Pattern synthesis method applied in designing HF superdirective receive arrays

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Abstract: Low side lobe level (SLL) and forming nulls can help to realize high-frequency (HF) superdirective receive arrays. In this letter, we reformulate the problem of designing an HF superdirective receive array as a problem of pattern synthesis. Based on the adaptive array theory, we present a numerical pattern synthesis algorithm combined with the design of HF superdirective receive array. Compared with the existing methods for superdirective array design, new method has more advantages in suppressing interferences and clutter. To confirm the effectiveness of novel method, numerical simulations are conducted.
Keywords: pattern synthesis, superdirective, HF receive array
Classification: Electromagnetic theory

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1 Introduction

HF (high-frequency) systems have been widely applied in military, remote sensing and communication. One of the key limitations of HF systems is that its receive array is too tremendous to afford. Superdirective array [1, 2, 3, 4, 5]

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is an irresistible way for developing HF receive array, as it claims that a small size array using superdirective beamforming can reach as high directivity as a large-size array using conventional beamforming. Although superdirective array is attractive, some inherent flaws such as low efficiency and sensitive to array perturbation limit its application in practice. In spite that Constrained Optimum Directive Gain (CODG) method in [3] can have lower demand on array perturbation, array calibration is essential in overcoming the limitation of array perturbation, which should be conducted in advance.

Aim at a small size array with accurately calibrated, the main challenge to realize a superdirective receive array is low array efficiency. Although the two methods introduced in [3, 5] can make the array efficiency reach the demanded level, their own flaws urge us to make improvements. In this paper, low side lobe level (SLL) and forming nulls are used to ameliorate array efficiency. Therefore, we present a numerical pattern synthesis algorithm, in which the problem of designing an HF superdirective receive array can be changed to a problem of pattern synthesis. Compared with the existing methods for superdirective array design, the proposed method has more advantages in suppressing interferences and clutter.

2 Prelimininaries

Applying a set of complex weights to sum the outputs of elements, beamforming can enhance the desired signal and suppress interferences and noise. Suppose $\mathbf{a}(\theta,\varphi)$ presents the array steering vector and $\mathbf{w}$ denotes the weight in the direction (θ_0,φ_0), the radiation pattern of beamforming can be defined as

$$F(\theta,\varphi)=\mathbf{w}^H(\theta_0,\varphi_0)\mathbf{a}(\theta,\varphi) \tag{1}$$

Signal-to-noise (SNR) can be viewed as one of the most important parameters to describe the performance of the receive array. According to the research of Newman [2], SNR can be proportional to the directive gain, provided that the system background noise dominates. Under this condition, SNR can be maximized by maximizing the directive gain. The directive gain in the direction (θ_0,φ_0) can be expressed as

$$G(\theta_0,\varphi_0)=\frac{4\pi|F(\theta_0,\varphi_0)|^2}{\int_0^{2\pi}\int_0^{\pi}\sin\theta|F(\theta,\varphi)|^2d\theta d\varphi} \tag{2}$$

Based on Eq. (1), the directive gain can be represented as

$$G(\theta_0,\varphi_0)=\frac{\mathbf{w}^HN\mathbf{w}}{\mathbf{w}^HR\mathbf{w}} \tag{3}$$

in which N and R can be written as

$$N=a(\theta_0,\varphi_0)a^H(\theta_0,\varphi_0) \tag{4}$$

and

$$R=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin\theta\mathbf{a}(\theta,\varphi)\mathbf{a}^H(\theta,\varphi)d\theta d\varphi \tag{5}$$

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The integral result of R can be a constant matrix. Assume k as the radio wave number and short vertical dipoles as antenna elements, the result of R can be denoted as:

$$R_{ij} = \begin{cases} \frac{2}{3} & \text{if } i = j, \\ \frac{kd_{ij}}{\sin kd_{ij}} - \frac{1}{(kd_{ij})^2}[\frac{kd_{ij}}{\sin kd_{ij}} - \cos kd_{ij}] & \text{if } i \neq j. \end{cases} \tag{6}$$

where d_{ij} is the distance between the ith element and the jth element.

Based on the distortionless constraint $\mathbf{w}^H a(\theta_0, \varphi_0) = 1$, the problem of maximizing the directive gain can be changed to an optimum problem, which can be expressed as follows:

$$\underset{\mathbf{w}}{min} \ \mathbf{w}^H R\mathbf{w} \ s.t \ \mathbf{w}^H a(\theta_0, \varphi_0) = 1 \tag{7}$$

Using Lagrange multiplier method, the solution of the optimization problem can be easily obtained. The optimal $\mathbf{w}_{opt}$ and the corresponding directive gain can be written as follows:

$$\mathbf{w}_{opt} = \frac{R^{-1}\mathbf{a}(\theta_0, \varphi_0)}{\mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0)}, \ G_{opt} = \mathbf{a}^H(\theta_0, \varphi_0)R^{-1}\mathbf{a}(\theta_0, \varphi_0) \tag{8}$$

The above method corresponds to the theoretical maximum directive gain, which can be called as Optimum Directive Gain (ODG) method. However, its array efficiency is very low, which can even make the attenuated external noise lower than internal receiver noise. In this case, the weight sum of array signals cannot be an enhancement of signals but get close to zero. Thus, to guarantee the performance of ODG method, the array efficiency must be higher than the demanded array efficiency.

Based on $\mathbf{w}^H a(\theta_0, \varphi_0) = 1$, the formulation of array efficiency [5] could be simplified as:

$$\eta = \frac{1}{M\mathbf{w}^H\mathbf{w}} \tag{9}$$

The demanded array efficiency η_0 can be calculated as follows. Assume external receiver noise at 10 MHz is 55 dB larger than internal receiver noise and the dipole element connected to a high-impedance preamplifier with a noise figure of 10 dB. To make sure that external noise dominates, we add 10 dB "cushion". Then η_0 at 10 MHz must have a minimum level of −35 dB.

To reach the demanded array efficiency, there were two kinds of methods. Obtaining the maximum directive gain, ODG method in [3] need to choose appropriate element number and element spacings. Using a proper loading value, the CODG method can make a much smaller size array reach the demanded array efficiency while the directive gain sacrifices. The detailed descriptions of ODG method and CODG method could be found in [3, 5].

3 Proposed method

Low side lobe level (SLL) and forming nulls are not only the basic tasks for array pattern synthesis but also the real requirements to suppress strong

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interferences and clutter. We find that low SLL and forming nulls can help to improve array efficiency. Therefore, the problem of designing an superdirective receive array can be changed to a problem of pattern synthesis.

To suppress the interference arriving from (θ_i, φ_i), a null with certain depth e_i is essential in the radiation pattern. The constraint can be written as

$$a^H(\theta_i, \varphi_i)\mathbf{w} = f_i \tag{10}$$

in which $f_i = 10^{e_i/20}$.

Assume the number of elements as M and N constraints to form nulls while N at most is equal to $M-1$. Based on Eq. (10), new linear constraint problem can be formulated as:

$$\underset{\mathbf{w}}{min}\ \mathbf{w}^H R\mathbf{w}\ s.t\ A^H\mathbf{w} = f \tag{11}$$

where $A = [a(\theta_1, \varphi_1), \ldots, a(\theta_N, \varphi_N)]$ and $f = [f_1, \ldots, f_N]^H$. The solution of this problem can be present as

$$\hat{\mathbf{w}} = R^{-1}A(A^H R^{-1} A)^{-1} f \tag{12}$$

Using Eq. (12), the problem of array efficiency and suppressing interferences can be solved simultaneously, provided that the condition $\hat{\mathbf{w}}^H\hat{\mathbf{w}} \leq \frac{1}{M\eta_0}$ reach. A null with certain width is a universal method to suppress interference. Herein, a broad null can be used to improve array efficiency. In the implementation, we just need to select several continuous angles around the desired angle to obtain a broad null.

Compared with forming nulls, low SLL can improve array efficiency more significant. However, the desired SLL ϵ cannot be achieved directly. According to adaptive array theory [6, 7] for pattern synthesis, the control of SLL can also be classified as a problem of choosing appropriate angles of interferences and forming nulls with proper depths. The iterative procedure introduced in [7] can help us to obtain the desired SLL. In the iteration, if there exists interferences, the weight $\hat{\mathbf{w}}$ after forming nulls is assumed as the initial weight $\vec{\mathbf{w}}_0$, else $\vec{\mathbf{w}}_0$ can be substituted by $\mathbf{w}_{opt}$ in the beginning. To make the side lobe peaks lower than ϵ, calculating the augmentation weight $\Delta\vec{\mathbf{w}}$ in each iteration is the key problem. $\Delta\vec{\mathbf{w}}$ can be obtained as follows:

$$\begin{array}{lll} \underset{\Delta\vec{\mathbf{w}}}{min} & \Delta\vec{\mathbf{w}}^H R\Delta\vec{\mathbf{w}} & \\ s.t & & \Delta\vec{\mathbf{w}}^H a(\theta_0, \varphi_0) = 0 \\ & & \Delta\vec{\mathbf{w}}^H \mathbf{a}(\theta_j, \varphi_j) = \Delta f_j,\ i = 1, 2, \ldots, P \end{array} \tag{13}$$

The scalar terms Δf_j is imposed on the response at the peak location (θ_j, φ_j), making it equal to ϵ after the weight $\vec{\mathbf{w}}$ has been updated. The appropriate Δf_j can be given as:

$$\Delta f_j = (\epsilon - |c_j|)\frac{c_j}{|c_j|} \tag{14}$$

where c_j is the response value of direction (θ_j, φ_j) in the current radiation pattern. As P at most is $M-1$, so P largest side lobe peaks in the specified region are used in (13). The desired SLL can be obtained after several

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iterations. In general, the condition $\vec{\mathbf{w}}^H\vec{\mathbf{w}} \leq \frac{1}{M\eta_0}$ can be reached, so long as ϵ and the specified region are properly chosen.

The whole procedure of novel method can be conducted as follows.

1. According to the practical situation, select Eq. (8) or Eq. (12) to calculate the initial weight $\vec{\mathbf{w}}_0$.

2. If the condition $\vec{\mathbf{w}}_0^H\vec{\mathbf{w}}_0 \leq \frac{1}{M\eta_0}$ satisfies, the following steps can be omitted; otherwise, go to the next steps to obtain low SLL.

3. Specify the desired SLL ϵ and the controlled SLL region.

4. Identify P largest side lobe peaks in the specified region and use Eq. (14) to determine the response values Δf_j.

5. Solve $\Delta\mathbf{w}$ by Eq. (12) and update the weight by $\vec{\mathbf{w}} = \vec{\mathbf{w}} + \Delta\vec{\mathbf{w}}$.

6. If the condition $\vec{\mathbf{w}}^H\vec{\mathbf{w}} \leq \frac{1}{M\eta_0}$ satisfies, stop and the final $\vec{\mathbf{w}}$ is the desired weight; otherwise, return to Step 3, adjusting ϵ and the controlled SLL region to go on.

4 Computer simulations

In the simulation, we assume the arrays work at 10 MHz and the demanded array efficiency can be -35 dB correspondingly. The receive arrays adopt a uniform circular array of $M = 7$ elements. Using the ODG method, the circular array must have a minimum radius of 2.85 m while its array efficiency is -34.9 dB. In the following, we provide several examples to confirm the effectiveness of the proposed method.

In Fig. 1, two programs based on the proposed method are presented to make a superdirective array reach the demanded array efficiency, low SLL and nulls at the same time. In program 1, the SLL assigned to the side lobe region $[0°, 140°] \cup [220°, 360°]$ is -30 dB. In program 2, six nulls in the directions $[10°, 65°, 110°, 270°, 295°, 315°]$ with a depth -75 dB are specified. As

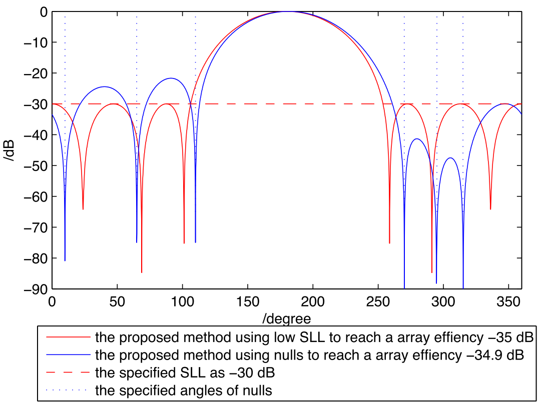

Fig. 1. Two programs based on the propose method are presented to make the circular array with $r = 2$ m reach the demanded array efficiency.

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Table I. Use the propose method with the specified SLL to make the superdirective arrays with different radius reach the demanded array efficiency.

Radius	The specified SLL	Directive Gain	Array efficiency
2.75 m	−12 dB	11.5 dB	−35.0 dB
2.50 m	−18 dB	11.1 dB	−34.9 dB
2.25 m	−24 dB	10.6 dB	−34.6 dB
2.00 m	−30 dB	10.2 dB	−35.0 dB
1.75 m	−41 dB	9.7 dB	−34.8 dB
1.50 m	−60 dB	9.2 dB	−34.9 dB

shown in Fig. 1, a proper selection of nulls and SLL can help a superdirective array to reach the demanded array efficiency.

To illustrate the proposed method can make a even smaller size array realize a superdirective array, we provide several representative results in Table I. For simplicity, we make the array reach the demanded array efficiency by controlling the SLL. From the results shown in Table I, it can be seen that novel method can make the aperture of HF receive array reduce further.

CODG method can also make a small size array reach the demanded array efficiency. To demonstrate the superiority of new method, we compare it with CODG method. Assume an interference arrives from 20° and the controlled SLL area is $[65°, 140°] \cup [220°, 285°]$. From Fig. 2, we can see that new method is more flexible and effective in controlling SLL and nulls.

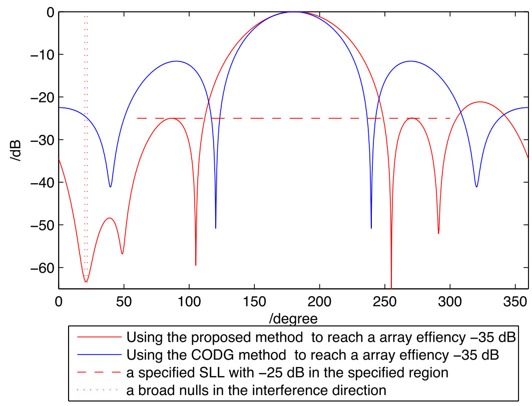

Fig. 2. The radiation patterns of two methods when they make the circular array with $r = 2$ m reach the demanded array efficiency.

5 Conclusion

Aim at a small size array with accurately calibrated, we present a numerical pattern synthesis algorithm, which extends the theory for designing HF superdirective receive array. Compared with the existing methods for superdirective array design, new method has more advantages in suppressing interferences and clutter. Through numerical examples in simulation, the superiority of novel method is demonstrated.

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