PAPER *Special Section on Formal Approach*

Protocol Inheritance Preserving Soundizability Problem and Its Polynomial Time Procedure for Acyclic Free Choice Workflow Nets

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SUMMARY A workflow may be extended to adapt to market growth, legal reform, and so on. The extended workflow must be logically correct, and inherit the behavior of the existing workflow. Even if the extended workflow inherits the behavior, it may be not logically correct. Can we modify it so that it satisfies not only behavioral inheritance but also logical correctness? This is named *behavioral inheritance preserving soundizability problem*. There are two kinds of behavioral inheritance: protocol inheritance and projection inheritance. In this paper, we tackled protocol inheritance preserving soundizability problem using a subclass of Petri nets called workflow nets. Limiting our analysis to acyclic free choice workflow nets, we formalized the problem. And we gave a necessary and sufficient condition on the problem, which is the existence of a key structure of free choice workflow nets called TP-handle. Based on this condition, we also constructed a polynomial time procedure to solve the problem.

key words: *workflow net, Petri net, behavioral inheritance, soundness, soundizability, polynomial time procedure*

1. Introduction

A workflow may be extended to adapt to market growth, legal reform, and so on. Business continuity requires the extended workflow to inherit the behavior of the existing workflow. The extended workflow must be logically correct, but it may be not logically correct even if it inherits the behavior.

Workflows can be modeled as a subclass of Petri nets [1], called workflow nets [2] (WF-nets for short). Van der Aalst [3] has proposed a criterion of logical correctness for WF-nets, called soundness. He has also shown that many actual workflows can be modeled as a subclass of WF-nets, called free choice WF-nets (FC WF-nets for short) [4], and that the soundness of FC WF-nets can be verified in polynomial time. Moreover, Van der Aalst et al. [5] have proposed a concept of behavioral inheritance between WF-nets. There are two kinds of behavioral inheritance: protocol inheritance and projection inheritance. Yamaguchi et al. [6] have proposed a necessary and sufficient condition on protocol inheritance between an acyclic FC WF-net and its extended net, and have shown that the protocol inheritance can be verified in polynomial time. The necessary and sufficient condition suggests that there exists a non-sound extended net even if it satisfies protocol inheritance. Can we modify the non-sound extended net so that it satisfies not only protocol inheritance but also soundness? This is named *protocol inheritance preserving soundizability problem*. Unfortunately, there has been no research tackling this problem.

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In this paper, we tackle the protocol inheritance preserving soundizability problem. Limiting our analysis to acyclic FC WF-nets, we formalize the problem. Next we propose a necessary and sufficient condition on the problem, which is the existence of a key structure of FC WF-nets called TP-handle. Based on this condition, we also construct a polynomial time procedure to solve the problem. After the introduction in Sect. 1, Sect. 2 gives the definition and properties of WF-nets. In Sect. 3, we give the definition and the necessary and sufficient condition on the problem. In Sect. 4, we present the polynomial time procedure for solving the problem. We also illustrate the procedure with examples. Section 5 gives the conclusion and future work.

2. WF-Net and Its Properties

(1) WF-Net

A (labeled) Petri net [1] is a four tuple (P, T, A, ℓ), where P and T are respectively disjoint finite sets of places and transitions, A $(\subseteq(P\times T)\cup(T\times P))$ is a set of arcs, and $\ell : T\rightarrow\mathcal{A}$ is a labelling function of transitions, where $\mathcal{A}$ denotes a set of labels. Let x be a node. $\overset{N}{\bullet}x$ and $x\overset{N}{\bullet}$ respectively denote $\{y|(y, x)\in A\}$ and $\{y|(x, y)\in A\}$.

Definition 1 (WF-net [3]): A Petri net N=(P, T, A, ℓ) is a (labeled) WF-net iff (i) N has a single source place p_I ($\overset{N}{\bullet}p_I$=$\emptyset$ and $\forall p\in(P-\{p_I\})$: $\overset{N}{\bullet}p\neq\emptyset$) and a single sink place p_O ($p_O\overset{N}{\bullet}$=$\emptyset$ and $\forall p\in(P-\{p_O\})$: $p\overset{N}{\bullet}\neq\emptyset$); and (ii) Every node is on a path from p_I to p_O. □

A marking of a WF-net N is a mapping M: $P\rightarrow\mathbb{N}$. We represent M as a bag over P, and write $[p^{M(p)}|p\in P, M(p)>0]$ for it. $[p_I]$ and $[p_O]$ are respectively the initial and final markings. Let M_X and M_Y be markings. M_X=M_Y denotes that $\forall p\in P$: $M_X(p)$=$M_Y(p)$. $M_X\geq M_Y$ denotes that $\forall p\in P$: $M_X(p)\geq M_Y(p)$. A transition t is said to be firable in a marking M if $M\geq\overset{N}{\bullet}t$. This is denoted by $M[N, t\rangle$. Firing t in M results in a new marking M' (=$M\cup t\overset{N}{\bullet}-\overset{N}{\bullet}t$). This is denoted by $M[N, t\rangle M'$. A marking M' is said to be reachable from a marking M if there exists a firing sequence of transitions transforming M to M'. The set of all possible markings reachable from M is denoted by $R(N, M)$. $R(N, M)$ can be represented as a graph, called reachability graph G=(V, E). The vertices represent markings generated from M and its successors, and each arc represents a transition firing[†].

N is said to be FC if $\forall p_1, p_2 \in P$: $p_1\overset{N}{\bullet} \cap p_2\overset{N}{\bullet} \neq \emptyset \Rightarrow |p_1\overset{N}{\bullet}| = |p_2\overset{N}{\bullet}| = 1$. If we add a transition t^* to N which connects p_O with p_I, then the resulting net $\overline{N}$ is strongly connected. We call $\overline{N}$ the short-circuited net of N. There are key structures that characterize FC: handle and bridge. A path (a circuit) is said to be elementary if no node appears more than once in the path (the circuit). Let ρ be an elementary path from a node x to another node y, and c an elementary circuit in $\overline{N}$. ρ is called a handle [1], [7] of c if ρ shares exactly two nodes, x and y, with c. Let h be a handle of c. ρ is called a bridge between c and h if each of c and h shares exactly one node, x or y, with ρ. A handle (a bridge) from a node x to another node y is called an XY-handle (an XY-bridge), where if $x \in P$ then X is P, otherwise X is T; if $y \in P$ then Y is P, otherwise Y is T. For example, a handle from a place to a transition is a PT-handle.

(2) Soundness

Soundness is a criterion of logical correctness defined for WF-nets.

Definition 2 (soundness [8]): A WF-net N is sound iff (i) $\forall M \in R(N, [p_I])$: $\exists M' \in R(N, M)$: $M' \geq [p_O]$; (ii) $\forall M \in R(N, [p_I])$: $M \geq [p_O] \Rightarrow M = [p_O]$; and (iii) There is no dead transition in $(N, [p_I])$. □

The following is the necessary and sufficient condition on the soundness problem of acyclic FC WF-nets. It can be checked in polynomial time.

Property 1: An acyclic FC WF-net N is sound iff (i) No circuit of $\overline{N}$ has TP-handles; and (ii) If $\overline{N}$ has PT-handles, then each PT-handle has a TP-bridge from the handle to the circuit. □

(3) Behavioral Inheritance

Behavioral inheritance is a relaxation of branching bisimilarity, which is an equivalence relation on WF-nets. Branching bisimilarity allows some transitions not to be observed. Such transitions are denoted by a designated label τ. Intuitively, branching bisimilarity equates WF-nets whose observable behaviors are the same.

Definition 3 (branching bisimilarity [9]): Let G_{N_X} and G_{N_Y} be respectively the reachability graphs of a WF-net $(N_X, [p_I^X])$ and another WF-net $(N_Y, [p_I^Y])$. A binary relation $\mathcal{R}$ $(\subseteq R(N_X, [p_I^X]) \times R(N_Y, [p_I^Y]))$ is branching bisimulation iff (i) if $M_X \mathcal{R} M_Y$ and $M_X[N_X, \alpha\rangle M_X'$, then $\exists M_Y', M_Y'' \in R(N_Y, [p_I^Y])$: $M_Y[N_Y, \tau^*\rangle M_Y''$, $M_Y''[N_Y, (\alpha)\rangle M_Y'$, $M_X \mathcal{R} M_Y''$, and $M_X' \mathcal{R} M_Y'$; (ii) if $M_X \mathcal{R} M_Y$ and $M_Y[N_Y, \alpha\rangle M_Y'$, then $\exists M_X', M_X'' \in R(N_X, [p_I^X])$: $M_X[N_X, \tau^*\rangle M_X''$, $M_X''[N_X, (\alpha)\rangle M_X'$, $M_X'' \mathcal{R} M_Y$, $M_X' \mathcal{R} M_Y'$; and (iii) if $M_X \mathcal{R} M_Y$ then $(M_X = [p_O^X] \Rightarrow M_Y[N_Y, \tau^*\rangle [p_O^Y])$ and $(M_Y = [p_O^Y] \Rightarrow M_X[N_X, \tau^*\rangle [p_O^X])$. $(N_X, [p_I^X])$ and $(N_Y, [p_I^Y])$ are said to be branching bisimilar, denoted by $(N_X, [p_I^X]) \sim_b (N_Y, [p_I^Y])$, iff there exists a branching bisimulation $\mathcal{R}$ between G_{N_X} and G_{N_Y}. □

To give the formal definition of protocol inheritance, we use encapsulation operator ∂. For a set H of observable labels, ∂_H removes transitions whose labels are included in H. Formally, $\partial_H: N \mapsto (P, T', A', \ell')$ such that $T' = \{t \in T | \ell(t) \notin H\}$, $A' = A \cap ((P \times T') \cup (T' \times P))$, and ℓ': t $(\in T') \mapsto \ell(t)$.

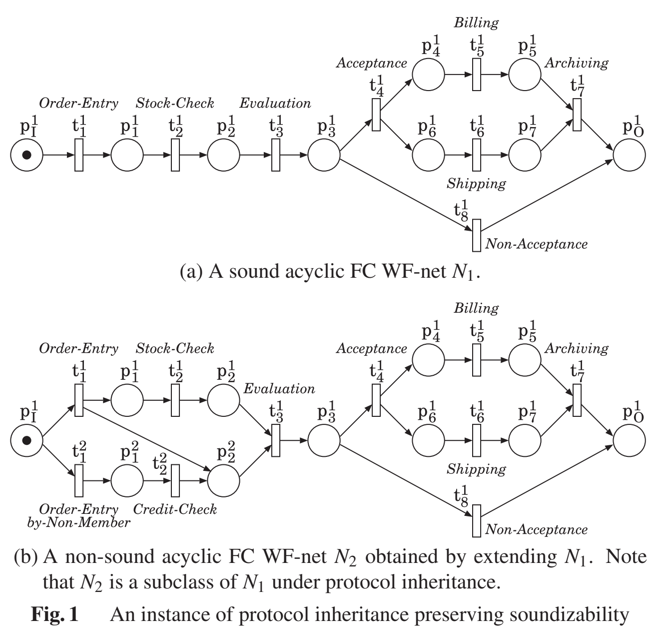
Definition 4 (protocol inheritance [5]): A WF-net N_X is a subclass of another WF-net N_Y under protocol inheritance iff $\exists H$: $(\partial_H(N_X), [p_I^X]) \sim_b (N_Y, [p_I^Y])$. □

Protocol inheritance relation is partial-order [5]. This means that protocol inheritance relation is transitive.

The following is the necessary and sufficient condition to verify protocol inheritance between an acyclic FC WF-net and its subnet [6]. It can be checked in polynomial time.

Property 2: Let N_X $(=(P_X, T_X, A_X, \ell_X))$ and N_Y $(=(P_Y, T_Y, A_Y, \ell_Y))$ be acyclic FC WF-nets, where N_X is a sound subnet of N_Y; and every transition in N_X (N_Y) represents a unique observable transition. N_Y is a subclass of N_X under protocol inheritance iff (i) $p_I^X = p_I^Y$ and $p_O^X = p_O^Y$; (ii) $\partial_{\ell_Y(T_Y - T_X)}(N_Y)$ equals N_X with 0 or more TT-handles with length 2; and (iii) $\partial_{\ell_Y(T_Y - T_X)}(N_Y)$ is sound, where isolated places in $\partial_{\ell_Y(T_Y - T_X)}(N_Y)$ are removed. □

This necessary and sufficient condition suggests that there exists a non-sound extended net even if it satisfies protocol inheritance. Let us consider an extension of an FC WF-net, which is shown in Fig. 1. Figure 1 (a) shows a



(a) A sound acyclic FC WF-net N_1.

(b) A non-sound acyclic FC WF-net N_2 obtained by extending N_1. Note that N_2 is a subclass of N_1 under protocol inheritance.

Fig. 1 An instance of protocol inheritance preserving soundizability problem.

[†]Formally, $V = R(N, M)$, $E = \{(M, \ell(t), M') | M, M' \in V, t \in T, M[N, t\rangle M'\}$. Let $M, M' \in V$, $\alpha \in \ell(T)$. We write $M[N, \alpha\rangle M'$ if M' is reachable from M by following an edge labeled as α. We write $M[N, \tau^*\rangle M'$ if M' is reachable from M by following any number of edges labeled as τ. We write $M[N, (\alpha)\rangle M'$ if either (i) $\alpha = \tau$ and $M = M'$; or (ii) $M[N, \alpha\rangle M'$.

sound acyclic FC WF-net N_1. Figure 1 (b) shows an acyclic FC WF-net N_2 obtained by extending N_1. N_2 is a subclass of N_1 under protocol inheritance but is not sound. The detail is described later.

3. Protocol Inheritance Preserving Soundizability Problem and Its Necessary and Sufficient Condition

In this section, we give the formal definition of protocol inheritance preserving soundizability problem of acyclic FC WF-net, and the necessary and sufficient condition on the problem.

3.1 Problem

The growth of business involves (i) extending the existing workflow. The extended workflow must (ii) inherit the behavior of the existing workflow, and further (iii) be logically correct. We model the existing workflow and the extended workflow as WF-nets N_X and N_Y, respectively, that satisfy (i) N_X is a subnet of N_Y; (ii) N_Y is a subclass of N_X under protocol inheritance; and (iii) N_X and N_Y are sound. Even if N_Y is a subclass of N_X under protocol inheritance, N_Y is not always sound. If N_Y is not sound, can we modify it so that it satisfies not only protocol inheritance but also sound? This can be shown as Fig. 2.

Van der Aalst [4] has shown that many actual workflows can be modeled as FC WF-nets. The Workflow Management Coalition [10] (WfMC for short), an international standardization organization on workflows, has identified four routing constructions: sequential, parallel, selective, and iterative. Acyclic FC WF-net can model workflows composed of the former three routing constructions. Therefore various extension of actual workflows would be modeled as acyclic FC WF-nets. Thus we limit our analysis to acyclic FC WF-nets and give the formal definition as follows.

Definition 5 (protocol inheritance preserving soundizability problem of acyclic FC WF-nets): Let N_X be a sound acyclic FC WF-net and N_Y a non-sound acyclic FC WF-net such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance. Is there a sound acyclic FC WF-net N_Z that satisfies the following? (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance. □

Let us consider two instances of the problem. In fact, those instances have different answers: The first is yes, but the second is no.

The first instance is shown in Fig. 1. N_2 is not sound because N_2 violates Condition (ii) of Property 1, i.e. $\overline{N_2}$ has a PT-handle $p_I^1 t_1^2 p_1^2 t_2^2 p_2^2 t_3^1$ which has no TP-bridge from the handle to the circuit. Can we modify N_2 as a sound acyclic FC WF-net which is a subclass of N_1 under protocol inheritance? The answer is yes. We have only to add a new TP-bridge from the handle to the circuit. The obtained WF-net N_3 is shown in Fig. 3 (a). N_3 is sound because each PT-handle in $\overline{N_3}$ has a TP-bridge from the handle to the circuit. Moreover, N_3 is a subclass of N_1 under protocol inheritance because N_3 satisfies the conditions of Property 2.

The second instance is shown in Fig. 3. N_4 is not sound because N_4 violates Condition (i) of Property 1, i.e. there is a TP-handle $t_1^1 p_1^1 t_1^4 p_O^1$. Can we modify N_4 as a sound acyclic FC WF-net which is a subclass of N_3 under protocol inheritance? The answer is no. This is because the TP-handle will never be removed.

We deduce from the analysis results that TP-handle plays a core role in the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

3.2 Necessary and Sufficient Condition

Based on our deduction, we divide the protocol inheritance preserving soundizabiltiy problem of acyclic FC WF-nets into two cases based on the existence of TP-handle. Let us

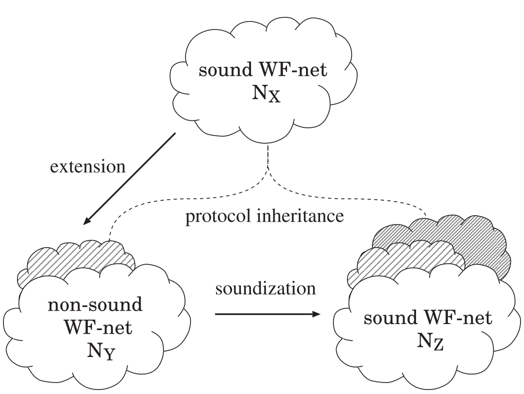
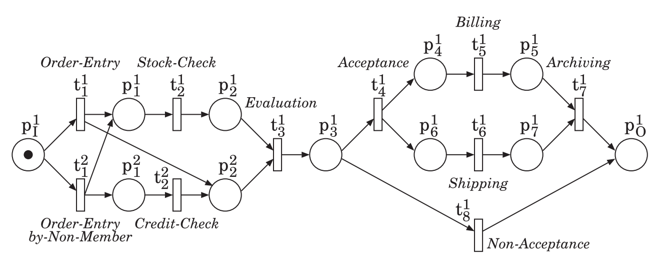
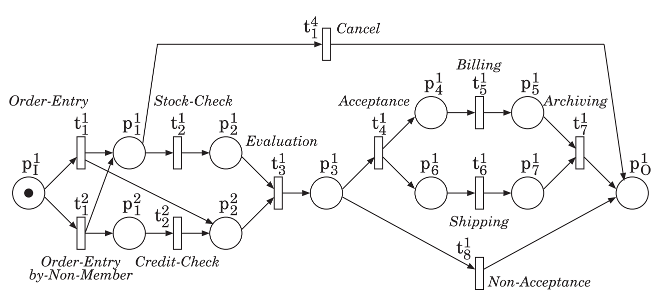


Fig. 2 Illustration of protocol inheritance preserving soundizability problem.



(a) A sound acyclic FC WF-net N_3. N_3 is a subclass under protocol inheritance of N_1.



(b) A non-sound acyclic FC WF-net N_4 obtained by extending N_3. Note that N_4 is a subclass of N_3 (and N_1) under protocol inheritance, but is not soundizable because $\overline{N_4}$ has TP-handles.

Fig. 3 Another instance of soundizability problem.

first consider the case with TP-handles.

Lemma 1: Let N_X be a sound acyclic FC WF-net and N_Y a non-sound acyclic FC WF-net such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance. If $\overline{N_Y}$ has a TP-handle h, there is no sound acyclic FC WF-net N_Z such that (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance. □

Proof: Assume N_Z exists. Since N_Y is a subnet of N_Z, h still exists in $\overline{N_Z}$. This implies N_Z is not sound from Condition (i) of Property 1. It is inconsistent with the assumption. **Q.E.D.**

This lemma means a necessary condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

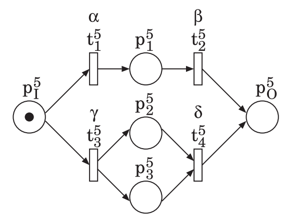
Next, let us consider the case with no TP-handle. Any non-sound acyclic FC WF-net satisfying protocol inheritance has a structural property as follows.

Property 3: Let N_X be a sound acyclic FC WF-net and N_Y a non-sound acyclic FC WF-net such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance. N_Y equals 'N_X with 0 or more TT-handles with length 2' with PP-handles and/or PP-bridges, where the PP-handles and/or PP-bridges are not included in 'N_X with 0 or more TT-handles with length 2' except for their terminal nodes. □

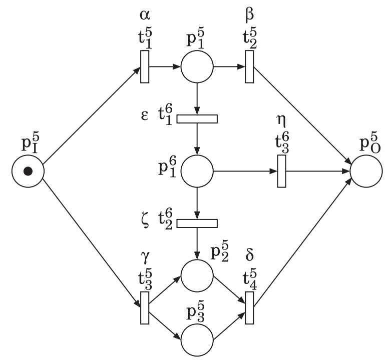
Proof: We show the contraposition. Assume that N_Y equals 'N_X with 0 or more TT-handles with length 2' with a PT/TP/TT-handle or PT/TP/TT-bridge ρ. Note that ρ is not included in 'N_X with 0 or more TT-handles with length 2' except for their terminal nodes. If ρ is a PT/TP-handle or PT/TP-bridge, since $\partial_{\ell_Y(T_Y-T_X)}$ makes a new source place or new sink place, N_Y is not a subclass of N_X under protocol inheritance (Refer to Lemmas 1 and 2 of Ref. [6]). If ρ is a TT-handle or TT-bridge with length 4 or more, for the similar reason N_Y is not a subclass of N_X under protocol inheritance (Refer to Lemma 5 of Ref. [6]). If ρ is a TT-handle with length 2, it would be included in 'N_X with 0 or more TT-handles with length 2', so this case does not occur. If ρ is a TT-bridge with length 2, since a new causality occurs between its terminal nodes, N_Y is not a subclass of N_X under protocol inheritance (Refer to Lemma 4 of Ref. [6]). Thus this property holds. **Q.E.D.**

This property means that all the transitions newly added to N_X are contained in the PP-handles and/or PP-bridges. We illustrate this property with an instance shown in Fig. 4. N_6 is an acyclic FC WF-net obtained by extending N_5, and is a subclass of N_5 under protocol inheritance. N_6, however, is not sound, because $\overline{N_6}$ has a PT-handle $p_I^5 t_1^5 p_1^5 t_1^6 p_1^6 t_2^6 p_2^5 t_4^5$ which has no TP-bridge from the handle to the circuit, i.e. the PT-handle violates Condition (ii) of Property 1. From Property 3, we can obtain that N_6 equals N_5 with a PP-handle $p_1^5 t_1^6 p_1^6 t_3^6 p_O^5$ and a PP-bridge $p_1^5 t_1^6 p_1^6 t_2^6 p_2^5$. Note that all the transitions newly added to N_5, i.e. t_1^6, t_2^6 and t_3^6, are contained in the PP-handle and/or PP-bridge.

From Condition (ii) of Property 1, $\overline{N_Y}$ must have a PT-handle which has no TP-bridge from the handle to the circuit. We say, a PT-handle is wrong if the handle has no TP-bridge from the handle to the circuit. Otherwise the PT-handle is said to be right. From Property 3, N_Y equals 'N_X with 0 or more TT-handles with length 2' with PP-handles and/or PP-bridges. 'N_X with 0 or more TT-handles with length 2' has no wrong PT-handle, because it is sound (Condition (iii) of Property 2). This means that a wrong PT-handle is caused by the PP-handles and/or PP-bridges added to 'N_X with 0 or more TT-handles with length 2'. In other words, the wrong PT-handle must include such a PP-handle or PP-bridge. Let us consider the instance shown in Fig. 4. $\overline{N_6}$ has a wrong PT-handle $p_I^5 t_1^5 p_1^5 t_1^6 p_1^6 t_2^6 p_2^5 t_4^5$. N_6 equals N_5 with a PP-handle $p_1^5 t_1^6 p_1^6 t_3^6 p_O^5$ and a PP-bridge $p_1^5 t_1^6 p_1^6 t_2^6 p_2^5$. The PT-handle includes the PP-bridge.



(a) A sound acyclic FC WF-net N_5.



(b) A non-sound acyclic FC WF-net N_6 obtained by extending N_5. Note that $\overline{N_5}$ has a PP-bridge.

Fig. 4 An instance including a PP-bridge of soundizability problem.

Intuitively, for each of the wrong PT-handles, if we add an arc as a TP-bridge from the handle to the circuit, we can make N_Y sound. If the TP-bridge starts from a transition added to N_X, it can be removed with the removal of the transition by encapsulation operator, so the obtained net is a subclass of N_X under protocol inheritance.

Lemma 2: Let N_X be a sound acyclic FC WF-net and N_Y a non-sound acyclic FC WF-net such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance. If $\overline{N_Y}$ has no TP-handle, there is a sound acyclic N_Z such that (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance. □

Proof: We construct a WF-net N by extending N_Y, and prove $N = N_Z$ by showing the following: (1) N is acyclic FC; (2) N is sound; and (3) N is a subclass of N_X under protocol inheritance.

First of all, we give how to construct N by extending N_Y. $\overline{N_Y}$ has wrong PT-handles. Let n be the number of

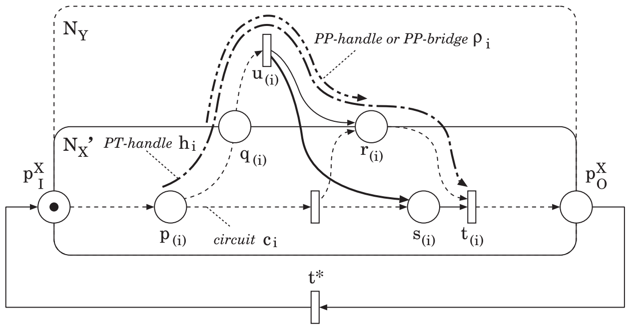


Fig. 5 Illustration of the proof of Lemma 2, where $N_X{}'$ denotes 'N_X with 0 or more TT-handles with length 2'.

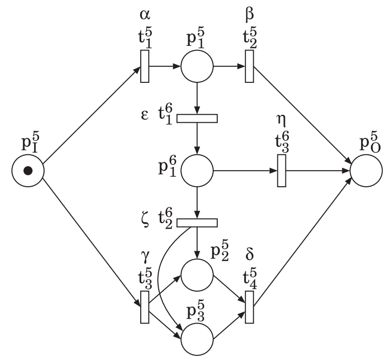


Fig. 6 A WF-net N_7 obtained by adding (t_2^6, p_3^5) to N_6. N_7 is a sound acyclic FC WF-net, and is a subclass of N_5 under protocol inheritance.

the wrong PT-handles. Let h_i $(i = 1, 2, \cdots, n)$ denote each wrong PT-handle, and c_i the circuit of h_i (See Fig. 5). For PT-handle h_i, let $p_{(i)}$ and $t_{(i)}$ denote respectively the start node and the end node, and $s_{(i)}$ denote the input place of $t_{(i)}$ which appears in c_i. Let ρ_i be a PP-handle or PP-bridge added to 'N_X with 0 or more TT-handles with length 2' such that ρ_i is a part of h_i. For PP-handle or PP-bridge ρ_i, let $q_{(i)}$ and $r_{(i)}$ denote respectively the start node and the end node, and $u_{(i)}$ denote the input transition of $r_{(i)}$ which appears in ρ_i. We construct N $(= (P, T, A, \ell))$ as follows: $P{=}P_Y$, $T{=}T_Y$, $A{=}A_Y{\cup}\{(u_{(i)}, s_{(i)})|i{=}1, 2, \cdots, n\}$, $\ell{=}\ell_Y$. N is obviously an extended WF-net of N_Y.

Firstly, we prove (1). The extended part, $\{(u_{(i)}, s_{(i)})|i{=}1, 2, \cdots, n\}$, consists of arcs from transition $u_{(i)}$ to place $s_{(i)}$. Therefore the free-choiceness is preserved. There is no path from $s_{(i)}$ to the nodes of ρ_i because such a path makes N non-FC. Therefore the part does not produce any circuit, i.e. N is acyclic.

Secondly, we prove (2). Arc $(u_{(i)}, s_{(i)})$ forms a TP-bridge from h_i to c_i. Note that $\overline{N_Y}$ originally has a TP-bridge from c_i to h_i. Without the TP-bridge, $\partial_{\ell_Y(T_Y - T_X)}$ would make a new source place, i.e. N_Y does not become a subclass of N_X under protocol inheritance. $\overline{N_Y}$ has no TP-handle. This means that the TP-bridge does not form any TP-handles. From the symmetry of structure, arc $(u_{(i)}, s_{(i)})$ also forms no TP-handle. Therefore N is sound.

Finally, we prove (3). Since $u_{(i)}$ is a transition in PP-handle or PP-bridge ρ_i, we can remove arc $(u_{(i)}, s_{(i)})$ because $u_{(i)}$ can be removed by encapsulation operator. N is a subclass of N_X under protocol inheritance.

Summarizing the above results, we have $N = N_Z$. Thus this lemma holds. **Q.E.D.**

This lemma means a sufficient condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

Let us consider again the instance shown in Fig. 4. We can say that N_6 is soundizable because $\overline{N_6}$ has no TP-handle. $\overline{N_6}$ has a wrong PT-handle $p_I^5 t_1^5 p_1^5 t_1^6 p_1^6 t_2^6 p_2^5 t_4^5$. The end node of the PT-handle is t_4^5. The input place of t_4^5 which appears in its circuit is p_3^5. The PT-handle includes a PP-bridge $p_1^5 t_1^6 p_1^6 t_2^6 p_2^5$. The end node of the PP-bridge is p_2^5. The input transition of p_2^5 which appears in the PP-bridge is t_2^6. An arc (t_2^6, p_3^5) forms a TP-bridge from the PT-handle to the circuit. Adding the arc to N_6, we can obtain WF-net N_7 shown in Fig. 6. N_7 is an sound acyclic FC WF-net, and is a subclass of N_5 under protocol inheritance.

From Lemmas 1 and 2, we can immediately obtain the following necessary and sufficient condition on the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

Theorem 1: Let N_X be a sound acyclic FC WF-net and N_Y a non-sound acyclic FC WF-net such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance. Iff $\overline{N_Y}$ has no TP-handle, there is a sound acyclic FC WF-net N_Z such that (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance. □

4. Polynomial Time Procedure and Examples

4.1 Polynomial Time Procedure

Based on Theorem 1, we construct a procedure for solving the protocol inheritance preserving soundizability problem of acyclic FC WF-nets.

≪Decision of Soundizability for Acyclic FC WF-nets≫

Input: Sound acyclic FC WF-net N_X and non-sound acyclic FC WF-net N_Y such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance.

Output: Is there a sound acyclic FC WF-net N_Z such that (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance.

1° Construct a flow network D_{N_Y} $(=(V_{N_Y}, E_{N_Y}))$ whose every edge has capacity 1, where $V_{N_Y}{=}P_Y{\cup}T_Y$, $E_{N_Y}{=}A_Y$.

2° For each vertex pair (v_i, v_j) $({\in}V_{N_Y}{\times}V_{N_Y})$, if

- v_i corresponds to a transition t of N_Y s.t. $|t\overset{N_Y}{\bullet}|{\geq}2$;
- v_j corresponds to a place p of N_Y s.t. $|\overset{N_Y}{\bullet}p|{\geq}2$; and
- The maximum value of flow between v_i and v_j exceeds 1,

then output no and stop.

3° Output yes and stop.

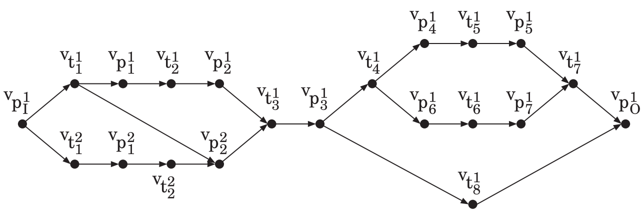


Fig. 7 The directed graph D_{N_2} of N_2.

Property 4: Algorithm ≪Decision of Soundizability for Acyclic FC WF-nets≫ outputs yes iff N_Y is soundizable, i.e. $\overline{N_Y}$ has no TP-handle. □

Proof: The max-flow min-cut theorem states that if every edge of a flow network has capacity 1, the number of the disjoint paths from a vertex to another vertex is equal to the maximum value of flow between those vertices [11].

The "if" part: Since $\overline{N_Y}$ has no TP-handle, the number of the disjoint paths from any transition t to any place p is at most one. The maximum value of flow between the vertices corresponding to t and p does not exceed 1. Therefore the procedure outputs yes.

The "only-if" part: If $\overline{N_Y}$ has a TP-handle, the number of the disjoint paths from the start transition to the end place of the TP-handle is two or more. The maximum value of flow between the corresponding vertices exceeds 1. Therefore the procedure outputs no. **Q.E.D.**

Property 5: The following problem can be solved in polynomial time: Given a sound acyclic FC WF-net N_X and a non-sound acyclic FC WF-net N_Y such that (i) N_X is a subnet of N_Y; and (ii) N_Y is a subclass of N_X under protocol inheritance, to decide whether there is a sound acyclic FC WF-net N_Z such that (i) N_Y is a subnet of N_Z; and (ii) N_Z is a subclass of N_X under protocol inheritance. □

Proof: Algorithm ≪Decision of Soundizability for Acyclic FC WF-nets≫ can run in polynomial time, because Step 1° takes time $O(|P_Y| + |T_Y| + |A_Y|)$ and Step 2° takes time $O(|T_Y||P_Y|(|P_Y| + |T_Y|)^3)$. Note that the computation of the maximum flow takes time $O((|P_Y| + |T_Y|)^3)$. **Q.E.D.**

4.2 Examples

We illustrate the proposed procedure with the instances shown in Figs. 1 and 3. Let us first consider the instance shown in Fig. 1. In Step 1°, we construct a flow network D_{N_2} shown in Fig. 7. In Step 2°, we compute the maximum values of flow from $v_{t_1^1}$ to $v_{p_2^2}$, from $v_{t_1^1}$ to $v_{p_O^1}$, and from $v_{t_4^1}$ to $v_{p_O^1}$. Since the maximum values are all 1, our procedure outputs yes and stops. In fact, there exists a sound acyclic FC WF-net N_3.

Next, let us consider the instance shown in Fig. 3. In Step 1°, we construct a flow network D_{N_4} shown in Fig. 8. In Step 2°, we obtain the maximum value of flow from $v_{t_1^1}$ to $v_{p_O^1}$ as 2. Therefore our procedure outputs no and stops.

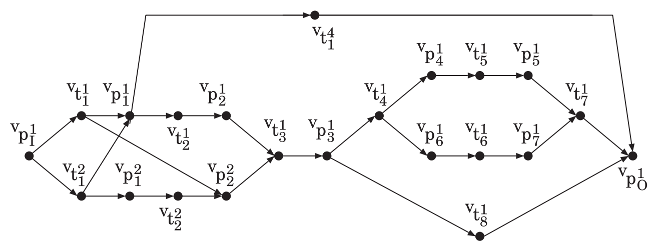


Fig. 8 The directed graph D_{N_4} of N_4. Note that the maximum flow from $v_{t_1^1}$ to $v_{p_O^1}$ is 2. This means that the path from t_1^1 to p_O^1 is a TP-handle.

5. Conclusion

In this paper, we have tackled the protocol inheritance preserving soundizability problem of acyclic FC WF-nets. We have first given the formal definition of the problem. Next we have proposed a necessary and sufficient condition on the problem. The condition is the existence of TP-handle. Based on this condition, we have also constructed a polynomial time procedure to solve the problem. The proposed procedure enables us to check it efficiently. It would contribute to strengthen the organization's competitive power in business environment that is changing rapidly.

This paper is the first step for soundization of WF-nets. The next step is to give how to soundize soundizable nets. A WF-net obtained by soundization is not unique in general, so it is desirable to have a minimal one. As a future work, we first propose a measure of quality of soundized nets, e.g. net size. Then considering the measure, we are going to construct a procedure of soundization.

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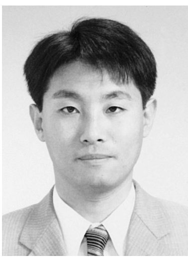

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