LETTER

IEICE Electronics Express, Vol.12, No.19, 1–6

# Equivalent circuit of external electromagnetic fields coupling to a transmission line above a lossy ground

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**Abstract:** Based on the vector fitting and the controlled sources in HSPICE, the delay extraction-based passive compact transmission-line (DEPACT) macromodeling for a transmission line above a lossy ground is presented. Combined with the equivalent sources of the external electromagnetic fields, the equivalent circuits about the scatted voltages and total voltages are proposed. The simulation results are compared with the inverse Fourier transform method and good agreement is obtained. At the same time, using this approach, the transient response for a transmission line above a lossy ground with nonlinear terminations excited by the external electromagnetic fields could be obtained easily and accurately.

**Keywords:** external electromagnetic fields, lossy ground, transmission line, equivalent circuit, nonlinear terminations, transient response

**Classification:** Electromagnetic theory

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DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015

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## 1 Introduction

The transient analysis of external electromagnetic fields coupling to a transmission line above a lossy ground can be performed by means of the inverse Fourier transform of the results obtained in the frequency domain and the finite difference time domain (FDTD) method [1, 2, 3, 4]. However, it becomes complex when the termination is nonlinear [5]. Macromodeling techniques attempt at solving this problem by building the equivalent circuit of the transmission line, which can then be directly linked to nonlinear SPICE-like circuit simulators [6, 7, 8, 9]. The method in [10] can handle a transmission line above a lossless ground excited by the external electromagnetic fields. The delay extraction-based passive compact transmission-line (DEPACT) macromodeling algorithm in [11] can solve the lossy transmission lines with incident fields. However, the equivalent circuit of external electromagnetic fields coupling to a transmission line above a lossy ground is not found.

The delay extraction-based passive compact transmission-line (DEPACT) macromodeling is that the equivalent circuit of the transmission line is viewed as a cascade of some transmission line subnetworks, and each subnetwork is represented by a cascade of one lossy and two lossless sections [12]. In this paper, the ground impedance is approximated as the rational function by the vector fitting (VF) method [13], and the equivalent circuit of the lossy section is built by the "Laplace Element" in HSPICE. The equivalent sources of the external electromagnetic fields are built, and the equivalent circuits about the scatted voltages and total voltages are presented.

## 2 Proposed equivalent circuit

Consider a transmission line above a lossy ground excited by the external electromagnetic fields, represented by Telegrapher's equations in the frequency-domain as

$$\frac{d}{dx}\begin{bmatrix} V^{sca}(x,s) \\ I(x,s) \end{bmatrix} = \boldsymbol{Q}(s)\begin{bmatrix} V^{sca}(x,s) \\ I(x,s) \end{bmatrix} + \tilde{\boldsymbol{F}}^{sca}(x,s) \tag{1}$$

where $\boldsymbol{Q}(s) = (\boldsymbol{A}(s) + s\boldsymbol{B})$,

$$\boldsymbol{B} = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix}, \quad \boldsymbol{A}(s) = \begin{bmatrix} 0 & -Z(s) \\ 0 & 0 \end{bmatrix}, \quad \tilde{\boldsymbol{F}}^{sca}(x,s) = \begin{bmatrix} E_x^{inc}(x,s) + E_x^{ref}(x,s) \\ 0 \end{bmatrix}.$$

$V^{sca}(x,s)$ is the scattered voltage, $I(x,s)$ represents the current, $L$ is the inductance, $L \approx (\mu_0/(2\pi))\ln(2h/r)$. $C$ is the conductance, $C = \varepsilon_0\mu_0/L$. $h$ is the height of the

© IEICE 2015
DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015

line above the ground. $r$ is the radius of the line. $Z(s)$ is the ground impedance, $Z(s) \approx (s\mu/(2\pi)) \ln((1+\gamma_g h)/(\gamma_g h))$ and $r_g = \sqrt{s\mu_0(\sigma_g + s\varepsilon_r\varepsilon_0)}$, in which $\sigma_g$ and $\varepsilon_r$ are respectively the ground conductivity and relative permittivity. $E_x^{inc}(x,s)$ is the horizontal component along the wire of the incident electric field. $E_x^{ref}(x,s)$ is the horizontal component along the wire of the reflecting electric field. The solution of Eq. (1) can be expressed in the frequency-domain as

$$\begin{bmatrix} V^{sca}(l,s) \\ I(l,s) \end{bmatrix} = e^{\boldsymbol{Q}(s)l} \begin{bmatrix} V^{sca}(0,s) \\ I(0,s) \end{bmatrix} + \boldsymbol{J}^{sca}(s) \tag{2}$$

$$\boldsymbol{J}^{sca}(s) = \begin{bmatrix} V^f(s) \\ I^f(s) \end{bmatrix} = \int_0^l e^{\boldsymbol{Q}(s)(l-x)} \tilde{\boldsymbol{F}}^{sca}(x,s)\mathrm{d}x \tag{3}$$

The equivalent circuit about Eq. (2) can be built by two steps. First the delay extraction-based passive compact transmission-line (DEPACT) macromodeling is built when $\boldsymbol{J}^{sca}(s)$ is ignored. Second, the equivalent sources about $\boldsymbol{J}^{sca}(s)$ are built. Then combine the equivalent sources and the delay extraction-based passive compact transmission-line (DEPACT) macromodeling.

The delay extraction-based passive compact transmission-line (DEPACT) macromodeling is a cascade of some transmission line subnetworks, and each subnetwork is represented by a cascade of one lossy and two lossless sections [12]. Supposed that the number of the subnetworks is $m$, The length of the lossless section is $d/2m$, and the equivalent circuit of lossless section as [9]. The length of lossy section is $d/m$, and the exponential stamp can be written as

$$e^{\frac{A(s)}{m}l} = \begin{bmatrix} 1 & -Z(s)l/m \\ 0 & 1 \end{bmatrix} \tag{4}$$

The terminal voltages and currents of the lossy section are related by

$$\begin{bmatrix} V(l_{k+1},s) \\ -I(l_{k+1},s) \end{bmatrix} = \begin{bmatrix} 1 & -Z(s)l/m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V(l_k,s) \\ I(l_k,s) \end{bmatrix} \tag{5}$$

Expanding Eq. (5) yields the following:

$$I(l_k,s) = (1/(Z(s)l/m))(V(l_k,s) - V(l_{k+1},s)) \tag{6}$$

The Eq. (6) can be expressed in the time domain as:

$$I(l_k,t) = (1/(Z(t)l/m)) * (V(l_k,t) - V(l_{k+1},t)) \tag{7}$$

The equivalent circuit of the lossy section can be represented using the Laplace element in HSPICE. $1/(Z(s)l/m)$ can be written in a rational function form by the vector fitting, and is realized using voltage controlled current source.

When the external electromagnetic field is the uniform plane wave, the geometry of external electromagnetic field and the overhead line is as Fig. 1. $\alpha$ is electric field polarization angle, $\phi$ is azimuthal angle, and $\psi$ is elevation angle. $Z_1$ and $Z_2$ are the terminations. $E_v$ and $E_h$ are respectively the vertical and horizontal electric field, and $E_v = E_0\cos\alpha$, $E_h = E_0\sin\alpha$. The $\boldsymbol{J}^{sca}(s)$ can be expressed as

$$\boldsymbol{J}^{sca}(s) = E_0(s)\boldsymbol{f}(s) \tag{8}$$

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DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015

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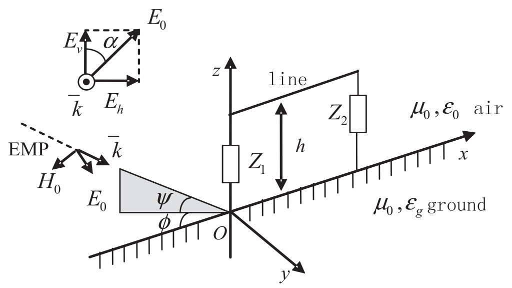


**Fig. 1.** The geometry of external electromagnetic field and the overhead line

Where

$$\begin{aligned} \boldsymbol{f}(s) &= \begin{bmatrix} f^V(s) \\ f^I(s) \end{bmatrix} \\ &= [\cos\alpha \sin\psi \cos\phi (e^{s\frac{\sin\psi h}{c}} - R_{ver} e^{-s\frac{\sin\psi h}{c}}) \\ &\quad + \sin\alpha \sin\phi (e^{s\frac{\sin\psi h}{c}} + R_{lev} e^{-s\frac{\sin\psi h}{c}})] \int_0^l e^{\boldsymbol{Q}(s)(l-x)} e^{-s\frac{\cos\psi \cos\phi x}{c}} \mathrm{d}x \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned} \tag{9}$$

The Eq. (9) can be expressed in the time domain as:

$$\boldsymbol{J}^{sca}(t) = \begin{bmatrix} V^f(t) \\ I^f(t) \end{bmatrix} = \begin{bmatrix} E_0(t) * f^V(t) \\ E_0(t) * f^I(t) \end{bmatrix} \tag{10}$$

$f^V(s)$ and $f^I(s)$ can be written in a rational function form by the vector fitting. $V^f(t)$ and $I^f(t)$ are realized respectively using voltage controlled voltage source and voltage controlled current source.

The results can be expressed back in terms of total voltages, as

$$\begin{bmatrix} V(l,s) \\ I(l,s) \end{bmatrix} = e^{\boldsymbol{Q}(s)l} \left\{ \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} - \begin{bmatrix} V_z^{ex}(0,s) \\ 0 \end{bmatrix} \right\} + \boldsymbol{J}^{sca}(s) + \begin{bmatrix} V_z^{ex}(l,s) \\ 0 \end{bmatrix} \tag{11}$$

where

$$\begin{aligned} V_z^{ex}(x,s) &= -\int_{-\infty}^{h} E_z^{ex}(x,z)dz \\ &= E_0(s) \Bigg\{ -\cos\alpha T_{ver} \cos\psi_t e^{-\gamma_g \cos\psi_t \cos\phi x} \frac{1}{\gamma_g \sin\psi_t} \\ &\quad - \cos\alpha \cos\psi e^{-s\frac{\cos\psi\cos\phi x}{c}} \left[ \frac{c}{s\sin\psi} (e^{s\frac{\sin\psi h}{c}} - 1) - R_{ver} \frac{c}{s\sin\psi} (e^{-s\frac{\sin\psi h}{c}} - 1) \right] \Bigg\} \end{aligned} \tag{12}$$

and $\gamma_g \sin\psi_t$, $\sin\psi \neq 0$.

The solution can be obtained as

$$\begin{aligned} V_z^{ex}(0,s) &= E_0(s) f^0(s) \\ &= E_0(s) \Bigg\{ -\cos\alpha T_{ver} \cos\psi_t \frac{1}{\gamma_g \sin\psi_t} \\ &\quad - \cos\alpha \cos\psi \left[ \frac{c}{s\sin\psi} (e^{s\frac{\sin\psi h}{c}} - 1) - R_{ver} \frac{c}{s\sin\psi} (e^{-s\frac{\sin\psi h}{c}} - 1) \right] \Bigg\} \end{aligned} \tag{13}$$

© IEICE 2015
DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015

IEICE Electronics Express, Vol.12, No.19, 1–6

$$
\begin{aligned}
V_z^{ex}(l,s) &= E_0(s) f^l(s) \\
&= E_0(s)\Bigg\{-\cos\alpha T_{ver}\cos\psi_t e^{-\gamma_g \cos\psi_t \cos\phi l}\frac{1}{\gamma_g \sin\psi_t} \\
&\quad -\cos\alpha\cos\psi e^{-s\frac{\cos\psi\cos\phi l}{c}}\left[\frac{c}{s\sin\psi}\left(e^{s\frac{\sin\psi h}{c}}-1\right)-R_{ver}\frac{c}{s\sin\psi}\left(e^{-s\frac{\sin\psi h}{c}}-1\right)\right]\Bigg\}
\end{aligned} \tag{14}
$$

The Eq. (13) and Eq. (14) can be expressed in the time domain as:

$$V_z^{ex}(0,t) = E_0(t) * f^0(t) \tag{15}$$

$$V_z^{ex}(l,t) = E_0(t) * f^l(t) \tag{16}$$

$f^0(s)$ and $f^l(s)$ can be written in a rational function form by the vector fitting. $V_z^{ex}(0,t)$ and $V_z^{ex}(l,t)$ are realized using voltage controlled voltage source.

The equivalent circuit about the total voltages is as Fig. 2. In the following, we will verify the equivalent circuit.

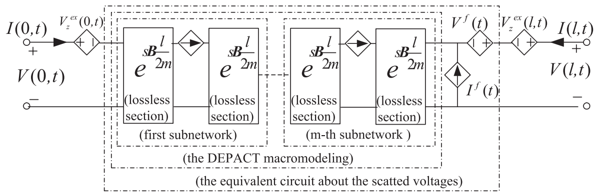


**Fig. 2.** The equivalent circuit about the total voltages.

## 3 Validation of the method

The parameters of a transmission line above a lossy ground are $l = 30\,\text{m}$, $h = 1\,\text{m}$ and $r = 1.52\,\text{mm}$. The parameters of the ground are $\sigma_g = 10^{-3}\,\text{S/m}$, $\varepsilon_r = 10$ and $\mu_r = 1$. $Z_1 = Z_2 = 100\,\Omega$. The waveform of the electric field is described by a double exponential function $E_0(t) = 1000(e^{-10000t} - e^{-400000t})\,\text{V/m}$ with $\alpha = 0$, $\varphi = 0$ and $\psi = \pi/6$. Fig. 3 shows the transient response of the voltage induced at the line end by the proposed method and the inverse Fourier transform method.

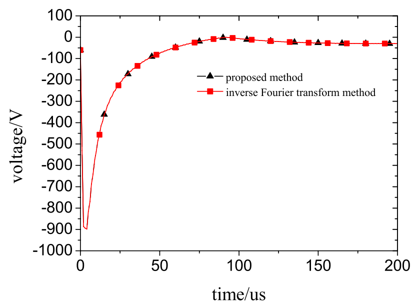


**Fig. 3.** The transient response when $Z_1$ and $Z_2$ are $100\,\Omega$.

From Fig. 3, we can see that the transient responses obtained by the proposed method are in excellent agreement with the results of the inverse Fourier transform method.

© IEICE 2015
DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015

IEICE Electronics Express, Vol.12, No.19, 1–6

When three different types of transient voltage suppressor are respectively connected in parallel with one of the load resistance, the transient response of the voltage induced at the line end by the proposed method is shown in Fig. 4. The types of transient voltage suppressor are 1.5KE440A, 1.5KE180A and 1.5KE39CA.

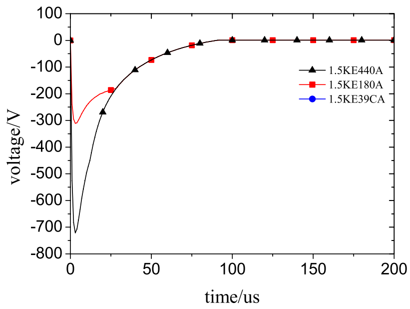


**Fig. 4.** The transient response when transient voltage suppressor is connected.

From Fig. 4 we can see that because the maximum clamping voltage of the transient voltage suppressor is different, the amplitude of the responses are different. They are all less than the maximum clamping voltage of the transient voltage suppressors and which agree well with the theoretical analysis results also.

## 4 Conclusion

The equivalent circuit of external electromagnetic fields coupling to a transmission line above a lossy ground is presented. When the loads are resistances, the transient response of the voltage induced at the line end by the equivalent circuit is compared to those obtained by the inverse Fourier transform method, and a good agreement is observed. The transient responses when three different types of transient voltage suppressor are respectively connected in parallel with one of the load resistance are agreed with the theoretical analysis results also. Consequently, using this approach, the transient response for a transmission line above a lossy ground with nonlinear terminations excited by the external electromagnetic fields could be obtained easily and accurately.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China (NO. 50977091).

© IEICE 2015
DOI: 10.1587/elex.12.20150474
Received May 26, 2015
Accepted June 4, 2015
Publicized June 19, 2015
Copyedited October 10, 2015