LETTER
IEICE Electronics Express, Vol.13, No.12, 1–6

A novel graphical method for dual-frequency two sections transformer

Bochao Zhao[1], **Peijun Ma**[1a)], **Yang Lu**[1], **Jiaxin Zheng**[2], **Hengshuang Zhang**[1], **Zuochen Shi**[1], **Xiaohua Ma**[2], **and Yue Hao**[1]

[1] *Key Laboratory for Wide Band-Gap Semiconductor Materials and Devices, School of Microelectronics, Xidian University, Xi'an 710071, P. R. China*

[2] *School of Advanced Materials and Nanotechnology, Xidian University, Xi'an 710071, P. R. China*

a) *pjma@xidian.edu.cn*

Abstract: The most useful transmission-line construct is used to realize impedance matching at dual-frequency. The usual algebraic method is to solve the transmission equation which is precise but lack of intuition. A novel graphical method which is simple, intuitive and has explicit physical meaning is proposed to solve the matching problem. The parameters can be determined by simple geometrical relationship. Simulation and experimental results show that the proposed method is convenient, precise and can be used to extend bandwidth as well.

Keywords: graphical method, dual-frequency, impedance matching, transformer, transmission lines

Classification: Microwave and millimeter-wave devices, circuits, and modules

References

[1] L. Liu, *et al.*: IEEE Trans. Antennas Propag. **61** (2013) 2304 (DOI: 10.1109/TAP.2013.2241716).
[2] K.-K. M. Cheng and F.-L. Wong: IEEE Microw. Wireless Compon. Lett. **17** (2007) 664 (DOI: 10.1109/LMWC.2007.903454).
[3] K.-A. Hsieh, *et al.*: IEEE Trans. Microw. Theory Techn. **60** (2012) 1649 (DOI: 10.1109/TMTT.2012.2191303).
[4] L. Wu, *et al.*: IEEE Microw. Wireless Compon. Lett. **15** (2005) 107 (DOI: 10.1109/LMWC.2004.842848).
[5] Y. Liu, *et al.*: Microelectronics J. **46** (2015) 674 (DOI: 10.1016/j.mejo.2015.05.005).
[6] Y. Liu, *et al.*: Prog. Electromagnetics Res. **126** (2012) 121 (DOI: 10.2528/PIER11121207).
[7] Y. L. Wu, *et al.*: IEEE Microw. Wireless Compon. Lett. **19** (2009) 792 (DOI: 10.1109/LMWC.2009.2034034).
[8] Y. L. Wu, *et al.*: IEEE Microw. Wireless Compon. Lett. **19** (2009) 77 (DOI: 10.1109/LMWC.2009.2034034).
[9] C. Monzon: IEEE Trans. Microw. Theory Techn. **51** (2003) 1157 (DOI: 10.1109/LMWC.2008.2011315).
[10] M. M. Radmanesh and A. M. Madni: *Radio Frequency and Microwave*

DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016

Electronics Illustrated (Prentice Hall PTR, N.J., U.S.A., 2001) 145, 148, and 231.

1 Introduction

Dual-frequency impedance transformer is widely used in microwave devices, such as antenna [1], power divider [2] and power amplifier [3]. Research on how to match the impedance at dual-frequency simultaneously has been carried out in [4, 5, 6, 7, 8, 9]. In [4], a dual-frequency method is carried out but the application is only a frequency and its first harmonic. In [5, 6], resonators are used to achieve dual-band impedance matching with the drawback of complex circuit configuration. In [7], two arbitrary complex impedances are matched at two frequencies however four sections transmission-line is used. The number of transmission line sections can decrease to two [8, 9], however the parameters of the above transformers are solved by algebraic method which is lack of intuition.

In the above dual-frequency matching method, we aim at the transformer in [9] which has a simple two sections transmission-line in structure. A novel graphical way is proposed to solve the problem. In Section 2, the matching process is described in graphical way and the parameters are worked out by simple graphic relationship. In Section 3, three groups of numerical results are given to present the matching characteristics and prove the validity of the proposed graphical method. Finally, conclusion is given in Section 4.

2 Proposed graphical method

The simple two sections transmission-line transformer is shown in Fig. 1. The impedance is transformed from Z_L to Z_S at two frequencies (f_1, f_2) through two transmission-lines (TL_1, TL_2) whose characteristic impedance and length are Z_1, L_1 and Z_2, L_2 respectively. The problem is how to determine the above four parameters (Z_1, L_1, Z_2, L_2) by the four given variables (Z_L, Z_S, f_1, f_2). Although four transmission equations (real and imaginary parts at two frequencies) can be listed [9], the solving process is a little intricate and not intuitive.

At Smith Chart plane, the matching problem becomes simple and intuitive. A transmission line transforms the impedance along the circle, and a quarter wave-

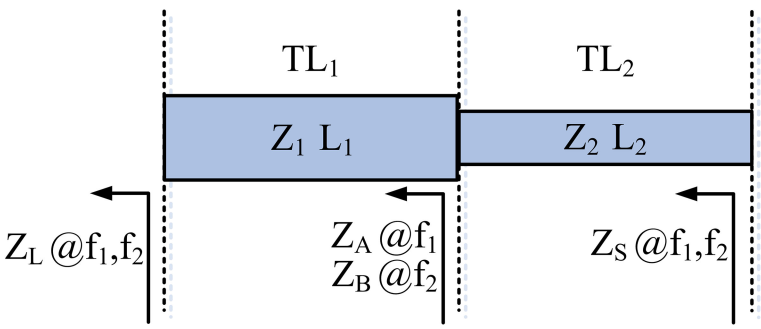

Fig. 1. Two sections dual-frequency transmission-line transformer. Z_L is transformed to Z_S by TL_1 and TL_2 at f_1 and f_2.

DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016

© IEICE 2016
DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016

length line corresponds to a half circle. As shown in Fig. 2, a certain point, for instance L, represents an impedance Z_L and a reflection coefficient Γ_L at the same time. The value can be read directly in stand Smith Chart. The relationship is [10]:

$$\begin{aligned} \Gamma_L &= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} \\ Z_L &= Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} \end{aligned} \tag{1}$$

where Z_0 is the system impedance of 50 Ohm. For matching at dual-frequency, we assume $f_1 < f_2$, and introduce a new variable the center frequency f_c:

$$f_c = (f_1 + f_2)/2 \tag{2}$$

The process of the proposed graphical method is shown in Fig. 2:

Firstly, at the center frequency f_c, a quarter transmission line TL_1 (electrical length is $\pi/2$) with the characteristic impedance of Z_1 transforms point L to point C. A circle which has the diameter of LC is painted. For a certain transmission line, electrical length is proportional to frequency. So for f_1 and f_2, the electrical length of transmission line TL_1 is:

$$\begin{aligned} \theta_1 &= \frac{f_1}{f_c}\frac{\pi}{2} < \frac{\pi}{2} \\ \theta_2 &= \frac{f_2}{f_c}\frac{\pi}{2} > \frac{\pi}{2} \end{aligned} \tag{3}$$

So at low frequency f_1, TL_1 transforms point L to point A (the red solid arc), and the arc angle ($\angle$AEL) is twice of electrical length (θ_1) [10]. At high frequency f_2, TL_1 transforms point L to point B (the blue solid arc). As relationship of (4) can be derived by (3), point A and B are symmetry thus the impedances are conjugate.

$$\frac{\pi}{2} - \theta_1 = \frac{\pi}{2} \times \frac{f_c - f_1}{f_c} = \frac{\pi}{2} \times \frac{f_2 - f_c}{f_c} = \theta_2 - \frac{\pi}{2} \tag{4}$$

Secondly, a same size circle is painted through point A, B and S. It presents the transmission line TL_2 with the same electrical length and different characteristic impedance of Z_2. As the radius of the two circles is the same, the two red arcs are equal (AE = AF, so $\angle$AEG = $\angle$AFG, therefore $\angle$AEL = $\angle$AFS) while two blue arcs are equal. So TL_2 with the same electrical length transforms point A to S (the red dot arc) at f_1 while it transforms point B to S (the blue dot arc) at f_2.

Thus this constructs realize dual-frequency matching from L to S. The transforming process is shown in Fig. 1 and Fig. 2. In a word, solid arcs are transforming process of TL_1 and dot arcs are process of TL_2. Red arcs represent the transforming process at f_1 while blue arcs represent process at f_2.

Thirdly, simple geometrical relationship is obtained:

$$LS = 2LG = 2r(1 + \cos(\angle AEG)) = 2r(1 + \cos(\pi - 2\theta_1)) \tag{5}$$

At reflection coefficient plane, point L and S are both on real axis, the length of segment LS can be present by Γ_S and Γ_L which can be read from Smith Chart, and θ_1 is shown in (3), the unique unknown parameter radius (r) can be solved:

$$r = \frac{|\Gamma_S - \Gamma_L|}{2(1 - \cos(2\theta_1))} \tag{6}$$

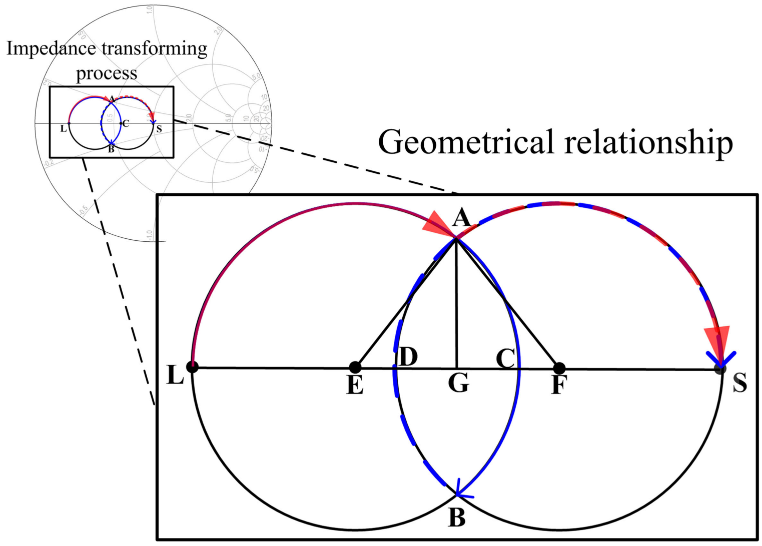

Fig. 2. Impedance transforming process and geometrical relationship. The red arc is the process at f_1 while the blue arc is the process at f_2.

So Γ_C and Γ_D can be expressed with r:

$$\begin{aligned} |\Gamma_C| &= |\Gamma_L| - 2r \\ |\Gamma_D| &= |\Gamma_S| + 2r \end{aligned} \tag{7}$$

If $Z_L < Z_S$, the points L D and C are on the left side of Smith Chart, the reflection coefficients are negative, otherwise the points are on the right side, and the reflection coefficients are positive.

At impedance plane, Z_C and Z_D can be obtained from Γ_C and Γ_D by (1). According to the quarter wavelength transformer, Z_1 and Z_2 are [10]:

$$\begin{aligned} Z_1 &= \sqrt{Z_L Z_C} \\ Z_2 &= \sqrt{Z_D Z_S} \end{aligned} \tag{8}$$

3 Numerical examples

The impedance transforming process is intuitively shown in Fig. 2, and the results of the graphical solution are shown in (3) and (6)–(8). This section explains some simulation and experimental results to valid the method.

For the first case, the two frequencies are fixed: $f_1 = 10$ GHz and $f_2 = 20$ GHz. Z_S is fixed to 50 Ohm, while Z_L varies from 10 Ohm to 30 Ohm. The parameters are shown in Table I and simulated S11 by ideal transmission line model is shown in Fig. 3. The data shows that the proposed graphical method can transform different impedances at dual-frequency.

The next example illustrates S11 as a function of f_2 for fixed $f_1 = 10$ GHz when $Z_L = 20$ Ohm and $Z_S = 50$ Ohm. For f_2 in the range of 12–20 GHz, the parameters obtained from (3) and (8) are listed in Table II and the corresponding simulated results are presented in Fig. 4. For any given two frequency, the proposed graphical method can realize matching in high precise.

Another case we consider here is the validation of frequency band extension which is always used in power amplifier design. Supposing the transistor load line

DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016

Table I. Parameters of dual-frequency in case-1

f_1	f_2	Z_L	Z_S	Z_1	Z_2	θ_1
GHz	GHz	Ohm	Ohm	Ohm	Ohm	°
10	20	10	50	17.32	28.87	60
10	20	15	50	22.53	33.29	60
10	20	20	50	27.21	36.75	60
10	20	25	50	31.53	39.64	60
10	20	30	50	35.58	42.15	60

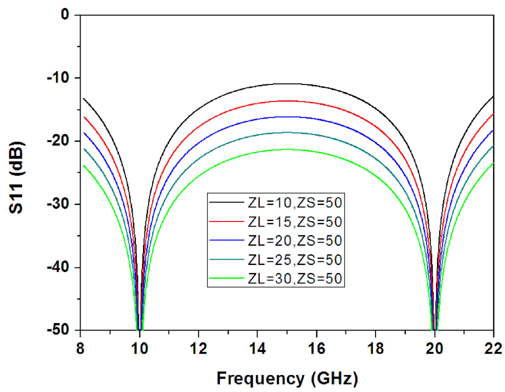

Fig. 3. Simulation results when $f_1 = 10$ GHz and $f_2 = 20$ GHz. Z_S is fixed to 50 Ohm while Z_L varies from 10–30 Ohm.

Table II. Parameters of dual-frequency in case-2

f_1	f_2	Z_L	Z_S	Z_1	Z_2	θ_1
GHz	GHz	Ohm	Ohm	Ohm	Ohm	°
10	12	20	50	25.27	39.57	82
10	14	20	50	25.58	39.09	75
10	16	20	50	26.02	38.43	69
10	18	20	50	26.57	37.64	64
10	20	20	50	27.21	36.75	60

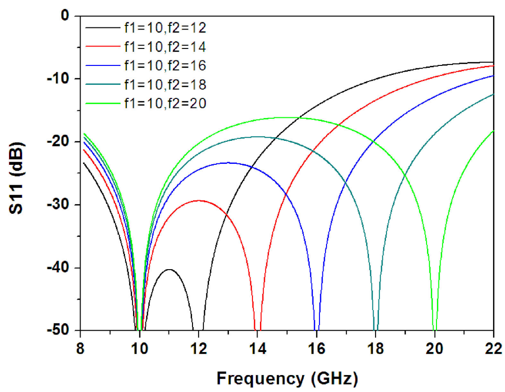

Fig. 4. Simulation results when $Z_L = 20$ Ohm and $Z_S = 50$ Ohm. f_1 is fixed to 10 GHz while f_2 varies from 12–20 GHz.

DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016

is 10 Ohm (Z_L = 10) and the system impedance is 50 Ohm (Z_S = 50). The center frequency is fixed to 10 GHz and the frequency band is extended from 0 to 6 GHz. To verify the graphical method, the two sections transmission line transformers are fabricated on AlN ceramic substrate with a dielectric constant of 9.9. The parameters are shown in Table III, microstrip line simulation and measured results are shown in Fig. 5. Single frequency transformer is a special case where the two circles are tangency, so TL_1 and TL_2 are two quarter wavelength lines. As f_1 and f_2 keep away from each other, S11 at center frequency is worse, but frequency band is extended. According to the different tolerance requirements, the bandwidth can be extended by tuning two frequency points.

Table III. Parameters of extending bandwidth in case-3

f_1	f_2	Z_L	Z_S	Z_1	Z_2	θ_1
GHz	GHz	Ohm	Ohm	Ohm	Ohm	°
10	10	10	50	15.81	35.36	90
9	11	10	50	15.96	35.02	81
8	12	10	50	16.44	33.96	72
7	13	10	50	17.38	31.96	63

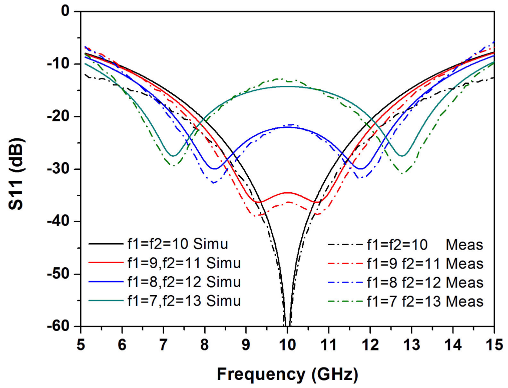

Fig. 5. Simulation and experimental results when Z_L = 10 Ohm and Z_S = 50 Ohm. The center frequency is fixed to 10 GHz. Bandwidth varies from 0–6 GHz.

4 Conclusion

A novel graphical method for dual-frequency transformer in two sections is proposed. The method not only describes the impedance transforming process in clear and intuitive way, but also solves the parameters by simple geometrical relationship. By simulations and experiments, the proposed graphical method is verified. It is believed that the method can be used widely in dual-frequency matching and bandwidth extension.

DOI: 10.1587/elex.13.20160509
Received May 18, 2016
Accepted May 25, 2016
Publicized June 9, 2016
Copyedited June 25, 2016