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A novel approach to pruning the general Volterra series for modeling power amplifiers

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Abstract: This paper proposes a recursive optimum-term selecting (ROS) approach to pruning the general Volterra series, by which we can achieve a custom-tailored model to characterize nonlinearity of wideband power amplifiers (PAs) with memory effects. The achieved model is more suitable for the individual PA than those static models, such as the MP and GMP models, as it selects the most efficient terms from the general Volterra series based on the theory of recursive correlation cancellation. Simulation results show that the approach is effective, and the pruned model developed by the proposed approach is efficient as well as adaptable.

Keywords: behavioral model, Volterra series, power amplifier modeling

Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

The Volterra series is a power series with memory [1], which is utilized to characterize nonlinearity of wideband power amplifiers (PAs) with memory effects. However, due to the high computational complexity, this kind of model is just suitable to modeling PAs with weak nonlinearity. For the purposes of application in the digital pre-distorter (DPD) to compensate for the strong nonlinearity with memory, it is necessary to find a practical and effective method to prune the general Volterra series according to the needed accuracy and the computational resources. There are several methods to actualize the simplification in the literature [2, 3, 4]. In [2], an approach called memory polynomial (MP) is introduced, which is a pruned Volterra model only with the diagonal terms. The generalized memory polynomial (GMP) model is an extension by adding cross terms between the signals and lagging [3]. Besides these, the dynamic deviation reduction (DDR) model in [4] is also a pruned Volterra model. These models are mostly based on an ideal assumption that the terms with particular structures, such as near-diagonality [5], are crucial to the nonlinearity of PAs. In fact, PAs are designed and manufactured with a great difference, so the models constructed statically only perform well on some special occasions.

Based on the general Volterra series, this paper presents a novel approach to achieve a custom-tailored model off-line according to the measurement, which is more suitable for the individual PA than those common ones. The custom-tailored model is achieved by a recursive optimum-term selecting (ROS) procedure, which selects more efficient terms from the general Volterra series. The simulation results in Section III show that the approach is effective, and the achieved model is efficient as well as adaptable.

2 The proposed approach

In the simulation of wireless communication systems, the modulated RF signal is usually represented by its complex envelope which contains all the information when the bandwidth is much smaller than the carrier frequency [6]. Here, suppose that $x(n)/y(n)$ is the real input/output signal of a PA, then we can express it in terms of complex baseband representation $\tilde{x}(n)/\tilde{y}(n)$ as: $x(n) = Re\{e^{j\omega_0 n}\tilde{x}(n)\}$, $y(n) = Re\{e^{j\omega_0 n}\tilde{y}(n)\}$, where $\omega_0 = 2\pi f_0$ with f_0 being the carrier frequency. Without loss of generality, we assume that the kernels are symmetric. So that the truncated Volterra series with M-delay memory and P-order nonlinearity can be written as [7]

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$$\tilde{y}(n)=\sum_{\substack{p=1\\p\ odd}}^{P}\sum_{m_1=0}^{M}\sum_{m_2=m_1}^{M}\cdots\sum_{m_{(p+1)/2}=m_{(p-1)/2}}^{M}$$
$$\sum_{m_{(p+3)/2}=0}^{M}\cdots\sum_{m_p=m_{p-1}}^{M}\tilde{h}_p(m_1,\dots,m_p) \tag{1}$$
$$\times\prod_{i=1}^{(p+1)/2}\tilde{x}(n-m_i)\prod_{j=(p+3)/2}^{p}\tilde{x}^*(n-m_j)$$

$$\Phi_x=\begin{pmatrix}\tilde{x}_1 & \dots & \tilde{x}_{1-M} & \tilde{x}_1\tilde{x}_1\tilde{x}_1^* & \dots & \tilde{x}_{1-M}\dots\tilde{x}_{1-M}\tilde{x}_{1-M}^*\dots\tilde{x}_{1-M}^*\\ \tilde{x}_2 & \dots & \tilde{x}_{2-M} & \tilde{x}_2\tilde{x}_2\tilde{x}_2^* & \dots & \tilde{x}_{2-M}\dots\tilde{x}_{2-M}\tilde{x}_{2-M}^*\dots\tilde{x}_{2-M}^*\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \tilde{x}_n & \dots & \tilde{x}_{n-M} & \tilde{x}_n\tilde{x}_n\tilde{x}_n^* & \dots & \tilde{x}_{n-M}\dots\tilde{x}_{n-M}\tilde{x}_{n-M}^*\dots\tilde{x}_{n-M}^*\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \tilde{x}_N & \dots & \tilde{x}_{N-M} & \tilde{x}_N\tilde{x}_N\tilde{x}_N^* & \dots & \tilde{x}_{N-M}\dots\tilde{x}_{N-M}\tilde{x}_{N-M}^*\dots\tilde{x}_{N-M}^*\end{pmatrix} \tag{2}$$

where $\tilde{h}_p(m_1,\dots,m_p)$ is the pth-order Volterra complex kernel. From (1), we can identify the linear relationship between the complex envelope $\tilde{y}(n)$ and the Volterra kernels, so it is feasible to extract the parameters by using linear system estimation theory such as least squares (LS) and its variants [6]. Given that the captured input and output data streams have N samples respectively, then the terms in the Volterra series can be written in terms of matrix as (2), where $()^*$ denotes the complex conjugate. The terms in the matrix Φ_x are all odd-order products and this means that the number of $\tilde{x}$ is more than that of its complex conjugate $\tilde{x}^*$ by exactly one [3]. Define a column vector $Y=(\tilde{y}_1,\tilde{y}_2,\dots,\tilde{y}_n,\dots,\tilde{y}_N)^T$, where $\tilde{y}_n$ indicates $\tilde{y}(n)$, and define another column vector $B=(\tilde{b}_1,\tilde{b}_2,\dots,\tilde{b}_s,\dots,\tilde{b}_S)^T$, where $\tilde{b}_s$ represents the sth Volterra kernel and S is the total number of the P-order Volterra kernels, which can be calculated as

$$S=\sum_{p=0}^{(P-1)/2}\binom{M+p}{p}\binom{M+p+1}{p+1} \tag{3}$$

then we can obtain a matrix equation that is equivalent to (1)

$$Y=\Phi_x B \tag{4}$$

Using the LS method, the linear matrix equation (4) can be solved into the form of

$$B=\Phi_x^{\dagger}Y \tag{5}$$

where $\Phi_x^{\dagger}=(\Phi_x^H\Phi_x)^{-1}\Phi_x^H$ is the Moore-Penrose pseudo-inverse of Φ_x, with $()^H$ indicating the Hermitian transpose.

Reviewing (2) and (4), if the matrix Φ_x is expressed in the form of row vector $\Phi_x=(\phi_1,\phi_2,\dots,\phi_s,\dots,\phi_S)$, where ϕ_s is a column vector composed of the products of the complex envelope $\tilde{x}$ and its corresponding complex

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conjugate $\tilde{x}^*$, then we can recognize that the vector Y is represented by a space which is spanned by the bases Φ_x and the element $\tilde{b}_s$ in the vector B is the coefficient corresponding to the bases Φ_x's. Because of the close correlation characteristics between the over sampled data, the bases Φ_x are correlated strongly. This means that the matrix equation (4) is redundant in a sense. So it is necessary to further identify the basis ϕ_s to select more efficient ones for approximating the vector Y. By cutting off less contributive bases, we can dramatically simplify Φ_x. In other words, the Volterra series will be pruned by abnegating those unimportant terms.

Here, we present an approach named recursive optimum-term selecting (ROS), which is based on the theory of correlation cancellation. Firstly, the inner products of the output vector Y and each basis vector ϕ_s are calculated to obtain a projection vector: $P_y = (\tilde{P}_1, \tilde{P}_2, ..., \tilde{P}_s, ...\tilde{P}_S)$, which represents the extent of correlation in complex. Secondly, the optimum term is selected for the most efficient term group Ψ_x according to the maximum absolute value criterion with respect to the vector P_y. At the same time, the other terms are updated by canceling correlation between them and the optimum term. Subsequently, the selected term is masked in Φ_x and the procedure repeats with the updated Φ_x. The recursive procedure continues until enough terms are selected. It should be noted that the number of terms in Ψ_x is decided by the needed accuracy and the computational resources. At last, we can achieve a group of terms in Ψ_x, which can characterize the PA as a behavioral model. Referring to the description above, suppose that the number of expected terms is K, then the proposed ROS approach can be written as the following algorithm

Algorithm: Recursive Optimum-terms Selecting (ROS)

$\Phi_x \leftarrow \tilde{x}(n)$
while $k \leftarrow 0$ **to** K **do**
  for $s \leftarrow 1$ **to** S **do**
    $P_y \leftarrow < Y, \phi_s >$
  end for
  $\Psi_x \leftarrow$ select the optimum term from Φ_x according to the maximum absolute value criterion with respect to the vector P_y
  $\Phi_x \leftarrow$ update the other terms by canceling correlation and mask it in Φ_x
  $k \leftarrow k + 1$
end while

Fig. 1 demonstrates the ROS process for the truncated Volterra model with one-delay memory and fifth-order nonlinearity as an example. The algorithm is revised by adopting the first-order terms directly to simplify the algorithm. Hence, we will only consider the third- and fifth-order terms, which leads to eighteen coefficients. The figure (a) shows the projections

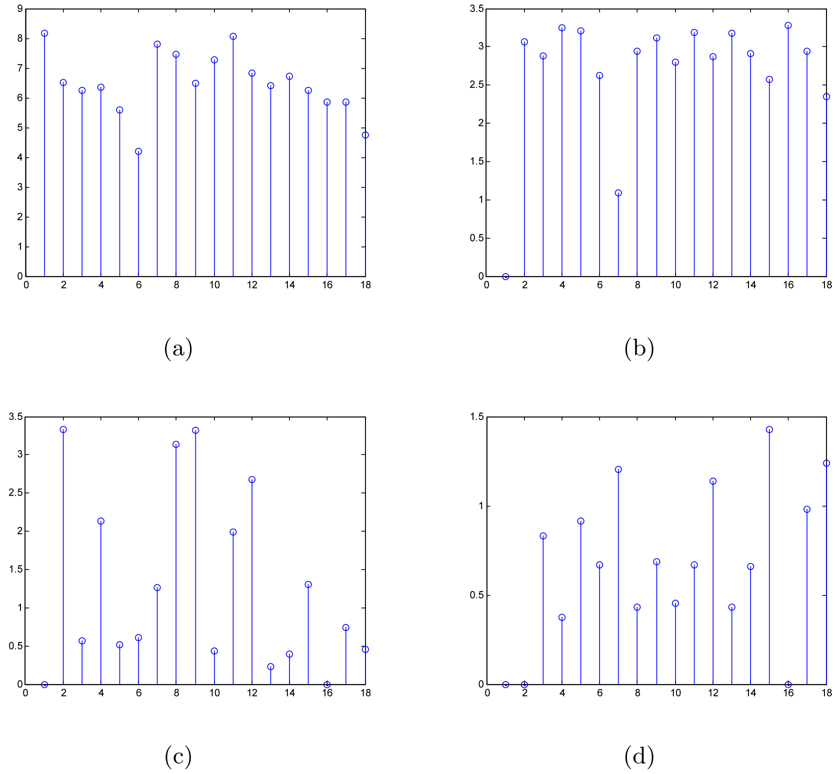

Fig. 1. The ROS process on a Volterra model (M=1, P=5)

of the output vector Y on the initial bases Φ_x, where the horizontal axis represents the index of the terms in Φ_x, and the vertical axis represents the absolute value of the projections. According to the criterion mentioned in the algorithm, the first term should be selected for the most efficient term group Ψ_x and masked in Φ_x, then the other terms would be updated by subtracting the correlation component between them and the first term. After calculating the inner products of the output vector Y and the updated Φ_x, we would obtain the figure (b). According to the same criterion, the sixteenth term would be selected. In this way, the second and the fifteenth terms would be selected, so the first, the sixteenth, the second and the fifteenth terms, in company with the first-order terms form a custom-tailored behavioral model, which is just the expected result of the proposed approach.

As shown in Table I, the MP/GMP and custom-tailored models are all the subsets of the Volterra model, where the marker ① denotes the terms in the MP model, the marker ② denotes the terms in the GMP model, and the marker $\star$ denotes the selected terms by ROS approach. The third- and fifth-order terms in the custom-tailored model differ from those in the MP/GMP model if they have the same number. This means that the developed model is different from the compared models with the same computational complexity, but it characterizes PA more accurately according to the simulation results in the following section.

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Table I. The Volterra model terms (M=1, P=5)

Index	The Volterra Model Terms	Marker
1	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}^*(n-0)$	① ② ⋆
2	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}^*(n-1)$	⋆
3	$\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}^*(n-0)$	②
4	$\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}^*(n-1)$	②
5	$\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-0)$	
6	$\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-1)$	① ②
7	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}^*(n-0)\tilde{x}^*(n-0)$	① ②
8	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}^*(n-0)\tilde{x}^*(n-1)$	
9	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}^*(n-1)\tilde{x}^*(n-1)$	
10	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-0)$	②
11	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-1)$	
12	$\tilde{x}(n-0)\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}^*(n-1)\tilde{x}^*(n-1)$	
13	$\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-0)$	
14	$\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-1)$	
15	$\tilde{x}(n-0)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-1)\tilde{x}^*(n-1)$	② ⋆
16	$\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-0)$	⋆
17	$\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-0)\tilde{x}^*(n-1)$	
18	$\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}(n-1)\tilde{x}^*(n-1)\tilde{x}^*(n-1)$	① ②

3 Approach evaluation

As illustrated in Fig. 2, we setup a test bench composed of a series of instruments and a solid PA with about 37 dB gain, which behaved almost flat in the frequency domain from 5 to 5.5 GHz. The baseband I/Q source data was produced in MATLAB and saved into a two bin file, which represented the I/Q signals. Then they were downloaded into the wideband arbitrary waveform generator (AWG), which operated at the sampling rate of 1.25 GSa/s. The baseband I/Q signals were generated by the AWG and transmitted into the vector signal generator (VSG) through dual differential channels, where the baseband I/Q signal was modulated at the carrier centered at frequency 5 GHz and led to the RF signal. In succession, the RF signal was amplified by the PA and went into an attenuator which weakened the amplified signal by 30 dB. The oscillograph sampled and digitalized the analog RF signal. At last, the digital signal was saved as a bin file and uploaded into PC via Ethernet.

In this study, we consider modeling PAs in high-speed wireless digital communication systems with implementing 16QAM modulated signals. Af-

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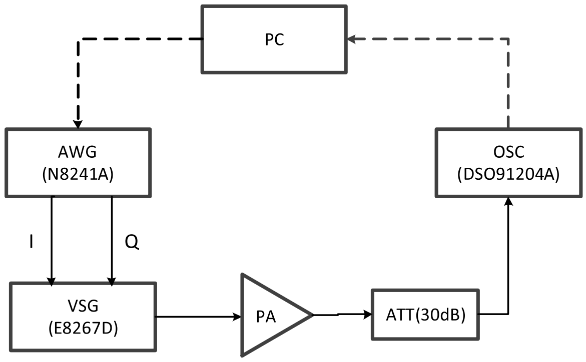

Fig. 2. Experimental test bench sketch map

Table II. Coefficients number with different memory delay

Memory Delay	1	2	3	4
MP	6	9	12	15
GMP	14	21	28	35
Volterra	20	81	244	605

ter obtaining the digitalized RF signal on PC, we down-mix and filter it to acquire the complex envelope in MATLAB, then align it with the baseband I/Q source data for model extraction. Using the ROS algorithm in Section II, we develop a custom-tailored model for the PA and solve coefficients of the inverse behavioral model. Then we can evaluate effectiveness as well as adaptability of the custom-tailored model by simulation with DPD architecture, which is an open-loop chain model introduced in [8].

3.1 Effectiveness evaluation

For the convenience of simulation, the general Volterra series is truncated to a model with four-delay memory and five-order nonlinearity. As shown in Table II, the number of coefficients in the Volterra model exponentially increases as memory length increases at one step, but the number of coefficients in the MP/GMP model increases slowly. For a fair comparison, we equalize the number of the total coefficients among MP, GMP, and our custom-tailored model. Applying the test bench mentioned above, and setting the exciting signal as a 80M-bandwidth 16QAM-modulation signal with -3 dBm power, we acquire around 20,000 data points which represent the input and output signals of the PA. After the development of the custom-tailored model, we extract coefficients and generate the post-distortion signal, which is equal to the predistortion according to the literature [1], then calculate the degree of difference between the input signal and the post-distortion signal in terms of NMSE, which is defined in [9].

As shown in Fig. 3, the custom-tailored model achieves lower NMSEs than the MP and GMP models on most occasions. There is an exception that the NMSE of the GMP model is lower than the Volterra and ROS(GMP) models when memory delay is one. This is caused by the next memory delay terms in the GMP model, which might contain more information about the output

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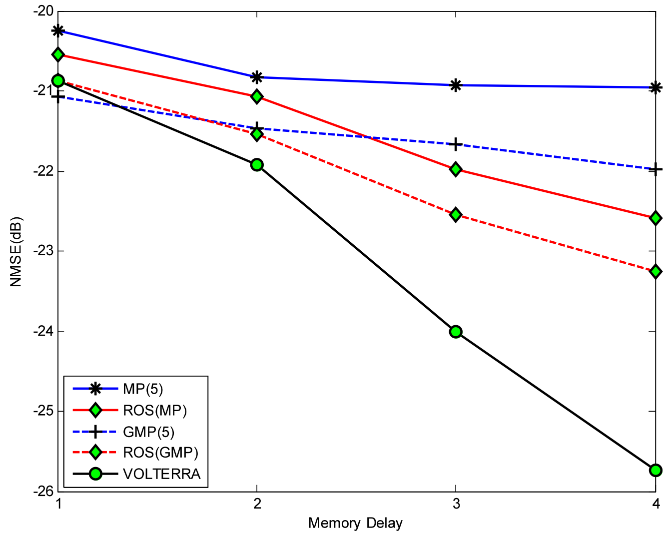

Fig. 3. Effectiveness comparison in terms of NMSE

Table III. Comparsion between the custom-tailored model and the MP model

Input Power	NMSE(MP)	NMSE(ROS)	NMSE Difference
0 dBm	−16.38	−17.20	−0.82
−3 dBm	−20.96	−22.58	−1.62
−6 dBm	−25.49	−27.12	−1.63

Table IV. Comparsion between the custom-tailored model and the GMP model

Input Power	NMSE(GMP)	NMSE(ROS)	NMSE Difference
0 dBm	−16.98	−18.02	−1.04
−3 dBm	−21.98	−23.25	−1.27
−6 dBm	−26.51	−27.68	−1.17

signal when memory length is not long enough. It is evident that the custom-tailored model behaves more excellently than the two models as memory length increases. When memory delay is four, the NMSE of the developed model is lower than that of the MP/GMP model by 1.6 dB/1.2 dB. So the custom-tailored model developed by proposed approach is more efficient than the two common models.

3.2 Adaptability evaluation

In order to evaluate the adaptability of the custom-tailored model, we adopt three input power levels in turn, which makes the PA to work in different regions. We simulate the predistortion signal with the data captured in the case of 0 dBm/−6 dBm input power by using the same term model developed for the −3 dBm-input-power case, and calculate the NMSEs for comparision. On this occasion, we also adopt a four-delay memory and five-order nonlinearity model. As shown in Table III and Table IV, we can conclude that

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the custom-tailored model developed for the −3 dBm-input-power case also performs well in the case of 0 dBm/−6 dBm input power, as it reduces the NMSEs from 0.82 dB to 1.63 dB compared with the MP model, and reduces the NMSEs from 1.04 dB to 1.17 dB compared with the GMP model. Therefore, the model developed by the proposed approach shows good performance at different input power levels.

4 Conclusion

This paper proposes an approach named ROS to pruning the general Volterra series. By using this method, we can develop a custom-tailored model to characterize the nonlinearity of PAs with memory effects, and also apply it to DPD for the linearization of PAs. It is shown that the proposed approach is effective and the developed model is efficient as well as adaptable. Owing to the recursive procedure, it is convenient to make trade-off between the needed accuracy and the computational complexity.

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