kQ-product formula for multiple-transmitter inductive power transfer system

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Abstract: This paper formulates the maximum efficiency of multiple-transmitter inductive power transfer system in terms of *system* kQ-product. We show that the cross-coupling among transmitters does not affect the maximum efficiency. More importantly, the square of *system* kQ-product is equal to the sum of squares of kQ-products of individual transmitter-receiver links. This result provides an intuitive insight into the characteristics of system efficiency.

Keywords: inductive power transfer, multiple transmitter, cross-coupling, kQ-product, maximum efficiency

Classification: Circuits and modules for electronic instrumentation

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1 Introduction

Maximum achievable efficiency is an important index to evaluate the performance of inductive power transfer (IPT) [1] systems. Maximum efficiency of single transmitter system has been investigated extensively in [2, 3, 4, 5]. Compared to one transmitter, multiple transmitters have more degree of freedom of the input current distribution and, therefore, provide the possibilities of efficiency enhancement as well as energy focusing toward target receiver. Maximum efficiency of multiple-transmitter system has been analyzed using linear programing or convex optimization [6, 7, 8]. However, these analyses give little physical insights into the system behavior, e.g., the relation between efficiency of the whole system and those of individual transmitter-receiver links. In this regard, these analyses might not be suitable for performance estimation of systems with many transmitters since they require full-size network's Z-matrix.

The purpose of this paper is to provide an intuitive analytical tool for estimating efficiency of multiple-transmitter IPT system at any scale. To this end, we formulate the maximum efficiency in terms of *system* kQ-product [4]. We observe that, regardless of the cross-coupling among transmitters [9], the square of *system* kQ-product is equal to the sum of squares of kQ-products of individual transmitter-receiver links. This observation suggests that maximum efficiency of IPT system, with any number of transmitters, can be estimated by measuring the kQ-products of individual transmitter-receiver links, using a simple two-port network analyzer. This observation also confirms the efficiency enhancing effect of multiple-transmitter system compared to the single-transmitter one [10].

2 Multiple-transmitter IPT system

We consider a single frequency IPT system with *N* transmitters and one receiver as illustrated in Fig. 1. The system is modeled by linear reciprocal (N + 1)-port network, where ports from 1 to *N* are the inputs, port 0 is the output. The system efficiency is defined as, the ratio of out power from port 0, to the total input power to ports from 1 to *N*. In this sense, the system efficiency is usually referred to as RF-to-RF efficiency. The efficiency depends on various parameters such as the input currents, the load impedance and network parameters. In order to derive the maximum efficiency, we assume that we are able to freely control the values of input currents I_n ($n \in \{1, ..., N\}$) and load impedance Z_0^L . In practice, these controls are typically carried out by current control circuits, matching networks, etc., which are inevitably lossy, and thus, cause some drops in overall efficiency.







Fig. 1. System model of general multiple-transmitter IPT.

However, as this paper focuses only on the RF-to-RF efficiency of the transceiving system, we hide all such components from Fig. 1 for the sake of simplicity.

Current-voltage relation of the system is expressed by Z-parameters as

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \\ V_0 \end{bmatrix} = \begin{bmatrix} R_{11} + jX_{11} & \dots & jX_{1N} & jX_{10} \\ \vdots & \ddots & \vdots & \vdots \\ jX_{N1} & \dots & R_{NN} + jX_{NN} & jX_{N0} \\ jX_{01} & \dots & jX_{0N} & R_{00} + jX_{00} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \\ I_0 \end{bmatrix}, \quad (1)$$

In Eq. (1) I_n , V_n are complex-valued current and voltage phasors of the input ports. I_0 and V_0 are respectively current and voltage phasors of the output. The coupling between input port n and the output is represented by the term jX_{n0} . The crosscoupling among transmitters is represented by the term jX_{nm} , where $m \in$ $\{1, \ldots, N\}$ and $m \neq n$. Load impedance at port 0 is determined by $Z_0^{L} = -V_0/I_0$.

Without loss of generality, we take I_0 as the reference phasor. From Eq. (1), the real part of V_0 is $V_0^{\text{re}} = -\sum_{n=1}^N X_{0n}I_n^{\text{im}} + R_{00}I_0$, where I_n^{re} and I_n^{im} are the real and imaginary parts of I_n , respectively. Real and imaginary parts of V_n are $V_n^{\text{re}} = -\sum_{m=1}^N X_{nm}I_m^{\text{im}} + R_{nn}I_n^{\text{re}}$, $V_n^{\text{im}} = \sum_{m=1}^N X_{nm}I_m^{\text{re}} + R_{nn}I_n^{\text{im}} + X_{n0}I_0$. Therefore, output power from port 0 is

$$P_{\text{out}} = -\frac{1}{2} V_0^{\text{re}} I_0 = -\frac{1}{2} \left(R_{00} I_0^2 - \sum_{n=1}^N X_{n0} I_0 I_n^{\text{im}} \right).$$
(2)

The input power to port *n* is $P_n = \frac{1}{2}(V_n^{\text{re}}I_n^{\text{re}} + V_n^{\text{im}}I_n^{\text{im}})$, thus the total output power $P_{\text{out}} = \sum_{n=1}^{N} P_n$ is determined by

$$P_{\rm in} = \frac{1}{2} \sum_{n=1}^{N} X_{n0} I_0 I_n^{\rm im} + \frac{1}{2} \sum_{n=1}^{N} R_{nn} [(I_n^{\rm re})^2 + (I_n^{\rm im})^2].$$
(3)

Finally, efficiency is formulated as

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{-R_{00}I_0^2 + \sum_{n=1}^N X_{n0}I_0I_n^{\text{im}}}{\sum_{n=1}^N X_{n0}I_0I_n^{\text{im}} + \sum_{n=1}^N R_{nn}[(I_n^{\text{re}})^2 + (I_n^{\text{im}})^2]}.$$
(4)



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Fig. 2. Phasor diagram of optimal input currents.

3 Maximum achievable efficiency

The maximum value of η is derived by applying the first-order necessary condition [11] on I_n^{re} and I_n^{im} , i.e., $\forall n \in \{1, ..., N\}$, we have

$$\frac{\partial \eta}{\partial I_n^{\rm re}} = \frac{(\partial P_{\rm out}/\partial I_n^{\rm re})P_{\rm in} - (\partial P_{\rm in}/\partial I_n^{\rm re})P_{\rm out}}{P_{\rm in}^2} = 0,$$
(5)

$$\frac{\partial \eta}{\partial I_n^{\rm im}} = \frac{(\partial P_{\rm out}/\partial I_n^{\rm im})P_{\rm in} - (\partial P_{\rm in}/\partial I_n^{\rm im})P_{\rm out}}{P_{\rm in}^2} = 0.$$
 (6)

Here, note that $P_{\text{out}}/P_{\text{in}} = \eta$, from Eq. (5) and (6), we have $I_n^{\text{re}} = 0$ and $I_n^{\text{im}} = (1 - \eta)X_{n0}I_0/(2\eta R_{nn})$. Substituting these equations into Eq. (4), we get a simple equation: $\chi \eta^2 - (4 + 2\chi)\eta + \chi = 0$, which yields one solution satisfying the condition $0 \le \eta < 1$. This solution is the maximum achievable efficiency

$$\eta_{\max} = 1 - \frac{2}{1 + \sqrt{1 + \chi}},\tag{7}$$

where χ is

$$\chi = \sum_{n=1}^{N} \frac{X_{n0}^2}{R_{00}R_{n0}}.$$
(8)

Optimal input currents are given by

$$I_n^{\rm re} = 0, \tag{9}$$

$$I_n^{\rm im} = \frac{1}{\sqrt{1+\chi}-1} \frac{X_{n0}}{R_{nn}} I_0,\tag{10}$$

meaning that the input currents must be, simultaneously, in-phase with each other and orthogonal to the output current I_0 . Phasor diagram of input currents at optimal point is illustrated in Fig. 2. Note that we do not restrict our analysis to a specific type of power source. The conditions given in Eq. (9), (10) are applied for all kinds of power supply.

Optimal load impedance is

$$Z_0^{\rm L} = -\frac{V_0}{I_0} = R_{00}\sqrt{1+\chi} - jX_{00},\tag{11}$$

meaning that the self-inductance of the receiver must be canceled out by a resonant capacitor and the load resistance must be $R_{00}\sqrt{1+\chi}$. Here, it is necessary that Eq. (9), (10) and (11) are simultaneously satisfied in order to achieve the maximum efficiency given in Eq. (7).







Fig. 3. Three-transmitter IPT systems.

4 kQ-product

It is well known that, for single-transmitter system, the maximum efficiency is dominated by the unloaded kQ-product [4]. Eq. (7) indicates that the maximum efficiency of the multiple-transmitter system has the same form with that of the single-transmitter case [3, 4]. Therefore, here we can define a *system* kQ-product α_{sys} for multiple-transmitter system, whose square is exactly equal to χ

$$\alpha_{\rm sys}^2 = \sum_{n=1}^N \frac{X_{n0}^2}{R_{00}R_{n0}} \,. \tag{12}$$

The system kQ-product does not contain the term X_{nm} , meaning that the crosscoupling does not affect the maximum efficiency. This can be understood that the optimization of input currents and load impedance—following Eq. (9), (10), (11)—has perfectly eliminated the impact of cross-coupling. Interestingly, $X_{n0}^2/(R_{00}R_{n0}) = \alpha_n^2$, where α_n is the kQ-product of the coupling between the *n*-th transmitter and the receiver. Therefore, regardless of the cross-coupling, relation between the system kQ-product and those of individual transmitter-receiver link is

$$\alpha_{\rm sys}^2 = \sum_{n=1}^N \alpha_n^2. \tag{13}$$

Here, it is convenient to view the term "kQ-product" following the extended definition based on Z-parameters presented in [4]. The term "kQ-product" should not be viewed as the multiplication of coupling coefficient and Q-factors of resonators as in conventional sense. This is because in the multiple-transmitter system, it is unclear whether each transmitter is resonating with the receiver.

5 Numerical analysis

Using moment method, we carry out numerical analysis of kQ-product for threetransmitter systems, placed in coaxial and coplanar arrangements as in Fig. 3. In this analysis, we consistently use single-turn circular coil with radius of 150 mm made from copper wire with diameter of 1 mm and with conductivity of 5.8×10^7 S/m. Distance between center of one transmitter to that of the other is 25 mm for the coaxial arrangement in Fig. 3(a) and 312 mm for the coplanar arrangement in Fig. 3(b). The relative positions among transmitters are unchanged while distance *d* from the group of transmitters to the receiver is varied. All transmitters simultaneously operate at the same frequency of 13.56 MHz.







- (a) Voltage source for port n
- (b) Optimal load for port 0
- Fig. 4. A realization by ideal voltage sources for optimal input currents and optimal load.



Fig. 5. Comparison between simulated and theoretical maximum efficiencies of systems in Fig. 3.

We use ideal voltage sources with 0Ω internal impedances to control the input currents. To determine the source structures, from Eq. (1), (9), (10), we derive the input impedance of port *n* at the optimal point as

$$Z_n^{\rm in} = R_{nn} \sqrt{1 + \alpha_{\rm sys}^2} + j \sum_{m=1}^N X_{nm} \frac{X_{m0} R_{nn}}{X_{n0} R_{mm}}.$$
 (14)

Therefore, as shown in Fig. 4(a), source for port *n* has a voltage V_n^S connected in series with a compensatory capacitance $-jX_n^S$. The values of V_n^S and X_n^S are

$$X_n^{\rm S} = {\rm Im}\{Z_n^{\rm in}\} = \sum_{m=1}^N X_{nm} \frac{X_{m0} R_{nn}}{X_{n0} R_{mm}},$$
(15)

$$V_n^{\rm S} = \operatorname{Re}\{Z_n^{\rm in}\}I_n = j \frac{\sqrt{1 + \alpha_{\rm sys}^2}}{\sqrt{1 + \alpha_{\rm sys}^2} - 1} X_{n0}I_0.$$
 (16)

Load impedance for the output port 0 is illustrated in Fig. 4(b).

Fig. 5 demonstrates the simulated and theoretical efficiencies of two arrangements in Fig. 3. In Fig. 5, "Simulation" stands for the efficiency achieved when input currents and load are optimized following equations (15), (16) and (11). "Theory" indicates the efficiency calculated by Eq. (7) with Z-parameters obtained via computer simulation. The very good agreement between the two values verifies the maximum efficiency formula in Eq. (7).

Fig. 6 shows the squares of kQ-products α_n^2 ($n \in \{1, 2, 3\}$) of individual transmitter-receiver links, their sum $\sum \alpha_n^2$ and the square of *system* kQ-product







Fig. 6. Relation between *system* kQ-product and those of individual couplings. The perfect matching between $\sum \alpha_n^2$ and α_{sys}^2 curves verifies Eq. (13).



Fig. 7. Efficiency comparison between three-transmitter system and individual transmitter-receiver links.

 α_{sys}^2 . The individual α_n^2 is calculated by applying kQ-product formula for singletransmitter system defined in [4]. In the calculation of α_n^2 , Z-parameters are measured for the network of port *n* and port 0 only, the other ports are opened. Value of α_{sys}^2 is determined by Eq. (12) using Z-parameters for the four-port network via computer simulation. The perfect agreement between $\sum \alpha_n^2$ and α_{sys}^2 curves confirms the correctiveness of Eq. (13).

Maximum efficiencies of the aforementioned three-transmitter systems and individual transmitter-receiver links are demonstrated in Fig. 7. The figure shows that efficiency of multiple-transmitter system is enhanced compared to the singletransmitter one.





6 Conclusion and discussion

This paper showed that the cross-coupling does not affect the maximum efficiency of multiple-transmitter IPT system. The maximum efficiency is characterized by a *system* kQ-product, whose square is equal to the sum of squares of all kQ-products of individual transmitter-receiver links. This result suggests that the system efficiency can be simply estimated by measuring the kQ-products of individual transmitter-receiver links. This result also confirms the efficiency enhancing effect of multiple-transmitter system compared to the single-transmitter one.

