LETTER

Outage Performance for Antenna Selection in AF Two-Way Relaying System with Channel Estimation Error

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SUMMARY This letter investigates the outage performance of a joint transmit and receive antenna selection scheme in an amplify-and-forward two-way relaying system with channel estimation error. A closed-form approximate outage probability expression is derived, based on which the asymptotic outage probability expression is derived to get an insight on system's outage performance at high signal-to-noise (SNR) region. Monte Carlo simulation results are presented to verify the analytical results.

key words: *two-way relaying, antenna selection, outage probability, diversity order*

1. Introduction

Two-way relaying has attracted enormous research interest from the wireless community in recent years, due to its ability to enhance spectral efficiency [1]. When the nodes in two-way relaying system are equipped with multiple antennas, the system's throughput and transmission reliability can be further enhanced [2], [3]. However, it is typically necessary to manipulate pre-coding and/or post-processing at sources and/or relay to fully exploit the merits of mounting multiple antennas, which imposes higher computational burden on sources and/or relay. To circumvent this drawback, the authors in [4], [5] have proposed a joint transmit and receive antenna selection (JTRAS) scheme for amplify-and-forward (AF) two-way relaying system with multiple antennas, aiming to achieve the full diversity order. However, the works in [4], [5] have assumed perfect channel state information (CSI) at both sources and the relay. To the best of our knowledge, the impact of channel estimation error (CEE) on the performance of the JTRAS scheme remains unknown in the available literature. Note that the effect of CEE in one-way relaying system has been investigated in [6]. Nevertheless, the analysis method for one-way relaying cannot be applied to two-way relaying directly due to the bidirectional transmission flows in two-way relaying system.

In this letter, we analyze the outage performance of JTRAS scheme proposed in [4], [5] by taking into account the CEE. Firstly, we derive a closed-form approximate outage probability expression. To get better insights, an asymptotic outage probability expression at high signal-to-noise ratio (SNR) region is derived to reveal the achievable diversity order. Finally, we conduct Monte Carlo simulations to verify the analytical results. Both analytical and simulation results reveal that (i) when the variance of CEE is a decreasing function of transmitting power, the system can still achieve full diversity order, and (ii) when the variance of CEE is independent of transmitting power, the diversity order tends to zero.

Manuscript received April 12, 2012.

Manuscript revised January 28, 2013.

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DOI: 10.1587/transinf.E96.D.1552

2. System Model

We consider an AF two-way relaying system that consists of two source nodes (denoted by S_1 and S_2) and one relay node (denoted by R). S_1 and S_2 exchange information via the aid of R, and thus, also act as destinations. Specially, S_1, S_2 and R are mounted with N_1, N_2 and N_R antennas, respectively. We assume that both sources and the relay operate in half-duplex mode, and no direct link between S_1 and S_2 exists due to high shadowing. The channels between S_l, $l = 1, 2$ and R undergo block, flat, Rayleigh fading. Assuming time-division duplex is adopted, then the channels are reciprocal, namely, the channel gains of links $S_l \rightarrow R$ and $R \rightarrow S_l$ are identical in one round of information exchange. Let the channel between the ith antenna of S_1 and the kth antenna of R be denoted as h_{ik}, and the channel between the jth antenna of S_2 and the kth antenna of R be denoted as g_{jk}, where $1 \leq i \leq N_1$, $1 \leq j \leq N_2$, $1 \leq k \leq N_R$. h_{ik} and g_{jk} are modeled as zero mean complex Gaussian random variables with variances Ω_h and Ω_g, respectively. We also assume that perfect information on noise powers at all nodes in the system is available.

One round of information exchange consists of two phases. In the first phase, S_1 and S_2 transmit simultaneously to R using the ith and jth antenna, respectively. Assuming that perfect synchronization among all the nodes in the system has been established, the received signal at the kth antenna of R can be expressed as

$$y_r = \sqrt{p_1} h_{ik} s_1 + \sqrt{p_2} g_{jk} s_2 + n_r \tag{1}$$

where p_l and s_l denote the transmitting power and unit energy modulated signal of S_l, $l = 1, 2$, n_r denotes the additive white Gaussian noise (AWGN) at R with zero mean and variance σ_n^2. In the second phase, R amplifies y_r by a factor

$$G = \sqrt{1 \Big/ \left[p_1 \left\| \hat{h}_{ik} \right\|^2 + p_2 \left\| \hat{g}_{jk} \right\|^2 + \sigma_n^2 \right]} \tag{2}$$

and broadcasts to both destinations using the kth antenna. $\hat{h}_{ik}$ and $\hat{g}_{jk}$ in (2) denote the estimates of h_{ik} and g_{jk}, respectively. Assuming a least mean squares estimator, we have the following relationship [6]

$$h_{ik} = \hat{h}_{ik} + e_{h_{ik}},\ g_{jk} = \hat{g}_{jk} + e_{g_{jk}} \tag{3}$$

where $e_{h_{ik}}$ and $e_{g_{jk}}$ denote the CEEs, and are modeled as zero mean complex Gaussian variables with variances $\sigma^2_{e_{h_{ik}}}$ and $\sigma^2_{e_{g_{jk}}}$, respectively. For simplicity, we make a reasonable assumption that $\sigma^2_{e_{h_{ik}}} = \sigma^2_{e_h}$, $\sigma^2_{e_{g_{jk}}} = \sigma^2_{e_g}\ \forall i, j, k$. Since $\hat{h}_{ik}$ is independent of $e_{h_{ik}}$ and $\hat{g}_{jk}$ is independent of $e_{g_{jk}}$ [6], $\hat{h}_{ik}$ and $\hat{g}_{jk}$ are also zero mean complex Gaussian variables with variances $\Omega_{\hat{h}} = \Omega_h - \sigma^2_{e_h}$ and $\Omega_{\hat{g}} = \Omega_g - \sigma^2_{e_g}$, respectively. As perfect synchronization is assumed, the received signal at S_1 can be written as

$$\begin{aligned} y_1 = {} & \sqrt{p_1 p_3} G(\hat{h}_{ik} + e_{h_{ik}})^2 s_1 + \sqrt{p_2 p_3} G(\hat{h}_{ik} + e_{h_{ik}}) \\ & \times (\hat{g}_{jk} + e_{g_{jk}}) s_2 + \sqrt{p_3} G(\hat{h}_{ik} + e_{h_{ik}}) n_r + n_1 \end{aligned} \tag{4}$$

in which p_3 denotes the transmitting power of R, and n_1 denotes the AWGN at S_1 with zero mean and variance σ^2_n. After self-information cancelation and some elementary manipulation, the effective SNR at S_1 can be expressed as

$$\gamma_1^{i,j,k} = \frac{\bar{p}_2 \bar{p}_3 \left\|\hat{h}_{ik}\right\|^2 \left\|\hat{g}_{jk}\right\|^2}{\left[\begin{array}{l} \left(4\bar{p}_1\bar{p}_3\sigma^2_{e_h} + \bar{p}_2\bar{p}_3\sigma^2_{e_g} + \bar{p}_3 + \bar{p}_1\right)\left\|\hat{h}_{ik}\right\|^2 \\ + \left(\bar{p}_2\bar{p}_3\sigma^2_{e_h} + \bar{p}_2\right)\left\|\hat{g}_{jk}\right\|^2 + 2\bar{p}_1\bar{p}_3\sigma^4_{e_h} \\ + \bar{p}_2\bar{p}_3\sigma^2_{e_h}\sigma^2_{e_g} + \bar{p}_3\sigma^2_{e_h} + 1 \end{array}\right]} \tag{5}$$

where $\bar{p}_l = p_l/\sigma^2_n$, $l = 1, 2, 3$. Since the last four terms in the denominator of (5) are of small values in practice where both estimation error and noise variances are small, they can be reasonably ignored and then (5) can be approximated as

$$\gamma_1^{i,j,k} = \frac{\left\|\hat{h}_{ik}\right\|^2 \left\|\hat{g}_{jk}\right\|^2}{a\left\|\hat{h}_{ik}\right\|^2 + b\left\|\hat{g}_{jk}\right\|^2} \tag{6}$$

$$a = \left(4\bar{p}_1\bar{p}_3\sigma^2_{e_h} + \bar{p}_2\bar{p}_3\sigma^2_{e_g} + \bar{p}_3 + \bar{p}_1\right)\Big/\bar{p}_2\bar{p}_3 \tag{7}$$

$$b = \left(\bar{p}_2\bar{p}_3\sigma^2_{e_h} + \bar{p}_2\right)\Big/\bar{p}_2\bar{p}_3 \tag{8}$$

Following the same approach as conducted to obtain (6) and assuming the variance of AWGN at S_2 is σ^2_n, we get the approximate effective SNR at S_2

$$\gamma_2^{i,j,k} = \frac{\left\|\hat{h}_{ik}\right\|^2 \left\|\hat{g}_{jk}\right\|^2}{c\left\|\hat{h}_{ik}\right\|^2 + d\left\|\hat{g}_{jk}\right\|^2} \tag{9}$$

$$c = \left(\bar{p}_1\bar{p}_3\sigma^2_{e_g} + \bar{p}_1\right)\Big/\bar{p}_1\bar{p}_3 \tag{10}$$

$$d = \left(4\bar{p}_2\bar{p}_3\sigma^2_{e_g} + \bar{p}_1\bar{p}_3\sigma^2_{e_h} + \bar{p}_3 + \bar{p}_2\right)\Big/\bar{p}_1\bar{p}_3 \tag{11}$$

In order to achieve the full diversity order and reduce the processing payload at both sources and the relay, the authors in [4], [5] have proposed the following antenna selection criterion

$$\{i^*, j^*, k^*\} = \mathop{\arg\max}_{1\le i\le N_1, 1\le j\le N_2, 1\le k\le N_R} \left[\min\left(\gamma_1^{i,j,k}, \gamma_2^{i,j,k}\right)\right] \tag{12}$$

where i^*, j^* and k^* are the best antenna indices at S_1, S_2 and R, respectively. In the following section, we analyze the outage performance of this antenna selection scheme in the presence of CEE. It is worthy noting that the differences between the analysis in this letter and that in [6] mainly consist of four aspects: 1) This letter focuses on two-way relaying, while the main concern of [6] is one-way relaying. Different from one-way relaying, the outage probability in two-way relaying is defined as the probability that the minimum of SNRs at two destinations falls below a predefined threshold [5]. Since the SNRs at two destinations are dependent, it is more complicated to investigate the effect of CEE on outage performance in two-way relaying, as compared to the analysis in [6]; 2) The most key part of analysis in [6] is to use $0.5\min(x, y) \le \frac{xy}{x+y} \le \min(x, y)$. However, this inequality cannot be used to derive the closed-form outage probability expression in this letter; 3) The sources/destinations in this letter are mounted with multiple antennas, while the source/destination in [6] is equipped with single antenna. This difference requires us to employ different method to derive the outage probability; 4) In two-way relaying, the CEE leads to residual self-noise, while no self-noise exists in one-way relaying [6]. As seen from (5)–(11), this difference results in different SNRs forms as compared to [6].

3. Outage Performance Analysis

3.1 Approximate Outage Probability

In the AF two-way relaying system, an outage event occurs when either $\gamma_1^{i^*,j^*,k^*}$ or $\gamma_2^{i^*,j^*,k^*}$ falls below a predefined threshold γ_{th}. So, the outage probability can be expressed as [5]

$$\begin{aligned} P_{out}(\gamma_{th}) &= \Pr\left[\min\left(\gamma_1^{i^*,j^*,k^*}, \gamma_2^{i^*,j^*,k^*}\right) < \gamma_{th}\right] \\ &= \Bigg\{\underbrace{\Pr\left[\min\left(\gamma_1^{i^*,j^*,k}, \gamma_2^{i^*,j^*,k}\right) < \gamma_{th}\right]}_{F(\gamma_{th})}\Bigg\}^{N_R} \end{aligned} \tag{13}$$

in which $\Pr(x)$ denotes the probability of a random variable x, $\gamma_1^{i^*,j^*,k}$ and $\gamma_2^{i^*,j^*,k}$ are given by

$$\gamma_1^{i^*,j^*,k} = \frac{X_k Y_k}{aX_k + bY_k},\ \gamma_2^{i^*,j^*,k} = \frac{X_k Y_k}{cX_k + dY_k} \tag{14}$$

where $X_k = \max_{1\le i\le N_1} \left\|\hat{h}_{ik}\right\|^2$, $Y_k = \max_{1\le j\le N_2} \left\|\hat{g}_{jk}\right\|^2$. Thus, the remaining problem is to solve $F(\gamma_{th})$. According to the results in [7], $F(\gamma_{th})$ in (13) can be approximated as

$$F(\gamma_{th}) = F_1(\gamma_{th}) + F_2(\gamma_{th}) - F_1(\gamma_{th}) F_2(\gamma_{th}) \tag{15}$$

where $F_l(\gamma_{th}) = \Pr\left(\gamma_l^{i^*,j^*,k} < \gamma_{th}\right)$ $(l = 1, 2)$ denotes the cumulative distribution function (CDF) of $\gamma_l^{i^*,j^*,k}$. Because $\gamma_1^{i^*,j^*,k}$ and $\gamma_2^{i^*,j^*,k}$ have similar forms, we focus on the derivation of $F_1(\gamma_{th})$ in the following.

We start our derivation by writing $\gamma_1^{i^*,j^*,k}$ in a more tractable form as follows:

$$\gamma_1^{i^*,j^*,k} = (A_k + B_k)^{-1} \tag{16}$$

where $A_k = b/X_k$, $B_k = a/Y_k$. By using order statistics [8], the probability density function (PDF) of X_k can be written as

$$f_{X_k}(x) = \frac{N_1}{\Omega_{\hat{h}}} \sum_{p=0}^{N_1-1} \binom{N_1-1}{p} (-1)^p e^{-\frac{p+1}{\Omega_{\hat{h}}}x} \tag{17}$$

By using Jacobian transformation between X_k and A_k, we get the PDF of A_k

$$f_{A_k}(x) = \frac{N_1}{\Omega_{\hat{h}}} \sum_{p=0}^{N_1-1} \binom{N_1-1}{p} (-1)^p \frac{b}{x^2} e^{-\frac{(p+1)b}{\Omega_{\hat{h}}x}} \tag{18}$$

Then, with the help of [9, Eq. (3.471.9)], the moment generating function (MGF) of A_k can be given by

$$M_{A_k}(s) = \frac{2N_1}{\Omega_{\hat{h}}} \sum_{p=0}^{N_1-1} \left\{ \begin{array}{l} \binom{N_1-1}{p} (-1)^p \sqrt{\frac{\Omega_{\hat{h}} bs}{p+1}} \\ \times K_1\left(2\sqrt{\frac{(p+1)bs}{\Omega_{\hat{h}}}}\right) \end{array} \right\} \tag{19}$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind [9, Eq. (8.432.6)]. Following the similar steps as (17)–(19), the MGF of B_k can be written as

$$M_{B_k}(s) = \frac{2N_2}{\Omega_{\hat{g}}} \sum_{q=0}^{N_2-1} \left\{ \begin{array}{l} \binom{N_2-1}{q} (-1)^q \sqrt{\frac{\Omega_{\hat{g}} as}{q+1}} \\ \times K_1\left(2\sqrt{\frac{(q+1)as}{\Omega_{\hat{g}}}}\right) \end{array} \right\} \tag{20}$$

By using the Theorem 1 in [10] and (19) and (20), the CDF of $\gamma_1^{i^*,j^*,k}$ can be expressed as

$$F_1(\gamma_{th}) = H(\gamma_{th}, a, b) \tag{21}$$

The function $H(\cdot,\cdot,\cdot)$ is presented at the top of next page.

Following the similar steps as (16)–(21), the CDF of $\gamma_2^{i^*,j^*,k}$ can be obtained as

$$F_2(\gamma_{th}) = H(\gamma_{th}, c, d) \tag{23}$$

Substituting (15), (21) and (23) into (13), we get the approximate outage probability $P_{out}^a(\gamma_{th})$, which is shown at the top of next page.

3.2 High SNR Analysis

For the purpose of getting better insights, we derive the asymptotic outage probability at high SNR region in the following. As the function $K_1(x)$ can be approximated as $1/x$ for small x [7], $F_1(\gamma_{th})$ can be approximated as

$$\begin{aligned} F_1(\gamma_{th}) = 1 - &\underbrace{N_1 \sum_{p=0}^{N_1-1} \binom{N_1-1}{p} \frac{(-1)^p}{p+1} e^{-\gamma_{th}\frac{(p+1)b}{\Omega_{\hat{h}}}}}_{E_1} \\ &\times \underbrace{N_2 \sum_{q=0}^{N_2-1} \binom{N_2-1}{q} \frac{(-1)^q}{q+1} e^{-\gamma_{th}\frac{(q+1)a}{\Omega_{\hat{g}}}}}_{E_2} \end{aligned} \tag{25}$$

Using Taylor series expansion, E_1 in (25) can be written by

$$E_1 = N_1 \sum_{p=0}^{N_1-1} \binom{N_1-1}{p} \frac{(-1)^p}{p+1} \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\gamma_{th}\frac{(p+1)b}{\Omega_{\hat{h}}}\right)^k \tag{26}$$

By using the following equations

$$N \sum_{p=0}^{N-1} \binom{N-1}{p} \frac{(-1)^p}{p+1} = 1 \tag{27}$$

$$\sum_{p=0}^{N-1} \binom{N-1}{p} (-1)^p (p+1)^k = 0 \, for \; k = 0 \sim N-2 \tag{28}$$

(The proof of (27), (28) is presented in Appendix) E_1 can be written as

$$\begin{aligned} E_1 = 1 + N_1 \sum_{p=0}^{N_1-1} \binom{N_1-1}{p} \frac{(-1)^{p+N_1}}{(p+1)N_1!} \left(\gamma_{th}\frac{(p+1)b}{\Omega_{\hat{h}}}\right)^{N_1} \\ + o\left[\left(\gamma_{th}\frac{(p+1)b}{\Omega_{\hat{h}}}\right)^{N_1+1}\right] \end{aligned} \tag{29}$$

where $o(x)$ represents $\lim_{x\to 0} \frac{o(x)}{x} = 0$. Following the similar steps as (26)–(29), E_2 can be written as

$$\begin{aligned} E_2 = 1 + N_2 \sum_{q=0}^{N_2-1} \binom{N_2-1}{q} \frac{(-1)^{q+N_2}}{(q+1)N_2!} \left(\gamma_{th}\frac{(q+1)a}{\Omega_{\hat{g}}}\right)^{N_2} \\ + o\left[\left(\gamma_{th}\frac{(q+1)a}{\Omega_{\hat{g}}}\right)^{N_2+1}\right] \end{aligned} \tag{30}$$

Substituting (29) and (30) into (25), $F_1(\gamma_{th})$ at high SNR region can be approximated by

$$F_1(\gamma_{th}) = C(a,b)\gamma_{th}^K \tag{31}$$

$$K = \min(N_1, N_2) \tag{32}$$

The function $C(\cdot,\cdot)$ is presented at the top of the next page. Following the similar steps as (25)–(31), $F_2(\gamma_{th})$ at high SNR region can be approximated as

$$F_2(\gamma_{th}) = C(c,d)\gamma_{th}^K \tag{34}$$

Substituting (15), (31) and (34) into (13), the asymptotic outage probability expression at high SNR is given by

$$P_{out}^{\infty}(\gamma_{th}) = (C(a,b) + C(c,d))^{N_R} \gamma_{th}^{KN_R} \tag{35}$$

Now, we employ (35) to analyze the impact of CEE on diversity order. Without loss of generality, we assume $\bar{p}_1 = \bar{p}_2 = \bar{p}_3 = \bar{p}_0$ and $\sigma_{e_h}^2 = \sigma_{e_g}^2 = \sigma^2$. We consider two CEE models: (i) the variance of CEE reduces with increasing SNR $\bar{p}_0$, and is formulated by $\sigma^2 = 1/(L\bar{p}_0)$, where L is the length of training sequence. By inspection on a, b, c, d and $P_{out}^{\infty}(\gamma_{th})$, it is observed that (35) can be written in the form $P_{out}^{\infty}(\gamma_{th}) = D(\bar{p}_0\gamma_{th})^{-KN_R}$, where D is a constant

$$H\left(\gamma_{th},a,b\right)=1-N_1N_2\sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1}\left\{\binom{N_1-1}{p}\binom{N_2-1}{q}(-1)^{p+q}\,e^{-\gamma_{th}\left(\frac{(p+1)b}{\Omega_{\hat{h}}}+\frac{(q+1)a}{\Omega_{\hat{g}}}\right)}\sqrt{\frac{4\gamma_{th}^2ab}{(p+1)(q+1)\Omega_{\hat{h}}\Omega_{\hat{g}}}}K_1\left(\sqrt{\frac{4\gamma_{th}^2(p+1)(q+1)ab}{\Omega_{\hat{h}}\Omega_{\hat{g}}}}\right)\right\} \tag{22}$$

$$P_{out}^{a}\left(\gamma_{th}\right)=\left\{1-N_1^2N_2^2\sum_{p=0}^{N_1-1}\sum_{q=0}^{N_2-1}\sum_{u=0}^{N_1-1}\sum_{v=0}^{N_2-1}\left\{\begin{array}{l}\binom{N_1-1}{p}\binom{N_2-1}{q}\binom{N_1-1}{u}\binom{N_2-1}{v}(-1)^{p+q+u+v}\,e^{-\gamma_{th}\left[\frac{(p+1)b+(u+1)d}{\Omega_{\hat{h}}}+\frac{(q+1)a+(v+1)c}{\Omega_{\hat{g}}}\right]}\\ \sqrt{\frac{16\gamma_{th}^4abcd}{(p+1)(q+1)(u+1)(v+1)\Omega_{\hat{h}}^2\Omega_{\hat{g}}^2}}K_1\left(\sqrt{\frac{4\gamma_{th}^2(p+1)(q+1)ab}{\Omega_{\hat{h}}\Omega_{\hat{g}}}}\right)K_1\left(\sqrt{\frac{4\gamma_{th}^2(u+1)(v+1)cd}{\Omega_{\hat{h}}\Omega_{\hat{g}}}}\right)\end{array}\right\}\right\}^{N_R} \tag{24}$$

$$C\left(a,b\right)=\begin{cases}N_1\sum_{p=0}^{N_1-1}\binom{N_1-1}{p}\frac{(-1)^{p+N_1+1}}{(p+1)\,N_1!}\left(\frac{(p+1)\,b}{\Omega_{\hat{h}}}\right)^{N_1} \quad for\,N_1<N_2, \quad N_2\sum_{q=0}^{N_2-1}\binom{N_2-1}{q}\frac{(-1)^{q+N_1+1}}{(q+1)\,N_2!}\left(\frac{(q+1)\,a}{\Omega_{\hat{g}}}\right)^{N_2} \quad for\,N_1>N_2\\ N\sum_{p=0}^{N-1}\binom{N-1}{p}\frac{(-1)^{p+N+1}}{(p+1)\,N!}\left[\left(\frac{(p+1)b}{\Omega_{\hat{h}}}\right)^N+\left(\frac{(p+1)a}{\Omega_{\hat{g}}}\right)^N\right] \quad for\,N_1=N_2=N\end{cases} \tag{33}$$

that is independent of $\bar{p}_0$ and γ_{th}. In this case, the system can achieve a diversity order of KN_R, namely, full diversity order. (ii) σ^2 is independent of SNR $\bar{p}_0$, $P_{out}^{\infty}\left(\gamma_{th}\right)$ will not approach to zero as $\bar{p}_0$ approaches infinity. This is because a, b, c, d have SNR independent constant terms, e.g., the term $5\sigma^2$ in $a=\left(4\bar{p}_1\bar{p}_3\sigma_{e_h}^2+\bar{p}_2\bar{p}_3\sigma_{e_g}^2+\bar{p}_3+\bar{p}_1\right)\Big/\bar{p}_2\bar{p}_3=5\sigma^2+2/\bar{p}_0$. In this case, the system's diversity order tends to zero as the SNR (or equivalently the transmitting power) approaches infinity.

4. Simulation Results

In this section, we conduct Monte Carlo simulations to verify the analytical results (shown in Fig. 1). The simulation parameters are as follows: $p_1=p_2=p_3=p_0$, $\sigma_{e_h}^2=\sigma_{e_g}^2=\sigma^2$, $\gamma_{th}=3$, $\Omega_h=2$, $\Omega_g=1$, $N_1=N_2=N_R=2$. The SNR in Fig. 1 equals to $\bar{p}_0=p_0/N_0$. We adopt two CEE models: (i) σ^2 is a decreasing function of received SNR, and formulated as $\sigma^2=1/\bar{p}_0$; (ii) σ^2 is independent of SNR $\bar{p}_0$. It is shown from Fig. 1 that the derived approximate and asymptotic expressions match well with the simulated results. We can observe that the curves corresponding to $\sigma^2=1/SNR$ have the same decreasing speed as those corresponding to $\sigma^2=0$ at high SNR. This observation illustrates that the CEE only degrades coding gain but not diversity order when the variance of CEE is a decreasing function of received SNR (or transmitting power). By comparing the curves corresponding to $\sigma^2=0.01$ with those corresponding to $\sigma^2=0$, an error floor is observed for $\sigma^2=0.01$, which means that the diversity order tends to zero when variance of CEE is fixed for the whole SNR. The curves with $\sigma^2=0.01, 0.001$ also show that increasing the variance of CEE leads to the increase of outage probability.

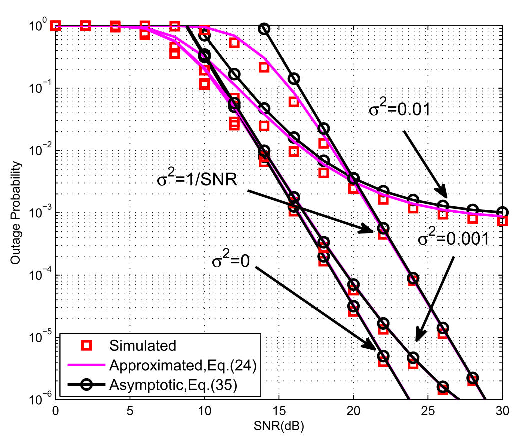

Fig. 1 Outage probability for JTRAS scheme in AF two-way relaying system with CEE.

5. Conclusions

In this letter, we have derived closed-form approximate and asymptotic outage probability expressions for JTRAS scheme in AF two-way relaying system by taking into account CEE. Monte Carlo simulation results have shown the correctness of our derived expressions. Both analytical and simulation results have revealed that the CEE had a detrimental effect on the system's outage performance. When the variance of CEE decreases as transmitting power (or received SNR) increases, the CEE degrades the coding gain while the system can still achieve the full diversity order. When the variance of CEE is fixed from low to high SNR, the system's diversity order tends to zero.

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Appendix: Proof of Eqs. (27) and (28)

In this appendix, we present the proof in brevity. (27) can be proved by using variable change $q = p+1$ and the following equation

$$h(x)\,|_{x=1} = (1-x)^{N-1}\,|_{x=1} = \sum_{p=0}^{N-1} \binom{N-1}{p} (-1)^p = 0 \tag{A$\cdot$1}$$

We use mathematical induction method to prove (28). By using variable change $q = p - 1$ and (A.1), we can prove (28) is true for $k = 0, 1$. Assuming (28) holds for $k = l$, then (28) can be proved through the following procedure:

$$\begin{aligned} &\sum_{p=0}^{N-1} \binom{N-1}{p} (-1)^p (p+1)^l = 0 \\ \Rightarrow &\sum_{p=0}^{N-1} \binom{N-1}{p} (-1)^p p^{l+1} = 0 \\ \Rightarrow &\sum_{p=0}^{N-1} \binom{N-1}{p} (-1)^p (p+1)^{l+1} = 0 \end{aligned} \tag{A$\cdot$2}$$