

DOA estimation of coherently distributed sources based on block-sparse constraint with measurement matrix uncertainty

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Abstract: In this letter, we proposed a new algorithm to solve the problem concerning the direction-of-arrivals (DOAs) estimation of coherently distributed sources based on the block sparse signal matrix uncertainty model of compressed sensing. Considering the measurement matrix corrupted by the unknown noise, the central DOA and the angular spread estimation may degrade considerably. But our method is robust with the measurement matrix uncertainty. Furthermore, the proposed method has better performance in low signal-to-noise (SNR). The effectiveness of our method is confirmed by simulation results.

Keywords: direction of arrival (DOA), distributed source, compressed sensing, block-sparse

Classification: Integrated circuits

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1 Introduction

The research of target localization with a set of sensors has promoted the study on the parameter estimation in signal processing. In rural or suburban areas, the fast fading problem existing in the wireless communications using the high base station antenna, which is caused by the local scattering in the vicinity of the mobile [1, 2, 3]. As a consequence, the source is no longer viewed by the array as a point source but as a spatially distributed source with mean direction-of-arrival (DOA) and angular spread. Depending on the relationship between the channel coherency time and observation period, the distributed sources have been classified into coherently distributed (CD) and incoherently distributed (ID) sources [4]. For a CD source, the signal components arriving from different angles within the extension width are modeled as the delayed and attenuated replicas of the same signal (coherent). For an ID source, these components are assumed to be uncorrelated.

The theory of compressed sensing suggests that the successful inversion of a sparse signal for its source modes and amplitudes can be achieved at measurement dimensions far lower than what might be expected from the classical theories of spectrum or modal analysis. CS theory [5] provides an efficient way to acquire and reconstruct a sparse signal from compressed measurements. A new array architecture which uses a small number of receivers to collect a large number of array outputs was put forward in [6] based on this theory. Recently, block-sparse signal models were proposed for practical applications according to sparse signals that have nonzero entries occurring in clusters in [7]. This theory shows better reconstruction property compared with the conventional method for the block-sparse signals. Then, a CS array that has fewer receivers than sensors was used to the DOA estimation with distributed sources in [8].

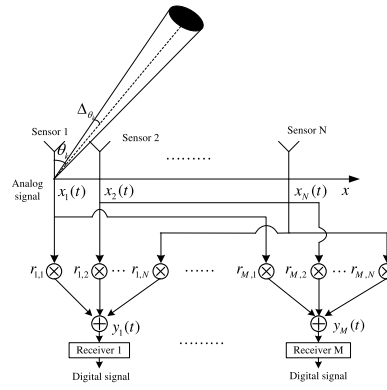


Fig. 1. Schematic diagram of compressed sensing array

2 Data model and previously work

As shown in Fig. 1, suppose that K far-field narrowband sources impinge onto the array of N sensors from directions of θ_k , $k = 1, 2, \dots, K$ with corresponding angular spread $\Delta\theta_k$. For simplicity, it is assumed that the sensors and the sources are in the same plane. For T snapshots, the output of the N sensors can be expressed as an $M \times T$ ($M \ll N$) matrix \mathbf{Y} of the form:

$$\mathbf{Y} = \mathbf{R}\mathbf{A}\mathbf{S} + \mathbf{N} \quad (1)$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_K \end{bmatrix} \\ \mathbf{S} &= \begin{bmatrix} \mathbf{s}(1) & \mathbf{s}(2) & \cdots & \mathbf{s}(T) \end{bmatrix} \\ \mathbf{A}_k &= \begin{bmatrix} \mathbf{a}(\theta_k + \tilde{\theta}_{k,1}) & \mathbf{a}(\theta_k + \tilde{\theta}_{k,2}) & \cdots & \mathbf{a}(\theta_k + \tilde{\theta}_{k,L_k}) \end{bmatrix}^T \\ \mathbf{s}_k(t) &= \begin{bmatrix} \chi_{k,1}s_k(t) & \chi_{k,2}s_k(t) & \cdots & \chi_{k,L_k}s_k(t) \end{bmatrix}^T, t = 1, 2, \dots, T \end{aligned}$$

And \mathbf{R} is measurement matrix. \mathbf{N} is an $N \times T$ additive gaussian white noise matrix. $s_{k,l}(t)$ is the l th reflected signal from the t th distribution source, L_k is the number of scattered multipath sources which are suggested around $150 \sim 200$ [4], $\chi_{k,l}(t)$ is corresponding stochastic gain of each bunch of signal, $\mathbf{n}(t)$ is the corresponding additive Gaussian white noise, \mathbf{A} is the array manifold matrix, $\tilde{\theta}_{k,l}(t)$ is the l th random angle migration relative to the central DOA θ_k . Assume that the mean value of $\tilde{\theta}_{k,l}(t)$ is zero and the standard deviation of $\tilde{\theta}_{k,l}(t)$ is $\sigma_{\tilde{\theta}_k}$, $p(\tilde{\theta}_k, \sigma_{\tilde{\theta}_k})$ is the probability density function of $\tilde{\theta}_{k,l}(t)$. Considering uniform linear array (ULA), $\mathbf{a}(\theta_k + \tilde{\theta}_{k,l})$ can be presented as follows:

$$\mathbf{a}(\theta'_{k,l}) = \begin{bmatrix} 1 & e^{-j2\pi d/\lambda \sin \theta'_{k,l}} & \cdots & e^{-j(N-1)2\pi d/\lambda \sin \theta'_{k,l}} \end{bmatrix}^T \quad (2)$$

where $\theta'_{k,l} = \theta_k + \tilde{\theta}_{k,l}$, d is the array element distance and λ is the wavelength of signals. The estimation of DOA is to solve following optimization:

$$\hat{\mathbf{B}} = \arg \min_{\mathbf{B}} \left\{ \|\mathbf{y}(t) - \mathbf{R}\Phi\mathbf{B}\|_2 + \eta_1 \sum_j \|\mathbf{B}_j\|_2 + \eta_2 \sum_{i=1}^I \|\text{vec}(\mathbf{G}_i\mathbf{B})\|_1 \right\} \quad (3)$$

where Φ is a $N \times N_a$ basic matrix, $\Phi = \begin{bmatrix} \mathbf{a}(\bar{\theta}_1) & \mathbf{a}(\bar{\theta}_2) & \cdots & \mathbf{a}(\bar{\theta}_{N_a}) \end{bmatrix}$ and:

$$\mathbf{G}_i^{N \times N} = \begin{bmatrix} \mathbf{G}_{i,1} \\ \mathbf{G}_{i,2} \end{bmatrix} \quad (4)$$

$$\mathbf{G}_{i,1} = \begin{bmatrix} \mathbf{1} & -\mathbf{1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \mathbf{1} & -\mathbf{1} & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \mathbf{1} & -\mathbf{1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & \mathbf{1} \end{bmatrix} \quad (5)$$

$$\mathbf{G}_{i,2} = \begin{bmatrix} -\mathbf{1} & \mathbf{1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\mathbf{1} & \mathbf{1} & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -\mathbf{1} & \mathbf{1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & -\mathbf{1} \end{bmatrix} \quad (6)$$

where $\mathbf{1}$ and $-\mathbf{1}$ is a $1 \times i$ vector with entire elements 1 and -1 . A series of software packages such as CVX and SeDuMi can work out the optimal solution [10]. Then, we make $\hat{\mathbf{B}} = \begin{bmatrix} \hat{\mathbf{b}}(1) & \hat{\mathbf{b}}(2) & \cdots & \hat{\mathbf{b}}(T) \end{bmatrix}$ as the recovered solution. For T snapshots, we can obtain the angle spectrum as:

$$P_{cs}(\theta) = \frac{1}{T} \sum_{t=1}^T \|\hat{\mathbf{b}}_{\theta}(t)\|^2, \theta = \bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_{N_a} \quad (7)$$

The central DOA and the angular spread with respect to the k th source can be calculated by $(\hat{\theta}_{\max}^k + \hat{\theta}_{\min}^k)/2$ and $(\hat{\theta}_{\max}^k - \hat{\theta}_{\min}^k)/2$, respectively, where $\hat{\theta}_{\max}^k$ and $\hat{\theta}_{\min}^k$ are corresponding maximum index value and minimum index value found by searching the spectral peak.

3 Proposed method

Recently, some researches have peoposed the problem of sparse signal recovery with matrix uncertainty [11, 12]. This model uncertainty also exists in the CS array. The model (1) is reformulated as:

$$\mathbf{Y} = (\mathbf{R} + \mathbf{\Xi}) \mathbf{A} \mathbf{S} + \mathbf{N} \quad (8)$$

where $\mathbf{\Xi}$ is a random noise matrix which corrupt the measurement matrix \mathbf{R} . Under matrix uncertainty, the LASSO and Dantzig selector turned out to be extremely unstable in recovering the sparsity pattern, even if the noise level is vary small. We proposed a new MU-selector (matrix uncertainty selector) to recover the block sparse matrix $\hat{\mathbf{B}}$. In what follows, without loss of generality, we mainly assume that $\mathbf{\Xi}$ are deterministic and satisfy the assumption:

$$|\mathbf{\Xi}|_2 \leq \delta \quad (9)$$

where $\delta \geq 0$. Here $\|\cdot\|_2$ stands for the l_2 norm. This MU-selector is based on the signal-subspace method. From (8), we can take the SVD of \mathbf{Y} to reduce the computational cost:

$$\mathbf{R}_{yy} = \frac{1}{T} \mathbf{Y} \mathbf{Y}^H = (\mathbf{R} + \mathbf{\Xi}) \mathbf{A} \mathbf{R}_{ss} (\mathbf{R} + \mathbf{\Xi})^H + \sigma_n^2 \mathbf{I}_M \quad (10)$$

Because \mathbf{R}_{SS} is a sparse matrix with few nonzero elements in block diagonal element, $\mathbf{R}_{SS}((\mathbf{R} + \mathbf{\Xi}) \mathbf{A})^H$ has few nonzero elements in some rows. Define $\mathbf{B} = \mathbf{R}_{SS}((\mathbf{R} + \mathbf{\Xi}) \mathbf{A})^H$, we write the model in (10) in sparse form as:

$$\mathbf{R}_{yy} = \frac{1}{T} \mathbf{Y} \mathbf{Y}^H = \mathbf{R} \mathbf{A} \mathbf{B} + \mathbf{\Xi} \mathbf{A} \mathbf{B} + \sigma_n^2 \mathbf{I}_M \quad (11)$$

This MU-selector is defined as a solution to the minimization problem:

$$\begin{aligned} \hat{\mathbf{B}} = \arg \min_{\mathbf{B}} \left\{ \|\mathbf{R}_{yy} - \mathbf{R} \Phi \mathbf{B}\|_2^2 + \delta \|\Phi \mathbf{B}\|_2^2 + \eta_1 \sum_j \|\mathbf{B}_j\|_2 \right. \\ \left. + \eta_2 \sum_{i=1}^I \|\text{vec}(\mathbf{G}_i \mathbf{B})\|_1 \right\} \end{aligned} \quad (12)$$

Since:

$$\|\mathbf{R}_{yy} - \mathbf{R} \Phi \mathbf{B} - \sigma_n^2 \mathbf{I}_M\|_2^2 = \|\mathbf{\Xi} \Phi \mathbf{B}\|_2^2 < \|\mathbf{\Xi}\|_2^2 \|\Phi \mathbf{B}\|_2^2 \leq \delta \|\Phi \mathbf{B}\|_2^2 \quad (13)$$

For matrix $\hat{\mathbf{B}}$, we can obtain the angle spectrum as:

$$P_{cs}(\theta) = \frac{1}{M} \sum_{t=1}^M \|\hat{\mathbf{b}}_\theta(t)\|^2, \theta = \bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_{N_a} \quad (14)$$

The central DOA and the angular spread can be estimated simultaneously by solving the convex optimization problem (12). The effectiveness of our method is confirmed by simulation results.

4 Performance analysis

To analyze the performance of our method, the notation is defined as: for a vector $\hat{\mathbf{B}} \in \mathbb{R}^{N_a \times M}$, $\hat{\mathbf{B}} = [\hat{\mathbf{b}}(1) \ \hat{\mathbf{b}}(2) \ \dots \ \hat{\mathbf{b}}(M)]$. For each $\hat{\mathbf{b}}(i)$, $i = 1, 2, \dots, M$ and a subset J of $\{1, \dots, N_a\}$, we denote $\mathbf{b}_J(i)$ and a subset J of $\{1, \dots, N_a\}$, we denote $\mathbf{b}_J(i)$ the vector in \mathbb{R}^{N_a} that has the same coordinates as \mathbf{b} on the set of indices J and zero coordinates on its complement J^c .

We will assume that the matrix $\mathbf{R} \Phi$ satisfies the following condition: There exists $\kappa > 0$ such that:

$$\min_{\Delta \neq 0: |\Delta_{J^c}|_1 \leq |\Delta_J|_1} \frac{|\mathbf{R} \Phi \Delta|}{\sqrt{M} |\Delta_J|_2} \geq \kappa \quad (15)$$

for all subsets J of $\{1, \dots, N\}$ of cardinality $|J| \leq s$.

Assume that there exists an block sparse solution, define that $\mathbf{r}(i)_{yy}$, $i = 1, 2, \dots, M$ is the i th column of \mathbf{R}_{yy} , we have $\mathbf{r}(i)_{yy} - \mathbf{R} \Phi \mathbf{b}(i)$. Then for any solution $\hat{\mathbf{b}}(i)$ of (12) we have the following inequalities:

$$\frac{1}{n} \left| \mathbf{R} \Phi (\hat{\mathbf{b}}(i) - \mathbf{b}(i)) \right| \leq 4\delta^2 \left| \hat{\mathbf{b}}(i) \right|_1^2 \quad (16)$$

If (14) holds, then:

$$\left| (\hat{\mathbf{b}}(i) - \mathbf{b}(i)) \right|_1 \leq \frac{4\sqrt{K}\delta}{\kappa} \left| \hat{\mathbf{b}}(i) \right|_1 \quad (17)$$

Proof. Set $\Delta = \hat{\mathbf{b}}(i) - \mathbf{b}(i)$ and $J = J(\mathbf{b}(i))$, denotes the set of nonzero coordinates of θ . Note that:

$$\begin{aligned} |\mathbf{R}\Phi\Delta|_2 &= |(\mathbf{R}\Phi + \Xi)\hat{\mathbf{b}}(i) - \mathbf{y} - \Xi\hat{\mathbf{b}}(i)|_2 \\ &\leq \sqrt{M} \left(\delta |\hat{\mathbf{b}}(i)|_1 + |\Xi|_2 |\hat{\mathbf{b}}(i)|_1 \right) \\ &\leq 2\delta\sqrt{M} |\hat{\mathbf{b}}(i)|_1 \end{aligned} \quad (18)$$

Which proves (15). Next, we have:

$$|\Delta_{J^c}|_1 \leq |\Delta_J|_1 \quad (19)$$

Thus,

$$|\hat{\mathbf{b}}(i) - \mathbf{b}(i)|_1 \leq 2|\Delta_J|_1 \leq 2\sqrt{K}|\Delta_J|_2 \leq \frac{2\sqrt{K}}{\kappa\sqrt{M}} |\mathbf{R}\Phi\Delta|_2 \quad (20)$$

Combining (18) and (19), we get (16).

In this section, we have investigated the sensitivity of compressed sensing to measurement matrix uncertainty. Our analysis pointed out the worst situation which is related with the known parameter κ .

5 Experiments

In this section, the proposed method is compared with the Gan's method [8] by simulations to demonstrate its superior performance.

The conventional array is an ULA with $N = 20$ array elements which are displaced with half the wavelength between adjacent elements. The number of compressed receivers is set to be $M = 5$. Each element in the random measurement matrix \mathbf{R} is consisted of random ± 1 which satisfy a Bernoulli distribution. The noise matrix Ξ obeys the Gaussian distribution when the variance is 0.3 and $\delta = |\Xi|_2$. The signal to noise ratio (SNR) is defined as $10\log_{10} \left(\frac{P_k}{\sigma_n^2} \right)$. We compare the Gan's method and the proposed method first. There are two correlated equal-power narrowband sources with $SNR = 15$ dB. We take $N_a = 360$ by searching from -90° to 89.5° with step size 0.5° . The parameters in Eq. (12) are set to be $\eta_1 = \eta_2 = 1$. Figure 2 shows the normalized angular spectrum of the Gan's method. Clearly, it cannot express the angular spreads, while the proposed method can reach a good result in Fig. 3.

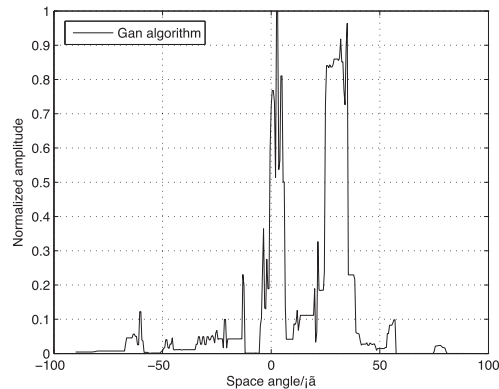


Fig. 2. Angle spectrum of Gan's method

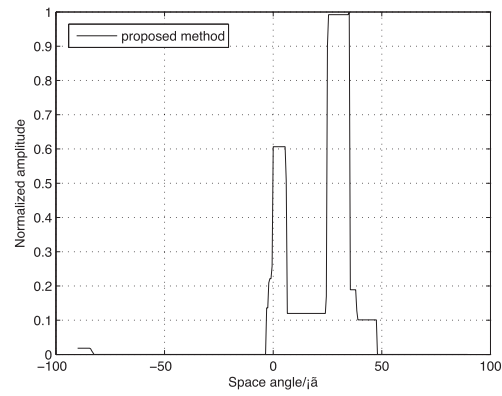


Fig. 3. Angle spectrum of proposed method

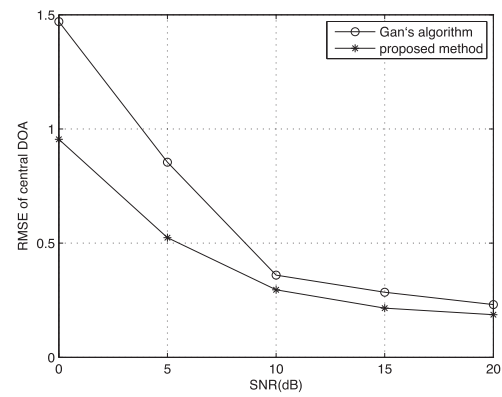


Fig. 4. The RMSE of the central DOA versus SNR

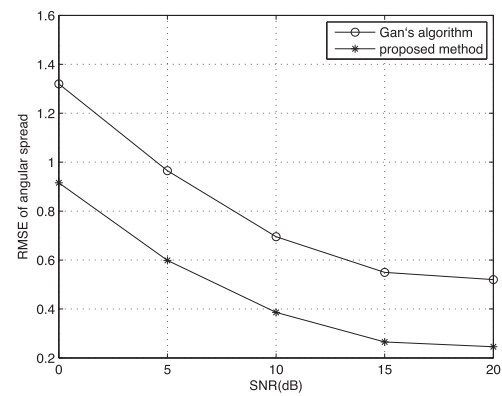


Fig. 5. The RMSE of the angular spread versus SNR

In the second test, all the conditions are as the same as the first test. The root mean square errors (RMSEs) of two parameters with respect to the proposed method and the Gan's method are shown in Figs.4 and 5 respectively. It is derived from 200 Monte-Carlo simulations when SNR varies from 0 to 20 dB. These two figures show that the RMSE of the proposed method are smaller than the Gan's method in different SNRs.

6 Conclusion

In this letter, we have proposed a matrix uncertainty selector (MU-selector) method to estimate the central DOAs and the angular spreads of the distributed sources. This method is based on the block-sparse signal recovery with unknown noise in measurement matrix. Compare to the method in [8], our method is robust to the measurement matrix uncertainty and has less computation. We also analyzed the worst performance with inequalities. Simulation results confirm that the proposed method was robust with the measurement matrix uncertainty.

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