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Parallel multi-rate compressed sampling with a sub-Nyquist sampling rate

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Abstract: The analog to information converter (AIC) based on compressed sensing (CS) is designed to sample the analog signals at a sub-Nyquist sampling rate. In this paper, we propose a novel parallel multi-rate compressed sampling (PMCS) system. It has several parallel paths and each path has several low-speed analog-to-digital converters (ADCs). This system has simple structure and low sampling rate, which makes it easy to be implemented on hardware. Simulation results show that signals can be reconstructed in high probability even though the sampling rate is much lower than the Nyquist sampling rate.

Keywords: analog to information converter (AIC), compressed sensing (CS), sub-Nyquist sampling rate, parallel multi-rate compressed sampling (PMCS)

Classification: Electron devices, circuits, and systems

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1 Introduction

In traditional digital signal processing system, analog-to-digital converter (ADC) which based on the Nyquist theorem is a key component. However

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with rapid development of wireless communication, the signal bandwidth becomes wider to satisfy the increasing data volume and the sampling rate may be beyond the capacity of ADCs.

Recent work in compressed sensing (CS) provides a way to sample sparse or compressible signals efficiently at a sub-Nyquist rate [1, 2]. Analog-to-information converter (AIC) has been proposed to obtain compressed samples directly from an analog signal [3, 4]. It consists of several parallel branches of mixers and integrators (BMIs) and the number of samples equals to the number of BMIs. The parallel segmented AIC (PS-AIC) is proposed to obtain more samples, where the signal is segmented and the equivalent measurement matrix (EMM) satisfies the restricted isometry property (RIP) with overwhelming probability if the original matrix of BMIs satisfies it [5, 6]. Reference [7] proposes a partial segmented AIC scheme where each BMI only works within partial time period. It provides a preferable trade off between the complexity and error performance.

In this paper, we propose a parallel multi-rate compressed sampling (PMCS) system, where each BMI integrates over the signal period T and has several parallel ADCs which respectively have different sampling rates. The former reduces the reset frequency of integrator to reduce errors. The latter further reduces the sampling rate of each ADC and at the same time the more ADCs in each BMI branch ensure more subsamples collected which improves the recovery probability when the number of BMI branch is determined.

The remainder of the paper is organized as follows. Section 2 provides a brief introduction to CS and describes the PS-AIC system. Section 3 introduces the proposed PMCS architecture. Simulation results are presented in section 4 and conclusion is made in section 4.1.

Notations: For a matrix $\mathbf{A}$, $\mathbf{A}(i,:)$ is the ith row of $\mathbf{A}$ and $\mathbf{A}_{i,j}$ means the entry of $\mathbf{A}$ in the ith row and the jth column. $\mathbf{A}^T$ stands for the transpose of $\mathbf{A}$.

2 Background

2.1 Compressed sensing

Define an N-point real-valued discrete-time signal $\mathbf{x} \in \mathbb{R}^N$. Then the signal $\mathbf{x}$ can be represented in an arbitrary basis $\{\psi_n\}_{n=1}^N$ for $\mathbb{R}^N$ with the weighting coefficients $\{\theta_n\}_{n=1}^N$ as $\mathbf{x} = \sum_{n=1}^N \theta_n \psi_n = \mathbf{\Psi\Theta}$, where $\mathbf{\Psi}$ is a matrix using ψ_n as columns, $\mathbf{\Theta}$ is the coefficient vector. If there are $K(K \ll N)$ nonzero elements in $\mathbf{\Theta}$, then signal is K-sparse.

The compressed sensing measurement process can be denoted as $\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi\Psi\Theta}$, where $\mathbf{y}$ is a column vector containing M measurements. When $\mathbf{V} = \mathbf{\Phi\Psi}$ satisfies the RIP, the sparse signal $\mathbf{x}$ can be recovered from $\mathbf{y}$ through solving l_1 norm minimization

$$\min \|\mathbf{\Theta}\|_1 \quad s.t. \quad \mathbf{y} = \mathbf{\Phi\Psi\Theta}. \tag{1}$$

If for every K-sparse signal $\mathbf{s} \in \mathbb{R}^N$, there exists $0 < \sigma_K < 1$ such that $(1-\sigma_K)\|\mathbf{s}\|_2^2 \le \|\mathbf{Vs}\|_2^2 \le (1+\sigma_K)\|\mathbf{s}\|_2^2$, then we can say $\mathbf{V}$ satisfies the RIP.

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2.2 Parallel segmented AIC

Parallel segmented AIC (PS-AIC) is proposed in [5, 6], where P BMIs are obtained. The basic idea is to split the signal period T in each BMI into Q segments and derive one subsample in each subperiod. Then the subsample obtained in qth segment of the pth BMI is $y_{p,q} = \int_{(q-1)T_c}^{qT_c} x(t)c_p(t)dt$, where $T_c = T/Q$ is the subperiod and $c_p(t)$ is a chipping sequence mixed by the pth BMI. These PQ subsamples are collected in the matrix $\mathbf{Y}$ and $y_{p,q}$ is the element in the pth row and qth column.

Before reconstruct the original signal, convert the matrix $\mathbf{Y}$ to a column vector $\widehat{\mathbf{Y}} = [\mathbf{Y}(1,:), \mathbf{Y}(2,:), \cdots, \mathbf{Y}(P,:)]^T$. Then the element of $\mathbf{V}$ in mth row and nth column, where $m = (p-1)Q + q$, can be expressed as $\mathbf{V}_{m,n} = \mathbf{V}_{(p-1)Q+q,n} = \int_{(q-1)T_c}^{qT_c} c_p(t)\psi_n(t)dt$. Therefore we can estimate $\boldsymbol{\Theta}$ by solving the problem in (1) and reconstruct the original signal.

3 Parallel multi-rate compressed sampling

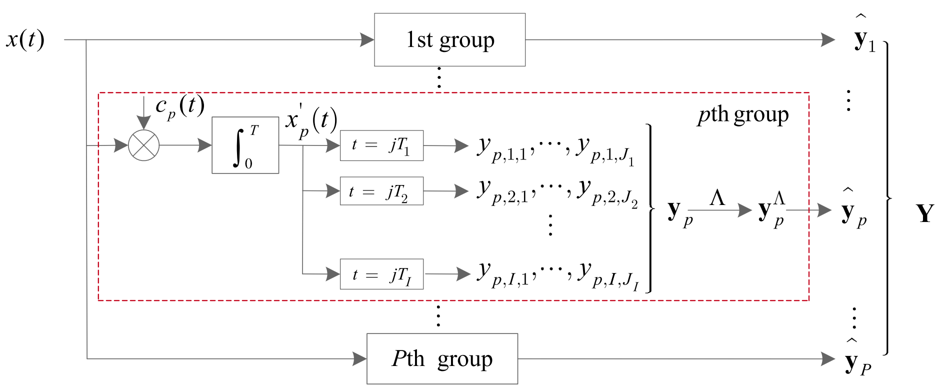

Fig. 1. The sampling process of PMCS

The sampling process of PMCS is illustrated in Fig. 1, where each BMI integrates over the signal period T and the result is sent into several parallel ADCs which have different sampling rates. The BMI and ADCs can be classified as one group. It is supposed that P groups are included in PMCS and each group has I ADCs.

The integration result of BMI in the pth group is given as

$$x'_p(t) = \int_0^T x(\tau)c_p(\tau)d\tau, \quad 0 \le t \le T. \tag{2}$$

Then the result is fed into I ADCs to be sampled at sub-Nyquist rates. It is supposed that the sampling period of ith ADC is T_i, then $J_i = int(T/T_i)$ subsamples are obtained during the signal period T by the ith ADC where int() means returnning the largest integer less than or equal to the specified expression. The output in the jth sampling interval of ith ADC in the pth group is

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$$\begin{aligned} y_{p,i,j} &= x'(t)|_{t=jT_i} \\ &= \int_0^{jT_i} c_p(\tau)x(\tau)d\tau \\ &= \sum_{n=1}^{N} \theta_n \int_0^{jT_i} c_p(\tau)\psi_n(\tau)d\tau, \end{aligned} \tag{3}$$

where $1 \le p \le P$, $1 \le i \le I$, $1 \le j \le J_i$. Then $M_{\rm G} = J_1 + J_2 + \cdots + J_I$ subsamples are collected by each group. According to (3), in order to make sure that less duplicate subsamples are obtained, the sampling times of I ADCs should not be repeated as possible. Then a simple condition should be satisfied is that any two ADCs' sampling periods can't be multiple relationships, which can avoid the subsamples obtained by one ADC are completely redundant.

The permutation Λ is a one-to-one mapping of the elements of some set to itself by simply changing the order of the elements. Then Λ_k stands for the index of the kth element in the permuted set.

It is defined that $\mathbf{T}_{\rm G} = [T_1, \cdots, J_1T_1, T_2, \cdots, J_2T_2, \cdots, T_I, \cdots, J_IT_I]$ and $\mathbf{y}_p = [y_{p,1,1}, \cdots, y_{p,1,J_1}, y_{p,2,1}, \cdots, y_{p,2,J_2}, \cdots, y_{p,I,1}, \cdots, y_{p,I,J_I}]^T$ in the pth group. It usually occurs that some elements of $\mathbf{T}_{\rm G}$ may take same value, which means ADCs sampling at same instant. This leads to the sampling is redundant and a sufficient number of effective samples may be not obtained. In order to make less elements have same value, the lowest common sampling instant of any two ADCs should be large and any two ADCs' sampling rate should not satisfy the multiple relationship.

Sort $\mathbf{T}_G$ in an ascending order, then $\mathbf{T}_{\rm G}^{\Lambda} = [\mathbf{T}_{\rm G}(\Lambda_1), \cdots, \mathbf{T}_{\rm G}(\Lambda_{M_{\rm G}})]$ is gotten, where Λ is the permutation set and $\mathbf{T}_{\rm G}(\Lambda_k)$ is the kth element of $\mathbf{T}_{\rm G}^{\Lambda}$. Apply the permutation function Λ to $\mathbf{y}_p$ and obtain $\mathbf{y}_p^{\Lambda} = [\mathbf{y}_p(\Lambda_1), \cdots, \mathbf{y}_p(\Lambda_{M_{\rm G}})]^T$, where $\mathbf{y}_p(\Lambda_k)$ is the kth element of $\mathbf{y}_p^{\Lambda}$.

Construct a new column vector $\widehat{\mathbf{y}}_p$, where the kth element is obtained by

$$\widehat{\mathbf{y}}_p(k) = \begin{cases} \mathbf{y}_p^{\Lambda}(k), & k = 1 \\ \mathbf{y}_p^{\Lambda}(k) - \mathbf{y}_p^{\Lambda}(k-1), & 1 < k \le M_{\rm G} \end{cases} \tag{4}$$

For brief, define $\mathbf{y}_p^{\Lambda}(0) = 0$. Then (4) can be simplified as $\widehat{\mathbf{y}}_p(k) = \mathbf{y}_p^{\Lambda}(k) - \mathbf{y}_p^{\Lambda}(k-1)$, $1 \le k \le M_{\rm G}$. According to (3) and (4), $\widehat{\mathbf{y}}_p(k)$ can be further expressed as

$$\begin{aligned} \widehat{\mathbf{y}}_p(k) &= \mathbf{y}_p^{\Lambda}(k) - \mathbf{y}_p^{\Lambda}(k-1) \\ &= \mathbf{y}_p(\Lambda_k) - \mathbf{y}_p(\Lambda_{k-1}) \\ &= \int_0^{\mathbf{T}_{\rm G}(\Lambda_k)} c_p(\tau)x(\tau)d\tau - \int_0^{\mathbf{T}_{\rm G}(\Lambda_{k-1})} c_p(\tau)x(\tau)d\tau \\ &= \int_{\mathbf{T}_{\rm G}(\Lambda_{k-1})}^{\mathbf{T}_{\rm G}(\Lambda_k)} c_p(\tau)x(\tau)d\tau \\ &= \sum_{n=1}^{N} \theta_n \int_{\mathbf{T}_{\rm G}(\Lambda_{k-1})}^{\mathbf{T}_{\rm G}(\Lambda_k)} c_p(\tau)\psi_n(\tau)d\tau, \end{aligned} \tag{5}$$

Then the element of matrix $\mathbf{V}_p$ in the kth row and nth column can be expressed as

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$$(\mathbf{V}_p)_{k,n} = \int_{T_{\mathrm{G}}(\Lambda_{k-1})}^{T_{\mathrm{G}}(\Lambda_k)} c_p(\tau)\psi_n(\tau)d\tau. \tag{6}$$

Combine all subsamples of P groups and construct a new column vector $\widehat{\mathbf{Y}}$ as $\widehat{\mathbf{Y}} = [\widehat{\mathbf{y}}_1^T, \widehat{\mathbf{y}}_2^T, \cdots, \widehat{\mathbf{y}}_P^T]^T$, which includes $M = PM_{\mathrm{G}}$ elements. Then the $M \times N$ matrix $\mathbf{V}$ is expressed as $\mathbf{V} = [\mathbf{V}_1^T, \mathbf{V}_2^T, \cdots, \mathbf{V}_P^T]^T$.

It is proposed that the more subsamples are obtained, the better performance will be gotten [5]. The PS-AIC system increased the number of subsamples by segmenting the original signal. The PMCS system further improves the performance by collecting more subsamples.

The number of subsamples obtained by the PMCS system is about

$$\begin{aligned} M &= PM_{\mathrm{G}} \\ &= P[int(T/T_1) + int(T/T_2) + \cdots + int(T/T_I)] \\ &\approx PT(f_1 + f_2 + \cdots + f_I) \end{aligned} \tag{7}$$

where f_i is the sampling frequency of the ith ADC in each group and $f_i \neq nf_j$ ($i \neq j$, $n = 1, 2, \cdots$). It is shown from (7) that three factors affect the system performance: the number of groups, the number of ADCs and the sampling rate of each ADC.

4 Simulations

In the simulations, we compare the performances of PS-AIC and PMCS under different conditions where the input signals is time-frequency signal which is modulated using different frequencies at different times, as in the case of frequency hopping radios. Detection percentage is exploited to reflect the system performance, which is defined as

$$P_{\mathrm{DP}} = \left(1 - \frac{\|\widetilde{\boldsymbol{\Theta}} - \boldsymbol{\Theta}\|}{\|\boldsymbol{\Theta}\|}\right) \cdot 100\%. \tag{8}$$

It is supposed that the time-frequency signal is composed by ten frequencies which are selected randomly from $(0, fs/2]$, where $fs = 200\,\mathrm{MHz}$.

Fig. 2 describes the performances of PMCS and PS-AIC under different signal-to-noise ratios (SNRs), where the sparsity of time-frequency signal is $K = 10$, and only $I = 1$ ADC is used by each group in the PMCS system. The sampling rate is 100 MHz, which is 50% of the Nyquist sampling rate f_s. These two systems have the same number of BMIs, and the number of BMIs is $P = 10$ and $P = 15$ respectively in two simulations. It shows that the performance of PMCS system is equal to that of PS-AIC system when only one ADC is used by each group.

It is assumed that the PMCS system has $P = 10$ groups and $I = 2$ ADCs which have different sampling rates are used in each group of PMCS. Fig. 3 gives the P_{DP} under three different conditions. In example 1, the sampling rates in each group are $f_1 = 60\,\mathrm{MHz}$ and $f_2 = 70\,\mathrm{MHz}$. Correspondingly, the sampling rates in example 2 and example 3 are $\{50, 60\}$ MHz and $\{40, 60\}$ MHz. It shows that the system performance in example 1 is best and in example 3 is worst. So the higher sampling rate of the total, the better system

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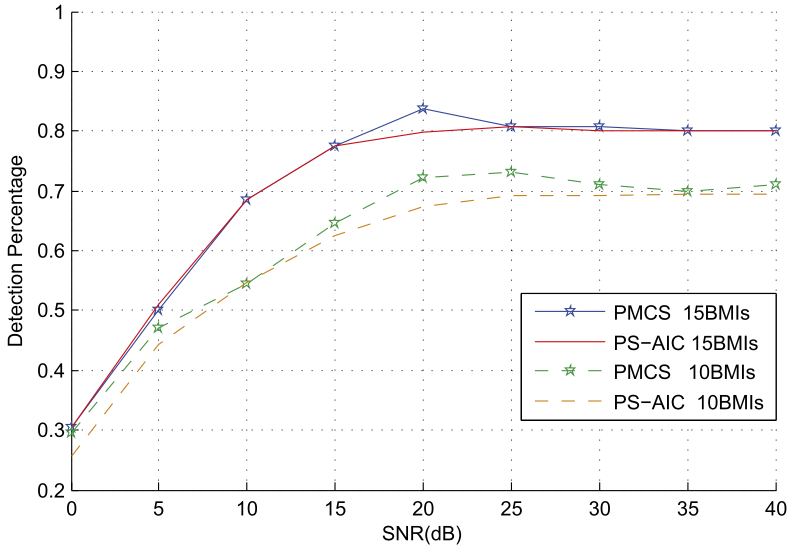

Fig. 2. The performances of PMCS and PS-AIC when only one ADC is used by each group of PMCS system

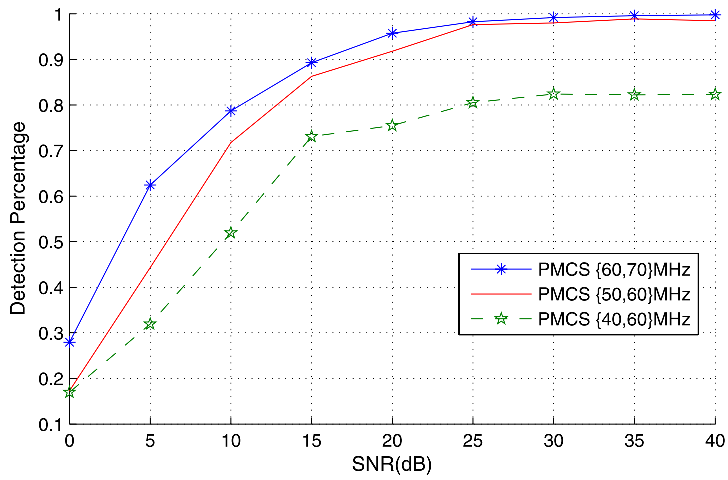

Fig. 3. Performance of PMCS system where two ADCs are used by each group

performance may be obtained. The performance of PMCS system is improved by using more parallel ADCs to increase the total sapling rate.

From Fig. 2 and Fig. 3, it is shown that when PS-AIC and PMCS both have the same number of BMIs, such as 10 BMIs are involved, and two ADCs are contained in each group of PMCS, although each ADC's sampling rate in example 2 and example 3 in Fig. 3 is much lower than that of PS-AIC in Fig. 2, the better performance is obtained because the total sampling rate is higher which makes more subsamples collected. Then the PMCS system reduces each ADCs sampling rate by increasing the number of ADC when AIC has the same number of BMIs.

Fig. 4 show the performance of PMCS system where two ADCs are used by each group and the sum of two ADCs' sampling rate is fixed. It is assumed this system has $P = 10$ groups and the sum of sampling rate is 100 MHz. In example 1, the number of common sampling instant of two ADCs is less than

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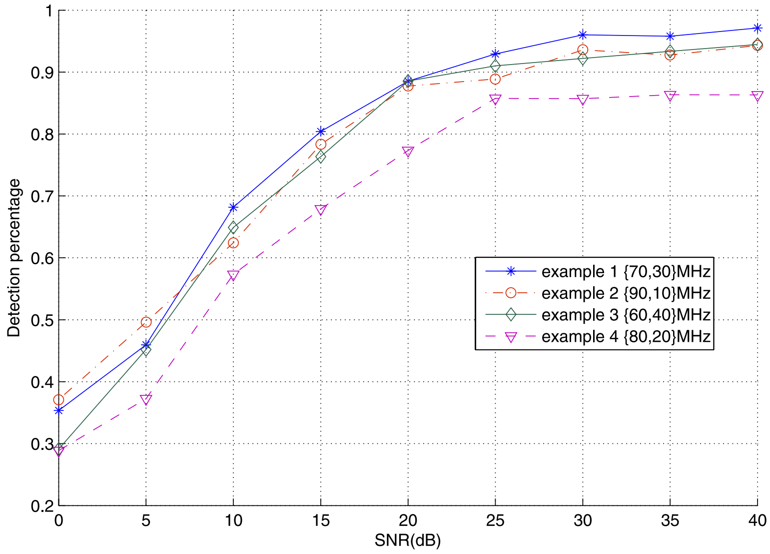

Fig. 4. Performance of PMCS system where two ADCs are used by each group and the sum of two ADCs' sampling rate is fixed

that of three other examples, so more effective samples are obtained and the performance is best. In example 2 and example 4, one ADC's sampling period is multiple of the other ADC's sampling period, which makes the samples by one ADC are fully redundant. However in example 2, the higher ADC's sampling rate is higher than that of example 4, so more effective samples are obtained and the performance is better than that of example 4. For example 2 and 3, the lowest common sampling instant in example 3 is larger than that of example 2. However during signal period, these two examples may have approximate number of common sampling instant, so they have similar performances. So in order to acquire enough effective samples to reconstruct the original signal, the ADCs should be carefully selected.

4.1 Conclusion

In this paper, we propose a parallel multi-rate compressed sampling system to reduce the sampling rate of each ADC and ensure a high recovery probability. The original signal is integrated over the signal period, and several ADCs which have different sampling rates are parallel used by each BMI branch. There will be a tradeoff between the number of BMIs and the total sampling rate of ADCs. Compared to PS-AIC, this system can further reduce the sampling rate and improve the performance.

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