Distribution

To express the spatial dependencies between sites in S of the mesh M, we build the dual graph G of M by representing each site in S by a node in G and defining the edges in G by adjacency relation in M of the sites of S like in [2].

Then we define the neighborhood system on the graph G: the neighborhood of site (i.e. node) *i* of is $N_i = \{j|i, j \text{ is connected by an edge in } G\}$, and the spatial dependencies between sites can be express as follow: site *i* is depend on site *j* in spatial if site *j* is a neighbor of site *i*. This neighborhood system defines a set of cliques, in which a clique is a fully connected sub graph.

The higher-level distribution determines the prior information of a configuration of the random field on the spatial dependencies between sites through the distribution of the configuration P(f). According to the Hamersley-Clifford theorem [11], P(f) is a Gibbs distribution:

$$P(f) = \frac{1}{Z} \exp(-U(f)) \tag{1}$$

where $U(f) = \sum_{c \in C} V_c(f)$ is a energy function defined as the sum of energy potentials function over all possible cliques C, $V_c(f)$ is the energy potential function for the configuration f define on spatial dependencies in the clique c, and $Z = \sum_f e^{-U(f)}$ is a normalization constant.

In our case, we use potts model to define the energy potential functions $V_{(i,j)}(f)$ on 2-site cliques (i.e. edges in the graph *G*):

$$V_{(i,j)}(f) = \begin{cases} 1, & if \ f_i \neq f_j \\ 0, & else \end{cases}$$
(2)

where $\{i, j\}$ is an edge in the graph G. And define the energy potential functions on other cliques equal to constant zero.

When the set of sites S = T, we can inject the prior information about smoothness of boundaries by defining a cost function for all edges in the mesh M (or a weight function for all edges in the graph G). Since the edge with a concave dihedral angle is better for a boundary, given an edge $\{i, j\}$, and $\theta_{\{i, j\}}$, the angle between the normal vectors of the two faces which are adjacent to the edge $\{i, j\}$, we can define the cost function to be: $Cost(i, j) = \ln(\alpha \frac{\theta_{[i, j]}}{\pi})$, where $\alpha = 1$ for the concave dihedral angle, and α equal to a small positive constant for the convex dihedral angle.

Then we can modify the energy potential functions $V_{(i,j)}(f)$ as:

$$V_{(i,j)}(f) = \begin{cases} Cost(i,j), & if \ f_i \neq f_j \\ 0, & else \end{cases}$$
(3)

Now, we can get the prior information of configurations: $P(f) = \frac{1}{Z} \exp\left(-\beta * \sum_{(i,j)\in C} V_{(i,j)}(f)\right)$, where β is an interaction coefficient controlling the weight of the prior information.

3.3 Lower-Level Distribution

To assign *K* labels to the set of sites *S* based on the correlated feature values, we fit *K* Gaussian to the histogram of feature values using Gaussian mixture model. First, we denote the random variable whose realization is the feature value of site *i* of the mesh as x_i , denote the family of these random variables on *S* as $X = \{x_1, x_2, \dots, x_N\}$, and denote the joint event $\{x_1 = x_1, x_2 = x_2, \dots, x_N = x_N\}$ as $x = \{x_1, x_2, \dots, x_N\}$. Then according to the GMM, we can express the density $p(x_i)$ as follow:

$$p(x_i) = \sum_{k=1}^{K} \pi_k N(x_i | \mu_k, \sigma_k)$$
(4)

where, μ_k is the mean value, σ_k is the standard deviation and π_k is the weight of the *k*-th Gaussian, *K* is the number of labels, and $N(\mathbf{x}_i|\mu_k, \sigma_k)$ is the probability of x_i conditioned on label *k*:

$$N(\mathbf{x}_i|\boldsymbol{\mu}_k, \boldsymbol{\sigma}_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(\mathbf{x}_i - \boldsymbol{\mu}_k)^2}{2\sigma_k^2}\right)$$
(5)

And the likelihood function for the sites set S is

$$\prod_{i=1}^{N} \left(\sum_{k=1}^{K} \pi_k N(\mathbf{x}_i | \mu_k, \sigma_k) \right)$$
(6)

The probability of that assigned a label to a site can be described as follow: $p(\mathbf{x}_i|f_i)$ obey a normal distribution $N(\mu, \sigma)$, and each label $k \in L = \{1, 2, \dots, K\}$ is represented by its mean value μ_k and standard deviation σ_k :

$$N(\mu_k, \sigma_k) = \frac{1}{\sqrt{2\pi\sigma_k}} \exp\left(-\frac{(x-\mu_k)^2}{2\sigma_k^2}\right)$$
(7)

We use EM (Expectation-Maximization) algorithm to estimate the weights $\pi = {\pi_1, \dots, \pi_K}$, the mean values $\mu = {\mu_1, \dots, \mu_K}$, and the standard deviations $\sigma = {\sigma_1, \dots, \sigma_K}$ by maximizing the log of likelihood function 6

$$\ln p(X|\mu\sigma) = \sum_{i=1}^{N} \ln\left(\sum_{k=1}^{K} \pi_k N(\mathbf{x}_i|\mu_k,\sigma_k)\right)$$
(8)

Then we can get the density of X conditioned on the configuration f:

$$p(X|f) = \prod_{i=1}^{N} p(\mathbf{x}_i|f_i) = \prod_{i=1}^{N} N(\mathbf{x}_i|\mu_k, \sigma_k)$$
(9)

3.4 Energy Function

In conclusion, given the feature values of all sites, our goal is to find the most probable configuration f^* such that the posterior probability $P(f^*|x)$ is max. As the feature values of all sites is given, p(x) is a constant. According to the Bayesian rule: $p(f|x) = \frac{p(f)p(x|f)}{p(x)}$, we can express f^* as : $f^* = \arg \max (p(f)p(x|f))$. Then, we denote U(f|x) as follow: $U(f|x) = \beta U(f) + U(x|f)$, where, $U(x|f) = -\sum_{i=1}^{N} \left(\ln \left(\sqrt{2\pi}\sigma_{f_i} \right) + \frac{(x_i - \mu_{f_i})^2}{2\sigma_{f_i}^2} \right)$.

Now the posterior probability is $p(f|x) \propto \exp(-U(f|x))$. Then we define the energy function E(f) as $E(f) = U(f|x) = \beta U(f) + U(x|f)$, and f^* as $f^* = \arg\min_f (E(f))$. According to [13], we can use graph cuts algorithm to minimize E(f).

4. Experimental Results and Analysis

4.1 Run Time Analysis of our Segmentation

In order to demonstrate the efficiency of our algorithm for 3D mesh segmentation based on MRF and graph cuts, we conduct experiments to segment different 3D meshes with different numbers of faces and for different numbers of labels, according to their SDF [5]. Table 1 detail the processing times for the different 3D meshes which are presented in Fig. 2, the first row are original 3D mesh models, and the second row are the result of our 3D mesh segmentation algorithm do with these models.

4.2 Comparison with K-Means Based Segmentation

We have also compared our 3D mesh segmentation algo-

 Table 1
 Processing time of our Segmentation algorithm based on MRF.

Name	Faces Num	Cluster Num	Time EM(s)	Time GC(s)
cow	5804	3	0.1	0.05
dinopet	8996	5	0.21	0.20
camel	19536	4	0.45	0.34
armadillo	52000	5	6.03	1.13



Fig. 2 Original 3D mesh model and the result of our algorithm with parameters: (a) $\beta = 7.0$; (b) $\beta = 8.0$; (c) $\beta = 15.0$; (d) $\beta = 10.0$.



Fig. 3 Compare with K-Means algorithm.



Fig. 4 Comparison with HFP segmentation algorithm.

rithm based on MRF with the algorithm based on K-Means. Figure 3 illustrates the clustering of the dinopet model (4 K vertices) according to its SDF, into 3 clusters. The noise introduced by the K-Means classification has been almost entirely removed by using our 3D mesh segmentation algorithm based on MRF.

4.3 Comparison with HFP Segmentation

To further illustrate the effectiveness of our schema, we compare it with the algorithm: Hierarchical mesh segmentation based on Fitting Primitives (HFP) [15]. The HFP algorithm implemented by the author are directly used to obtain the segmentation results and compare with our schema in this paper. Here, we configure three fitting primitives with plane, cylinder and sphere together. Obviously, for the two objects dinopet and horse, HFP-based segmentation can not provide better geometric and semantic segmentation. Different from HFP, as shown in Fig. 4, our schema (MRF) with different interactive control parameters β provides more convenient and different perceptual and geometric results. We can select different parameters β to achieve different segmentation results. From the overall segmentation effect, our segmentation can present a better semantic and geometric results than HFP. Therefore, MRF based segmentation can give better segmentation results than HFP algorithm.

5. Conclusions and Future Research

This paper presents a definition of mesh segmentation and a new 3D mesh segmentation algorithm based on Markov Random Fields. The algorithm makes it possible to combine both attribute values and spatial constraints in the labeling. In a global optimization process, two levels Gibbs Random Field are used to model the geometric information and the topology information of 3D mesh. Experiment results demonstrate the efficiency and effectiveness of our algorithm. We can also select different attribution of faces, edges or vertices to test our MRF-based segmentation model in the future.

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