

# SPICE model of memristive device using Tukey window function

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**Abstract:** This paper proposes a memristor SPICE model using the Tukey window (or tapered cosine window) function. Compared with the previously proposed models based on the boundary function, the proposed model is resistant to numerical errors. From the SPICE result of a relaxation oscillator, we observe that the proposed model can be effectively used to minimise the effect of round-off errors.

**Keywords:** memristor, modeling, Tukey window, SPICE

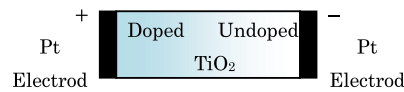
**Classification:** Electron devices, circuits, and systems

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## 1 Introduction

In 1971, Chua pointed out the existence of the fourth passive component called memristor [1]. This element is defined by a non-linear relationship between the charge and the flux, and it can be used as a non-linear resistance memory. After 37 years, the first experimental realisation of the memristor was demonstrated in a solid-state thin-film two-terminal device (as shown in Fig. 1) by HP Labs [2]. The memory effect has been achieved by moving the doping front along the device. This discovery has attracted significant research attention and many potential applications have been reported, e.g. memories for low-cost technology [3, 4], programmable logic [5], and reconfigurable logic [6].



**Fig. 1.** HP Lab's memristor.

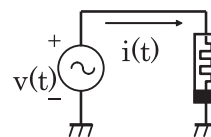
To design/simulate many applications, several SPICE models of the memristor have been presented, for example in [7, 8], and [9]. In these models, the boundary conditions of the memristor are solved using the window function. However, Biolek et al. [10] reported that, because the sensitivity of such boundary function models depends on the truncation error, the analysis may produce a false result.

In this paper, we propose a new memristor SPICE model with a window function for achieving numerical stability. The proposed window function is applied with a tapered cosine window (Tukey window) function. Compared with the results obtained by the conventional model, the results achieved using the proposed model are stable and unaffected by round-off errors.

## 2 Memristor

### 2.1 Theory

In [1], a charge-controlled memristor, shown in Fig. 2, is defined as a two-terminal element in which the flux  $\phi$  between the terminals is a function of the amount of electric charge  $q$ . The voltage  $v(t)$  across the memristor is given by  $v(t) = M(q(t))i(t)$ , where  $M(q(t)) = d\phi(t)/dq(t)$  is the memristance. When the memristance becomes a constant, the element functions as a resistance owing to the linear relationship. However,  $\phi$ – $q$  relation is non-linear and the element is referred to as a memristance, which can be charge-controlled.



**Fig. 2.** Two terminal memristor circuit for I-V characteristics measurement.

## 2.2 SPICE model of HP's memristor

A memristor consisting of a thin layer of  $\text{TiO}_2$  and a second oxygen-deficient layer of  $\text{TiO}_{2-x}$  sandwiched between two Pt nanowires was fabricated [2]. The voltage–current relationship of this memristor is modeled as

$$M(q(t)) = \left[ R_{\text{ON}} \frac{w(t)}{L} + R_{\text{OFF}} \left( 1 - \frac{w(t)}{L} \right) \right], \quad (1)$$

where  $R_{\text{ON}}$  is the resistance of the completely doped memristor,  $R_{\text{OFF}}$  is the resistance of the undoped region, and  $L$  is thickness of the  $\text{TiO}_2$  film. The width of the doped region  $w(t)$  is given by

$$\frac{dx(t)}{dt} = \mu \frac{R_{\text{ON}}}{L^2} i(t), \quad (2)$$

where  $\mu$  represents the average dopant mobility. Although Eqs. (1) and (2) can yield a linear equation between memristance  $M$  and charge  $q$ , this model does not consider the boundary non-linear dopant drift. To overcome this drawback, a window function  $w(D - w) = D^2$  is multiplied to the right side of (2). Let the state variable  $x = w(t)/D$ . The HP physical model with the window function can now be described as

$$v(t) = (R_{\text{ON}}x + R_{\text{OFF}}(1 - x))i(t), \quad (3)$$

$$\frac{dx(t)}{dt} = \alpha i(t)f(x), \quad (4)$$

where the window function  $f(x) = x(1 - x)$ , and the constant coefficient  $\alpha = \mu\nu R_{\text{ON}}/D^2$ . In general, a class of memristor models can be obtained using different window functions.

## 3 Proposed memristor model with Tukey window function

The proposed memristor model using the Tukey window function is expressed as follows:

$$w(x) = \begin{cases} \frac{1}{2} \left\{ 1 + \cos \left( \frac{2\pi}{r} \left[ x - \frac{r}{2} \right] \right) \right\} & \left( 0 < x \leq \frac{r}{2} \right) \\ 1 & \left( \frac{r}{2} < x \leq 1 - \frac{r}{2} \right) \\ \frac{1}{2} \left\{ 1 + \cos \left( \frac{2\pi}{r} \left[ x - 1 + \frac{r}{2} \right] \right) \right\} & \left( 1 - \frac{r}{2} < x \leq 1 \right) \end{cases} \quad (5)$$

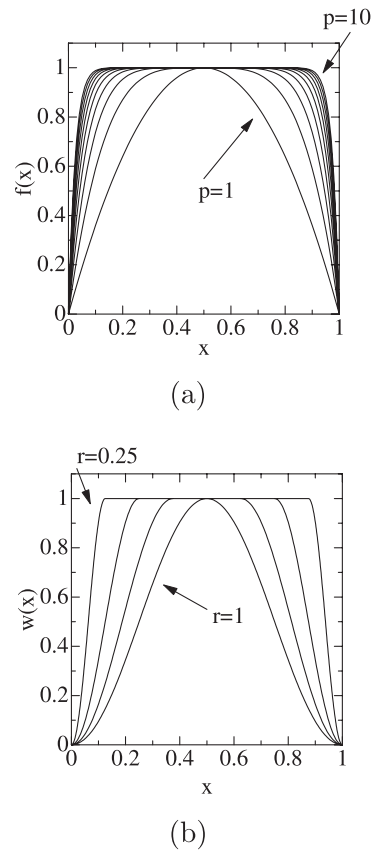
Using the proposed windows function, the memristor model is described as

$$v(t) = (R_{\text{ON}}x + R_{\text{OFF}}(1 - x))i(t), \quad (6)$$

$$\frac{dx(t)}{dt} = \alpha i(t)w(x). \quad (7)$$

Fig. 3(a) and (b) show the conventional window model [8] and the proposed model respectively. Unlike the conventional model, the advantage of the proposed model is that the expression of memristance  $M(q)$  can be easily obtained and the numerical value of  $M(q)$  can be readily calculated from this expression. Because the conventional window function as shown in Fig. 3(a) is  $f(x) = 1 - (2x - 1)^{2p}$  (where  $p$  is a positive integer number), the numerical analysis depends on the value

of  $p$ . This means that the convergence of numerical solutions degrades when  $p$  has a large value.



**Fig. 3.** Conventional window function: (a), and proposed window function: (b).

Table I summarizes a comparison between different window functions shown in [12]. Compared with the other window function models, window function of the proposed model is a simple numerical equation. And the proposed model has also simple bound constraints. This characteristic is useful for simulations, where the bounds can be exceeded due to the discrete nature of simulation engines.

A complete SPICE model based on the proposed window function is listed in Table II. In the next section, we will show the difference in the responses related to the window functions obtained through SPICE simulations.

## 4 Simulation results

### 4.1 Voltage–current plane of memristor model

To confirm the pinched hysteresis loop of the memristor, the voltage–current ( $v-i$ ) plane is observed using SPICE. The memristor is driven by a sinusoidal voltage source:  $1.5 \sin(\omega t)$ . The parameter values of the memristor are listed in Table II.

Fig. 4(a) illustrates the simulation result of the memristor model based on the window function [8]. Simulation results show a significant difference between the state variable curves of the two excitation cycles. This is caused by the numerical error that occurs during simulation.

**Table I.** Comparison of different memristor models using window function.

	Joglekar [8]	Biolek [9]
<b>Window function</b>	$f(w) = 1 - (2w/D - 1)^{2p}$	$f(w) = 1 - (w/D - \text{stp}(-j))^{2p}$
<b>Symmetric</b>	Yes	Yes
<b>Resolve boundary conditions</b>	No	Yes
<b>Impose non-linear drift</b>	Partially	Partially
<b>Scale factor (<math>f_{max} &lt; 1</math>)</b>	No	No
<b>Fits memristive device model</b>	L/N/TEAM	L/N/TEAM
Prodromakis [11]	Kvatinsky [12]	Proposed
$f(w) = j[1 - \{(w - 0.5)^2 + 0.75\}^p]$	$f_{\text{on, off}} = e\{-e( x - x_{\text{on, off}}  - w_c)\}$	Eq. (5)
Yes	Not necessarily	Yes
Practically Yes	Practically Yes	Practically Yes
Partially	Yes	Partially
Yes	No	No
L/N/TEAM	TEAM	L/N/TEAM

Note: L = Linear ion drift, N = Non-linear ion drift, TEAM = Threshold adaptive memristor

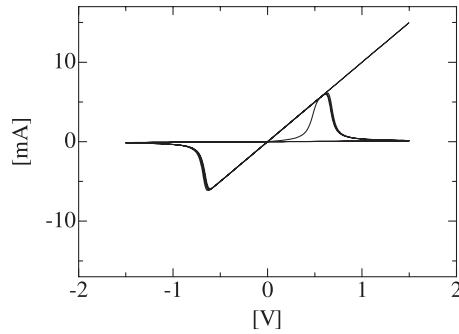
**Table II.** SPICE netlist of memristor model using Tukey window function.

```
*****
* HP Memristor SPICE Model
* Original created by Zdenek and D. Biolek
* Improved by HIRABAYASHI Motonari
*****
* Ron, Roff - Resistance in ON / OFF States
* Rinit - Resistance at T=0
* D - Width of the thin film
* uv - Migration coefficient
* r - Parameter of the window function
* x - W/D Ratio, W is the actual width
* of the doped area (from 0 to D)
*****
.SUBCKT memristor Plus Minus
.PARAMS Ron=100 Roff=16K Rinit=1K D=10N
+ uv=24F r=0.8

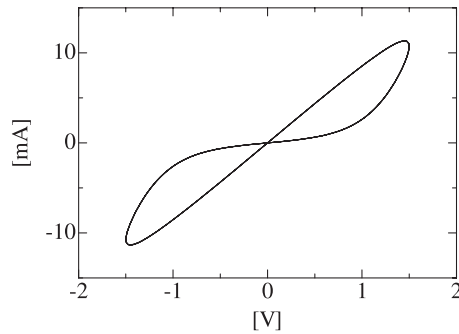
*****
* DIFFERENTIAL EQUATION MODELING
*****
Gx 0 x value={I(Emem)*uv*Ron/D**2*f(V(x),r)}
Cx x 0 1 IC={Roff-Rinit)/(Roff-Ron)}
Raux x 0 1T

*****
* RESISTIVE PORT OF THE MEMRISTOR
*****
Emem plus aux value={-I(Emem)*V(x)*(Roff-Ron)}
Roff aux minus {Roff}

*****
* PROPOSED WINDOW FUNCTION
*****
.func f(x,r)={if(x>0 &
x<=r/2,1,0)*1/2*(1+cos(2*pi/r*(x-r/2)))+
if(x>r/2 & x<=1-r/2,1,0)+if(x>1-r/2 &
x<=1,1,0)*1/2*(1+cos(2*pi/r*(x-1+r/2)))}
.ENDS
```



(a) Conventional window function model.



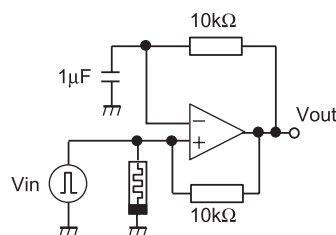
(b) Proposed window function model.

**Fig. 4.** Simulation results of the memristor model when driving source  $v(t) = 1.5 \sin(\omega t)$ .

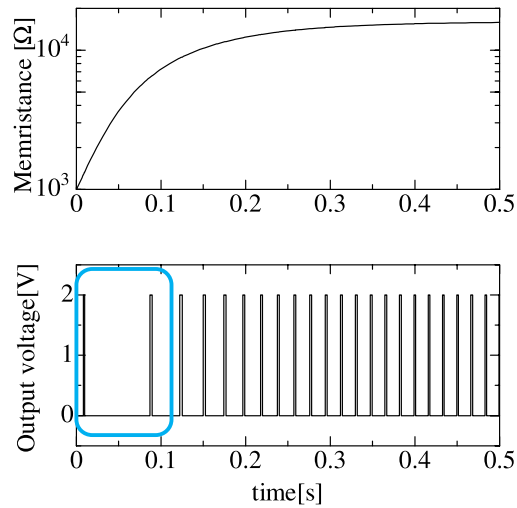
On the other hand, Fig. 4(b) shows the  $v$ - $i$  curve results of the proposed model. From the simulation results, we summarise that the expression of memristance  $M(q)$  can be easily obtained and the numerical value of  $M(q)$  can be readily calculated using the proposed model.

#### 4.2 Application: relaxation oscillator

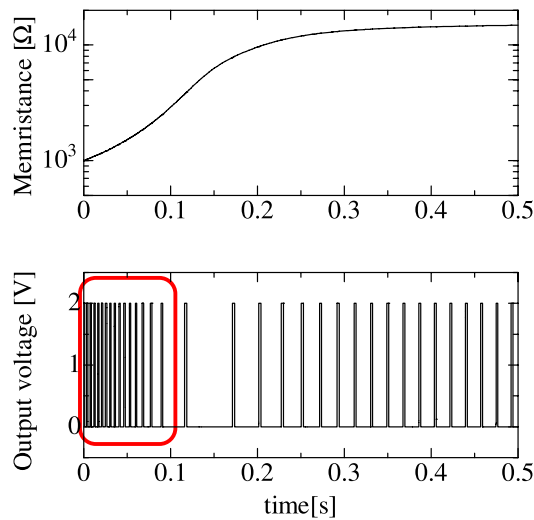
Fig. 5 shows a target application circuit: a relaxation oscillator using a memristor. In this circuit, the oscillation frequency depends on the variation in the memristance value. Fig. 6 shows the simulation results obtained using conventional and proposed memristor models. As shown by the red rectangle marked in Fig. 6(b), we can minimise the effect of round-off errors using the proposed model. On the other hand, in the case of the conventional model, round-off errors are not minimised; hence, the output voltage, as shown by the blue rectangle marked in Fig. 6(a), is always zero despite the continuing rise in memristance.



**Fig. 5.** Relaxation oscillator using memristor.



(a) Conventional window function model.



(b) Proposed window function model.

**Fig. 6.** Simulation results of the relaxation oscillator.

## 5 Conclusion

In this paper, the window function for a memristor model was presented. The proposed model used the tapered cosine window (Tukey window) function. Using SPICE, the proposed model is shown to be numerically more stable compared to conventional model.

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